On a finite $2, 3$-generated group of period 12

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St Andrews

A group G has period n if $x^n = 1$ for all $x \in G$
(equivalently, if $\exp(G)$ is finite and divides n)

Groups of small period are of interest in light of the Burnside problem.

- All groups period $n = 1, 2, 3, 4, 6$ are locally finite.
- Groups of large period n need not be locally finite.

$n = 12$ is one of the smallest unknown cases.

Definition. A $2, 3$-*generated* group is a quotient of $\mathbb{Z}_2 * \mathbb{Z}_3$.
A $2, 3$-generated group of period 12 is called a $(2, 3;\ 12)$-*group*.

$B = B(2, 3; 12)$ is the free $(2, 3;\ 12)$-group. $\quad (B = \mathbb{Z}_2 *_{\mathfrak{B}_{12}} \mathbb{Z}_3)$

- It is unknown if B is finite.

We study the finite quotients of B $\quad (\sim$ finite $(2, 3;\ 12)$-groups $)$

- B has a largest finite quotient $B_0 = B_0(2, 3;\ 12)$
(a consequence of the restricted BP)

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Theorem. Let $B_0 = B_0(2, 3;\ 12)$. Then the structure of B_0 is known. In particular, the following facts hold:

- $|B_0| = 2^{66} \cdot 3^7 \approx 1.6 \cdot 10^{23}$.
- B_0 is solvable of derived length 4 and Fitting length 3; $Z(B_0) = 1$.
- The quotients of the derived series for B_0 are $\mathbb{Z}_6$, $\mathbb{Z}_{12}^2$, $\mathbb{Z}_2^{61}$, $\mathbb{Z}_3^4$.
- A Sylow 2-subgroup of B_0 has nilpotency class 5 and rank 7.
- A Sylow 3-subgroup of B_0 has nilpotency class 2 and rank 4.
- $\mathrm{O}_2(B_0)$ has order 2^{62} and nilpotency class 2.
- $\mathrm{O}_{2,3}(B_0)/\,\mathrm{O}_2(B_0) \cong \mathbb{Z}_3^6$.
- $B_0/\,\mathrm{O}_{2,3}(B_0) \cong \mathrm{SL}_2(3) \star \mathbb{Z}_4$; in particular, the 3-length of B_0 is 2.
- $\mathrm{O}_3(B_0) \cong \mathbb{Z}_3^4$.
- $\mathrm{O}_{3,2}(B_0)/\,\mathrm{O}_3(B_0)$ has order 2^{65} and nilpotency class 4.
- $B_0/\,\mathrm{O}_{3,2}(B_0) \cong 3^{1+2} : 2$; in particular, the 2-length of B_0 is 2.

B_0 is constructed explicitly in GAP and Magma

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B_0 is obtained as a homomorphic image of

$$G = \langle\, a, b \mid 1 = a^2 = b^3 = w_1^{12} = \ldots = w_{18}^{12} \,\rangle, \qquad (*)$$

where $w_1 = ab,\ w_2 = abab^2, \ldots$ are explicitly given words.

- The largest *known* finite quotient of G is B_0.
- The relators w_i^{12} are all *essential* (if any one is omitted, the resulting group will have finite homomorphic images of exponent 24).

The following questions concerning G are of interest:

- Is B_0 the largest finite quotient of G ?
- Does the least number of words w_i in $(*)$ that define a group with no finite quotients bigger than B_0 equal 18 ?
- Is G finite?

A positive answer to the last question would imply that $B = B_0$

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- Is B_0 the largest finite quotient of G?
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A positive answer to the last question would imply that $B = B_0$

B_0 is obtained as a homomorphic image of

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$F = \langle x, y \rangle$ is a free 2-generator group.

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- Take from F only words w expressible in $s = ab$ and $t = ab^2$
- Omit cyclically permuted words and inversions

