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On connected transversals to nilpotent subgroups

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A and B are H-connected transversals in G.

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2. Let $G = AH = BH$ and $[A, B] \leq H$ and $G = \langle A, B \rangle$. Under which conditions on H does it follow that H is subnormal in G?
3. Let $G = AH = BH$ and $[A, B] \leq H$ and $G = \langle A, B \rangle$. Under which conditions on H does it follow that $G' \leq N_G(H)$.

Results for the abelian case

1. Kepka, Niemenmaa (1994): If G is a finite group and H is abelian, then G is solvable.

   Proof: *First assume that H is maximal in G and use Herstein (1958). If H is not maximal in G, then we proceed by induction.*

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2. If G is finite, H is abelian and $G = \langle A, B \rangle$, then H is subnormal in G.

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6. Csörgö (2007): $|G| = 2^{13}$, H is elementary abelian of order 2^6, $G = \langle A, B \rangle$ and $G' \leq N_G(H)$ does not hold.

Problems

What happens if $G = \langle A, B \rangle$ and

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Do we need some extra conditions (like $A = B$ or $A = A^{-1}$ and $B = B^{-1}$) to get $G' \leq N_G(H)$?

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2. Niemenmaa (2009): If G is finite, H is nilpotent and $G = \langle A, B \rangle$, then H is subnormal in G.