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(*fl*. China, 1280–1303),

*mathematics*.

Chu Shih-chieh (literary name, Han-ch'ing; appellation, Sung-t'ing) lived in Yen-shan (near modern Peking). George Sarton describes him, along with Ch'in Chiu-shao, as "one of the greatest mathematicians of his race, of his time, and indeed of all times." However, except for the preface of his mathematical work, the *Ssu-yüan yü-chien* ("Precious Mirror of the Four Elements"), there is no record of his personal life. The preface says that for over twenty years he traveled extensively in China as a renowned mathematician; thereafter he also visited Kuang-ling, where pupils flocked to study under him. We can deduce from this that Chu Shih-chieh flourished as a mathematician and teacher of mathematics during the last two decades of the thirteenth century, a situation possible only after the reunification of China through the Mongol conquest of the Sung dynasty in 1279.

Chu Shih-chieh wrote the *Suan-hsüeh ch'i-meng* ("Introduction to Mathematical Studies") in 1299 and the *Ssu-yüan yü-chien* in 1303. The former was meant essentially as a textbook for beginners, and the latter contained the so-called "method of the four elements" invented by Chu. In the *Ssu-yüan yü-chien*, Chinese algebra reached its peak of development, but this work also marked the end of the golden age of Chinese mathematics, which began with the works of Liu I, Chia Hsien, and others in the eleventh and the twelfth centuries, and continued in the following century with the writings of Ch'in Chiu-shao, Li Chih, Yang Hui, and Chu Shih-chieh himself.

It appears that the *Suan-hsüeh ch'i-meng* was lost for some time in China. However, it and the works of Yang Hui were adopted as textbooks in Korea during the fifteenth century. An edition now preserved in Tokyo is believed to have been printed in 1433 in Korea, during the reign of King Sejo. In Japan a punctuated edition of the book (Chinese texts were then not punctuated) under the title *Sangaku keimo kunten*, appeared in 1658; and an edition annotated by Sanenori Hoshino, entitled *Sangaku keimo chūkai*, was printed in 1672. In 1690 there was an extensive commentary by Katahiro Takebe, entitled *Sangaku keimō genkai*, that ran to seven volumes. Several abridged versions of Takebe's commentary also appeared. The *Suan-hsüeh ch'i-meng* reappeared in China in the nineteenth century, when Lo Shih-lin discovered a 1660 Korean edition of the text in Peking. The book was reprinted in 1839 at Yangchow with a preface by Juan Yuan and a colophon by Lo Shih-lin. Other editions appeared in 1882 and in 1895. It was also included in the *ts'e-hai-shan-fang chung-hsisuan-hsüeh ts'ung-shu* collection. Wang Chien wrote a commentary entitled *Suan-hsüeh ch'i-meng shu i* in 1884 abd Hsu Feng-k'ao produced another, *Suan-hsüeh ch'i-meng t'ung-shih*, in 1887.

The *Ssu-yüan yü-chien* also disappeared from China for some time, probably during the later part of the eighteenth century. It was last quoted by Mei Kuch'eng in 1761, but it did not appear in the vast imperial library collection, the *Ssu-k'u ch'üan shu*, of 1772; and it was not found by Juan Yuan when he compiled the *Ch'ou-jen chuan* in 1799. In the early part of the nineteenth century, however, Juan Yuan found a copy of the text in Chekiang province and was instrumental in having the book made part of the *Ssu-k'u ch'üan-shu*. He sent a handwritten copy to Li Jui for editing, but Li Jui died before the task was completed. This handwritten copy was subsequently printed by Ho Yüan-shih. The rediscovery of the *Ssu-yüan yü-chien* attracted the attention of many Chinese mathematicians besides Li Jui, Hsü Yu-jen, Lo Shih-lin, and Tai Hsü. A preface to the *Ssu-yüan yü-chien* was written by Shen Ch'in-p'ei in 1829. In his work entitled *Ssu yüan yü-chien hsi ts'ao* (1834), Lo Shih-lin included the methods of solving the problems after making many changes. Shen Ch'in-p'ei also wrote a so-called *hsi ts'ao* ("detailed workings") for this text, but hsi work has not been printed and is not as well known as that by Lo Shih-lin. Ting Ch'ü-chung included Lo's *Ssu-yüan yü-chien hsi ts'ao* in his *Pai-fu-t'ang suan hsüeh ts'ung shu* (1876). According to Tu Shih-jan, Li Yen had a complete handwritten copy of Shen's version, which in many respects is far superior to Lo's.