$$L = \{s; st; s^2; s^3t, s^2t^2; s^4t, s^3t^2, (st)^2s; \ldots\}$$

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- Start with $G = \langle a, b \mid 1 = a^2 = b^3 = r^{12},\ \forall r \in R \rangle$, where $R = \varnothing$, and with a known quotient G_0 of G of period 12, say $G_0 = \mathbb{Z}_6$.
- Try "Solvable Quotient" to find a bigger quotient G_1 of G of order $2^e|G_0|$ successively for $e = 1, 2, \ldots$ until either $e \leqslant$ `max_e` or G_1 is found.
- If period of G_1 is 12 then replace $G_0 := G_1$ and start over.
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Input:
    l := 10; // Maximal length in the alphabet {s, t} of elements in L
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Multiplier:
    p:=2 or 3; e:=1; // Searching for a new quotient of G0 of order d0 * p^e
    max_e:=10; max_t:=100 h; max_m:=16 G; // Maximal exponent e, time, memory
Start:
    G := < a,b | 1 = a^2 = b^3 = w^12, w in R >;
Quotient:
    d1 := d0 * p^e; // New order of a quotient to search for
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    If time > max_t Or memory > max_m Then -> Output;
    If G1 = fail Then { e := e+1; If e > max_e Then -> Output;
                                            Else -> Quotient; }
    If period(G1) = 12 Then { G0 := G1; d0 := d1; e := 1; -> Quotient; }
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Search:
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- GAP: found G_0 of order $2^{24} \cdot 3^7$, but exceeded time for $2|G_0|$ and $3|G_0|$
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Maximality of G_0 of order $2^{66} \cdot 3^7$.

Suppose there is a bigger $(2,3;12)$-group E.

$$1 \to V \to E \to G_0 \to 1$$

- V is elementary abelian p-group with $p = 2, 3$.
- V is irreducible as $\mathbb{F}_p G_0$-module.

Restrictions on V:

$p = 3$: $x \in G_0,\ |x| = 3 \ \Rightarrow \ x^2 + x + 1 = 0$ on V (*quadratic* action)

$p = 2$: $x \in G_0,\ |x| = 4 \ \Rightarrow \ x^3 + x^2 + x + 1 = 0$ on V (*cubic* action)

Lemma. Let H be a periodic group of p-period p^l and let V be a p-elementary abelian invariant section of H viewed as an $\mathbb{F}_p H$-module. Then, for every $h \in H$ of order p^l, we have

$$1 + h + h^2 + \ldots + h^{|h|-1} = 0$$

on V.

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Lemma. Let H be a periodic group of p-period p^l and let V be a p-elementary abelian invariant section of H viewed as an $\mathbb{F}_p H$-module. Then, for every $h \in H$ of order p^l, we have

$$1 + h + h^2 + \ldots + h^{|h|-1} = 0$$

on V.

Maximality of G_0 of order $2^{66} \cdot 3^7$.

Suppose there is a bigger $(2, 3;\, 12)$-group E.

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p	2					3		
V	$V_1^{(2)}$	$V_2^{(2)}$	$V_3^{(2)}$	$V_4^{(2)}$	$V_5^{(2)}$	$V_1^{(3)}$	$V_2^{(3)}$	$V_3^{(3)}$
$\dim V$	1	2	2	4	6	1	1	4
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Let $B = B(2, 3; 12)$, $K = \mathbb{F}_3$, and $W = KB/(x^8 + x^4 + 1 \mid x \in B)$

- W is a free cyclic KB-module with quadratic action of elts. of order 3.

Lemma. $\dim W = 16$.

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Suppose V is fixed. Check, for every extension

$$1 \to V \to E \to G_0 \to 1,$$

- if E has period 12 ($V \leftthreetimes G_0$ automatically does)
- if E is $2,3$-generated

Extensions E are parameterized by elements of $H^2(G_0, V)$.

For small $H^2(G_0, V)$, one could use exhaustive search

For bigger $H^2(G_0, V)$, the action of $\mathrm{Comp}(G_0, V) \leqslant \mathrm{Aut}(G_0) \times \mathrm{Aut}(V)$

For big $H^2(G_0, V)$, the search can be linearized (ex. $|H^2(G_0, V)| = 2^{34}$)

Result:

- Most extensions E have exponent exceeding 12 ($\Rightarrow$ period $\neq 12$)
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Theorem. $G_0 = B_0$.

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For small $H^2(G_0, V)$, one could use exhaustive search

For bigger $H^2(G_0, V)$, the action of $\mathrm{Comp}(G_0, V) \leqslant \mathrm{Aut}(G_0) \times \mathrm{Aut}(V)$

For big $H^2(G_0, V)$, the search can be linearized (ex. $|H^2(G_0, V)| = 2^{34}$)

Result:

- Most extensions E have exponent exceeding 12 ($\Rightarrow$ period $\neq 12$)
- Those of period 12 are not $2,3$-generated

Theorem. $G_0 = B_0$.

ИМ СО РАН

Suppose V is fixed. Check, for every extension

$$1 \to V \to E \to G_0 \to 1,$$

- if E has period 12 ($V \leftthreetimes G_0$ automatically does)
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