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(*fl. ca.* 190 b.c.)

mathematics.

The few facts known about Diocles' life are derived entirely from his one surviving work. *On Burning Mirrors* (Π∊ρί πνρίων). His date can be determined approximately from his acquaintance with the mathematician Zenodorus, who is known to have lived in the early second century b.c. This date accords well with the terminology and treatment of conic sections in Diocles' work, which shows little or no trace of influence by Apollonius' *Conics,* since it makes him an exact contemporary of Apollonius. Diocles mentions only mathematicians contemporary with or earlier than Archimedes (except for Zenodorus). He was living in Arcadia when Zenodorus visited him.

Until the recent discovery of the Arabic translation of *On Burning Mirrors,* it was lost except for three excerpts in the commentary by Eutocius on Archimedes' *Sphere and Cylinder.* Since Eutocius did not quote Diocles verbatim, but reformulated his proofs (for instance, introducing references to Apollonius' *Conies*), modern inferences about Diocles' date and place in the history of the theory of conics are misleading and usually wrong. The following account is based on the Arabic text.

The work consists of an introduction and sixteen propositions, of which numbers 6, 9, and 14 are spurious (probably interpolated in the Arabic transmission). The title *On Burning Mirrors* is somewhat misleading, as it applies only to the first five propositions. Numbers 7 and 8 deal with a problem in Archimedes' *Sphere and Cylinder,* and propositions 10- 16 with the problem of "doubling the cube." The book as a whole has no unity except that it deals with" higher geometry"; as is natural for a Hellenistic Greek work, much of it is concerned with conic sections.

Diocles starts from two problems, the first posed by one Pythion (otherwise unknown) to Conon of Samos: What mirror surface will reflect the sun's rays to the circumference of a circle? The second was posed by Zenodorus to Diodes: What mirror surface will reflect the sun's rays to a point? Diocles says that the second problem was solved by Dositheus (well-known as a correspondent of Archimedes). This implies that the focal property of the parabola was recognized by about the middle of the third century b.c. Diocles indicates, however, that he himself is the first to give a formal proof of the property. After an obscure but historically interesting discussion of the application of burning mirrors to sundial construction, he proves (prop. 1) the focal property of the parabola, and shows how Pythion's and Zenodorus' problems can be solved by suitable rotation of the parabola.

In propositions 2 and 3 Diocles shows that it is useless to construct a spherical burning mirror from an arc greater than 60°. and proves that all rays reflected from such an arc will pass through a section less than 1/24 the diameter of the mirror.

Propositions 4 and 5 are of great historical interest. They address the problem of constructing a parabolic mirror of given focal length. Diocles' solution is as follows (see Figure 1). If the given focal length is *AB*, complete the square *ABEF*, extend *AF* to *K* so that *FK* =*AF*, join *KE*. and produce it to meet *AB* produced in *R*. Take arbitrary points *D,G* on *AB*. draw *DH, GZ* parallel to *AF*, and produce them to meet *KE* in *L,M.* Then, with center *A* and radius *DL*, draw a circle to cut *DL*, in *N* and (on the other side in)φ. Similarly, with center *A* and radius *GM*, draw a circle to cut *GM* in Θ and Ψ Make *AX* = *AK*. Then points *K, N,Θ,B,Ψ,X* lie on a parabola with *A* as focus. This construction is equivalent to the construction of the parabola from focus and directrix– as is obvious if, like Diocles, we complete the square *ARSK* and drop onto *SR* the perpendiculars *LQ, NO, MC,ΘP.* For *NA=LD* by construction, and *NA=LQ=LD*,so *NA=NO*, or the distance of the point *N* from *A* (the focus) is equal to its vertical distance from the line *SR* (the directrix). Similarly for the other points *K, Θ*, and so on. It is noteworthy that Diocles proves not only this, but also that a curve so generated is indeed a parabola (according to the classical Greek definition, by the equivalent of the relationship $y^2=px$ The obvious inference is that Diocles himself discovered the focus-directrix property of the parabola.[1]

In propositions 7 and 8 Diocles discusses a problem arising out of Archimedes' *Sphere and Cylinder* II,4: to divide a sphere in a given ratio. The problem involves, in modern terms, a cubic equation, which Diocles solves by the intersection of a hyperbola and an ellipse. His solution was already known from Eutocius, who also gives solutions by Dionysodorus and (possibly) Archimedes that likewise employ the intersection of two conies.