Following the publication of Lo Shih-lin's *Ssu-yüan yü-chien hsi-ts'ao*, the "method of the four elements" began to receive much attention from Chinese mathematicians. I Chih-han wrote the *K'ai-fang shih-li* ("Illustrations of the Method of Root Extraction"), which has since been appended to Lo's work. Li Shan-lan wrote the *Ssu-yüan chieh* ("Explanation of the Four Elements") ans included it in his anthology of mathematical texts, the *Tse-ku-shih-chai suan-hsüeh*, first published in Peking in 1867. Wu Chia-shan wrote the *Ssu-yüan ming-shih shih-li* ("Examples Illustrating the Terms and Forms in the Four Elements Method"), the *Ssu-yüan ts'ao* ("Workings in the Four Elements Method"), and the *Ssu-yüan ch'ien-shih* ("Simplified Explanations of the Four Elements Method"), and incorporated them in his *Pai-fu-t'ang suan-hsüeh ch'u chi* (1862). In his *Hsüeh-suan pi-t'an* ("Jottings in the Study of Mathematics"), Hua Heng-fang also discussed the "method of the four elements" in great detail.

A French translation of the *Ssu-yüan yü-chien* was made by L. van Hée. Both George Sarton and Joseph Needham refer to an English translation of the text by Ch'en Tsai-hsin. Tu Shih-jan reported in 1966 that the manuscript of this work was still in the Institute of the History of the Natural Sciences, Academia Sinica, Peking.

In the *Ssu-yüan yü-chien* the "method of the celestial element" (*t'ien-yuan shu*) was extended for the first time to express four unknown quantities in the same algebraic equation. Thus used, the method became known as the "method of the four elements" (*su-yüan shu)*—these four elements were *t'ien* (heaven), *ti* (earth), *jen* (man), and *wu* (things or matter). An epilogue written by Tsu I says that the "method of the celestial element" was first mentioned in Chiang Chou's *I-ku-chi*, Li Wen-i's *Chao-tan*, Shih Hsin-tao's *Ch'ien-ching*, and Liu Yu-chieh's *Ju-chi shih-so*, and that a detailed explanation of the solutions was given by Yuan Hao-wen. Tsu I goes on to say that the "earth element" was first used by Li Te-tsai in his *Liang-i ch'un-ying chi-chen* while the "man element" was introduced by Liu Ta-chien (literary name, Liu Junfu), the author of the *Ch'ien-k'un kua-nang;* it was his friend Chu Shih-chieh, however, who invented the "method of the four elements." "Except for Chu Shih-chieh and Yüan Hao-wen, a close friend of Li Chih, wer know nothing else about Tsu I and all the mathematicians he lists. None of the books he mentions has survived. It is also significant that none of the three great Chinese mathematicians of of the thirteenth century—Ch'in Chiu-shao, Li Chih, and Yang Hui—is mentioned in Chu Shih-chieh's works. It is thought that the "method of the celestial element" was known in China before their time and that Li Chih's *I-ku yen-tuan* was a later but expanded version of Chiang Chou's *I-ku-chi*.

Tsu I also explains the "method of the four elements," as does Mo Jo in his preface to the *Ssu-yüan yü-chien*. Each of the "four elements" represents an unkown quantity—$u$, $v$, $w$, and $x$, respectively. Heaven ($u$) is placed below the constant, which is denoted by *t'ai*, so that the power of $u$ increases as it moves downward; earth ($v$) is placed to the left of the constant so that the power of $v$ increases as it moves toward the left; man ($w$) is placed to the right of the constant so that the power of $w$ increases as it moves toward the right; and matter ($x$) is placed above the constant so that the power of $x$ increases as it moves upward. For example, $u + v + w + x = 0$ is represented in Fig. 1.

Chu Shih-chieh could also represent the products of any two of these unknowns by using the space (on the countingboard) between them rather as it is used in Cartesian geometry. For example, the square of

$$(u + v + w + x) = 0,$$

i.e.,

$$u^2 + v^2 + w^2 + x^2 + 2ux + 2vw + 2ux + 2wx = 0,$$

can be represented as shown in Fig. 2 (below). Obviously, this was as far as Chu Shih-chieh could go, for he was limited by the two-dimensional space of the countingboard. The method cannot be used to represent more than four unknowns or the cross product of more than two unknowns.

Numerical equations of higher degree, even up to the power fourteen, are dealt with in the *Suan-hsüeh ch'i-meng* as well as the *Ssu-yüan yü-chien*. Sometimes a transformation method (*fan fa*) is employed. Although there is no description of this transformation method, Chu Shih-chieh could arrive at the transformation only after having used a method similar to that independently rediscovered in the early nineteenth century by Horner and Ruffini for the solution of cubic equations. Using his method of *fan fa*, Chu Shih-chieh changed the quartic equation.

$$x^4 - 1496x^2 - x + 558236 = 0$$

to the form

$$y^4 - 80y^3 + 904y^2 - 27841y - 119816 = 0.$$

Employing Horner's method in finding the first approximate figure, 20, for the root, one can derive the coefficients of the second equation as follows:

Eigher Chu Shih-chieh was not very particular about the signs for the coefficients shown in the above example, or there are printer's errors. This can be seen in another example, where the equation $x^2 - 17x - 3120 = 0$ became $y^2 + 103y + 540 = 0$ by the *fan fa* method. In other cases, however, all the signs in the second equations are correct. For example,

$$109x^2 - 2288x - 348432 = 0$$

gives rise to

$$109y^2 + 10792y - 93312 = 0$$

and

$9x^4 - 2736x^2 - 48x + 207936 = 0$

gives rise to

$9y^4 + 360y^3 + 2664y^2 - 18768y + 23856 = 0.$

Where the root of an equation was not a whole number, Chu Shih-chieh sometimes found the next approximation by using the coefficients obtained after applying Horner's method to find the root. For example, for the equation $x^2 + 252x - 5292 = 0$, the approximate value $x_1 = 19$ was obtained; and, by the method of *fan fa*, the equation $y^2 + 290y - 143 = 0$. Chu Shih-chieh then gave the root as $x = 19(143/1 + 290)$. In the case of the cubic equation $x^3 - 574 = 0$, the equation obtained by the *fan fa* method after finding the first approximate root, $x_1 = 8$, becomes $y^3 + 24y^2 + 192y - 62 = 0$. In this case the root is given as $x = 8(62/1 + 24 + 192) = 8\ 2/7$. The above was not the only method adopted by Chu Shih-chieh in cases where exact roots were not found. Sometimes he would find the next decimal place for the root by continuing the process of root extraction. For example, the answer $x = 19.2$ was obtained in this fashion in the case of the equation

$135x^2 + 4608x - 138240 = 0.$

For finding square roots, there are the following examples in the *Ssu-yüan yü-chien:*

Like Ch'in Chiu-shao, Chu Shih-chieh also employed a method of substitution to give the next approximate number. For example, in solving the equation $-8x^2 + 578x - 3419 = 0$, he let $x = y/8$. Through substitution, the equation became $-y^2 + 578y - 3419 \times 8 = 0$. Hence, $y = 526$ and $x = 526/8 = 65–3/4$. In another example, $24649x^2 - 1562500 = 0$, letting $x = y/157$, leads to $y^2 - 1562500 = 0$, from which $y = 1250$ and $x = 1250/157 = 7\ 151/157$. Sometimes there is a combination of two of the above-mentioned methods. For example, in the equation $63x^2 - 740x - 432000 = 0$, the root to the nearest whole number, 88, is found by using Horner's method. The equation $63y^2 + 10348y - 9248 = 0$ results when the *fan fa* method is applied. Then, using the substitution method, $y = z/63$ and the equation becomes $z^2 + 10348z - 582624 = 0$, giving $z = 56$ and $y = 56/63 = 7/8$. Hence, $x = 88\ 7/8$.

The *Ssu-yüan yü-chien* begins with a diagram showing the so-called Pascal triangle (shown in modern form in Fig. 3), in which

$(x + 1)^4 = x^4 + 4x^3 + 6x^2 + 4x + 1.$

Although the Pascal triangle was used by Yang Hui in the thirteenth century and by Chia Hsien in the twelfth, the diagram drawn by Chu Shih-chieh differs

from those of his predecessors by having parallel oblique lines drawn across the numbers. On top of the triangle are the words *pen chi* ("the absolute term"). Along the left side of the triangle are the values of the absolute terms for $(x + 1)^n$ from $n = 1$ to $n = 8$, while along the right side of the triangle are the values of the coefficient of the highest power of $x$. To the left, away from the top of the triangle, is the explanation that the numbers in the triangle should be used horizontally when $(x + 1)$ is to be raised to the power $n$. Opposite this is an explanation that the numbers inside the triangle give the *lien*, i.e., all coefficients of $x$ from $x^2$ to $x^{n-1}$. Below the triangle are the technical terms of all the coefficients in the polynomial. It is interesting that Chu Shih-chieh refers to this diagram as the *ku-fa* ("old method").