The rest of the book is devoted to the problem of doubling the cube, to which much attention was paid by Greek mathematicians from the fifth century b.c. on. Like everyone else in antiquity. Diocles in fact solves the equivalent problem of finding two mean proportionals between two given magnitudes.[2] His first solution, employing the intersection of two parabolas, was already known (in an altered form) from Eutocius; but since Eutocius did not mention the author, in modern times it has been almost universally misattributed to Me-naechmus.

The second solution is both more interesting and more influential (see Figure 2). In a circle, with diameters *AB, GD* intersecting at right angles, there are marked off from *D* equal small arcs *DZ, ZH,HΘ* … and *DN, NS,SO*… on the other side of *D*. Drop onto *AB*the perpendiculars *ZK, HL,ΘM*.… and join*BN, US, BO*.… Mark the points *P, Q, R*… where *BN*cuts *ZK*.… Then it can be shown that.

That is, *KZ* and *KB* are two mean proportionals between *AK*and *KP*. Similarly for point *Q, LH* and *LB* are two mean proportionals between *AL* and *LQ*, and so on. The points *P,Q,R* … are joined in a smooth curve *DPQRB,* which can be used to find two mean proportionals between any two magnitudes, and to solve related problems, as Diocles demonstrates at length in propositions 11-16.

Despite his contributions to the theory of conics, there is no mention of Diocles in surviving Greek mathematical works until late antiquity. In the sixth century his work was used by Eutocius and. about the same time, by the unknown author of the "Bobbio Mathematical Fragment."[3] It seems likely that Diocles had a considerable indirect influence on medieval discussions of the parabolic burning mirror. There is only one known explicit reference to his work in Islamic literature.[4] It is, nevertheless, very probable that it is one of the sources of Ibn al-Haytham's *On Paraboloidal Burning Mirrors*,[5] which was well-known not only in the Islamic world but also in the West after its translation into Latin. Diocles was known by name in the West, however, only through the extracts in Eutocius, whose commentary on Archimedes attracted the attention of mathematicians of the late sixteenth and seventeenth centuries particularly for its discussion of curves used by the ancient geometers to solve the problem of doubling the cube. Among these was Diodes' curve (see Figure 2 above), part of the discussion on which had been excerpted by Eutocius. This curve was dubbed "cissoid" in the seventeenth century,[6] It was discussed by some of the most notable mathematicians of that time, including Fermat, Descartes, Roberval, Huygens, and Newton. To them we owe the generalization of the curve, the discovery of its infinite branch, and the revelation of many of its beautiful properties.

NOTES

1. The extension to all three conic sections is found in Pappus. VII, 312-3 IS. Hulisched., II, 1004-1014. It was probably made in the later Hellenistic period. The argument that it was known as early as Euclid cannot be sustained: see Toomer ed. of Diodes, 17.

2. The problem of doubling the cube had been reduced to finding two mean proportionals between two lines, one of which was double the other, by Hippocrates of Chios (late fifth century b.c.)

3. The author mentions a work, "On the Burning Mirror." which he attributes to Apollonius. For arguments that this is in fact Diodes' work, see Toomer ed. of Diocles, 20- 21.

4. In the encyclopedic work by al-Akfānī (fourteenth century). Sprenger ed.. 82. All other references known to me are derived from Entocius, whose commentary on Archimedes' *Sphere and Cylinder* was also translated into Arabic.

5. For arguments in favor of this see Toomer ed. of Diodes. 22.

6. Because it was identified with a class of curves known as κισσοειδής (“ivy-shaped”) from ancient sources. The identification is almost certainly wrong, as I have argued in my ed. of Diocles, 24.

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G. J. Toomer