The interest of Chinese mathematicians in problems involving series and progressions is indicated in the earliest Chinese mathematical texts extant, the *Choupei suan-ching* (*ca*. fourth century b.c.) and Liu Hui's commentary on the *Chiu-chang suan-shu*. Although arithmetical and geometrical series were subsequently handled by a number of Chinese mathematicians, it was not until the time of Chu Shih-chieh that the study of higher series was raised to a more advanced level. In his *Ssu-yüan yü-chien* Chu Shih-chieh dealt with bundles of arrows of various cross sections, such as circular or square, and with piles of balls arranged so that they formed a triangle, a pyramid, a cone, and so on. Although no theoretical proofs are given, among the series found in the *Ssu-yüan yü-chien* are the following:

After Chu Shih-chieh, Chinese mathemathicians made almost no progress in the study of higher series. It was only after arrival of the Jesuits that interest in his work was revived. Wang Lai, for example, showed in his *Heng-chai suan hsüeh* that the first five series above can be represented in the generalized form

where $r$ is a positive integer.

Further contributions to the study of finite integral series were made during the nineteenth century by such Chines mathematicians as Tung Yu-ch'eng, Li Shan-lan, and Lo Shih-lin. They attempted to express Chu Shih-chieh's series in more generalized and modern forms. Tu Shih-jan has recently stated that the following relationship, often erroneously attributed to Chu Shih-chieh, can be traced only as far as the work of Li Shan-lan.

If , where $r$ and $p$ are positive integre, then

($a$)

with the examples

and

($b$)

where $q$ is any other positive integer.

Another significant contribution by Chu Shih-chieh is his study of the methods of *chao ch'a* ("finite differences"). Quadratic expression had been used by Chinese astronomers in the process of finding arbitrary constants in formulas for celestial motions. We know that his methods was used by Li Shun-feng when he computed the Lin Te calender in a.d. 665. It is believed that Liu Ch'uo invented the *chao ch'a* method when he made the Huang Chi calender in a.d. 604, for he established the earliest terms used to denote the differences in the expression

$S = U_1 + U_2 + U_3 \ldots + U_n$,

calling $\Delta = U_1$*shang ch'a* ("upper difference"),

$\Delta^2 = U_2 - U_1$*erh ch'a* ("second difference"),

$\Delta^3 = U_3 - (2\Delta^2 + \Delta)$ *san ch'a* ("third difference"),

$\Delta^4 = U_4 - [3(\Delta^3 + \Delta^2) + \Delta]$ *hsia ch'a* ("lower difference").

Chu-Shih-chieh illustrated how the method of finite differences could be applied in the last five problems on the subject in chapter 2 of *Ssu-yüan yü-chien:*

If the cube law is applied to [the rate of] recruiting soldiers, [it is found that on the first day] the *ch'u chao* [$\Delta$] is equal to the number given by a cube with a side of three feet and the *tz'u chao* [$U_2 - U_1$] is a cube with a side one foot longer, such that on each succeeding day the difference is given by an cube with a side one foot longer that that of the preceding day. Find the total recruitment after fifteen days.

Writing down $\Delta$, $\Delta^2$, $\Delta^3$, and $\Delta^4$ for the given number we have what is shown is Fig. 4 Employing the Conventions of Liu Ch'uo, Chu Shih-chieh gave *shang ch'a* ($\Delta$)= 27 *erh ch'a* ($\Delta^2$) = 37; *san ch'a* ($\Delta^3$) = 24;

and *hsia ch'a* ($\Delta^4$) = 6. He then proceeded to find the number of recruits on the *n*th day, as follows:

Take the number of day [$n$] as the *shang chi*. Subtracting unity from the *shang chi* [$n - 1$], one gets the last term of a *chiao ts'ao to* [a pile of balls of triangular cross section, or $S = 1 + 2 + 3 + \ldots + (n - 1)$]. The sum [of the series] is taken as the *erh chi*. Subtracting two from the *shang chi* [$n - 2$], one gets the last term of a *san chiao to* [a pile of balls of pyramidal cross section, or $S = 1 + 3 + 6 + \ldots + n(n - 1)/2$]. The sum [of this series] is taken as the *san chi*. Subtracting three from the *shang chi* [$n - 3$], one gets the last term of a *san chio lo i to* series

The sum [of this series] is taken as the *hsia chi*. By multiplying the differences *[ch'a]* by their respective sums *[chi]* and adding the four results, the total recruitment is obtained.

From the above we have:

*Shang chi* = $n$

Multiplying these by the shang *ch'a erh ch'a san ch'a*, and *hsia ch'a* respectively, and adding the four terms, we get

.

The following results are given in the same section of the *Ssu yüan yü-chien:*

The *chai ch'a* method was also employed by Chu's contemporary, the great Yuan astronomer, mathematician, and hydraulic engineer Kuo Shou-ching, for the summation of power progressions. After them the *chao ch'a* method was not taken up seriously again in China until the eighteenth century, when Mei Wen-ting fully expounded the theory. Known as *shōsa* in Japan, the study of finite differences also received considerable attention from Japanese mathematicians, such as Seki Takakazu (or Seki Kōwa) in the seventeenth century.

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