

# On a finite 2, 3-generated group of period 12

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A group  $G$  has period  $n$  if  $x^n = 1$  for all  $x \in G$   
 (equivalently, if  $\exp(G)$  is finite and divides  $n$ )

Groups of small period are of interest in light of the Burnside problem.

- All groups period  $n = 1, 2, 3, 4, 6$  are locally finite.
- Groups of large period  $n$  need not be locally finite.

$n = 12$  is one of the smallest unknown cases.

Definition. A *2,3-generated* group is a quotient of  $\mathbb{Z}_2 * \mathbb{Z}_3$ .

A 2,3-generated group of period 12 is called a *(2,3;12)-group*.

$B = B(2,3;12)$  is the free (2,3;12)-group.  $( B = \mathbb{Z}_2 *_{\mathfrak{B}_{12}} \mathbb{Z}_3 )$

- It is unknown if  $B$  is finite.

We study the finite quotients of  $B$   $( \sim \text{finite } (2,3;12)\text{-groups} )$

- $B$  has a largest finite quotient  $B_0 = B_0(2,3;12)$   
 ( a consequence of the restricted BP )



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**Theorem.** Let  $B_0 = B_0(2, 3; 12)$ . Then the structure of  $B_0$  is known. In particular, the following facts hold:

- $|B_0| = 2^{66} \cdot 3^7 \approx 1.6 \cdot 10^{23}$ .
- $B_0$  is solvable of derived length 4 and Fitting length 3;  $Z(B_0) = 1$ .
- The quotients of the derived series for  $B_0$  are  $\mathbb{Z}_6, \mathbb{Z}_{12}^2, \mathbb{Z}_2^{61}, \mathbb{Z}_3^4$ .
- A Sylow 2-subgroup of  $B_0$  has nilpotency class 5 and rank 7.
- A Sylow 3-subgroup of  $B_0$  has nilpotency class 2 and rank 4.
- $O_2(B_0)$  has order  $2^{62}$  and nilpotency class 2.
- $O_{2,3}(B_0)/O_2(B_0) \cong \mathbb{Z}_3^6$ .
- $B_0/O_{2,3}(B_0) \cong \text{SL}_2(3) \star \mathbb{Z}_4$ ; in particular, the 3-length of  $B_0$  is 2.
- $O_3(B_0) \cong \mathbb{Z}_3^4$ .
- $O_{3,2}(B_0)/O_3(B_0)$  has order  $2^{65}$  and nilpotency class 4.
- $B_0/O_{3,2}(B_0) \cong 3^{1+2} : 2$ ; in particular, the 2-length of  $B_0$  is 2.

$B_0$  is constructed explicitly in GAP and Magma

We prove that this group is indeed  $B_0(2, 3; 12)$



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$B_0$  is obtained as a homomorphic image of

$$G = \langle a, b \mid 1 = a^2 = b^3 = w_1^{12} = \dots = w_{18}^{12} \rangle, \quad (*)$$

where  $w_1 = ab$ ,  $w_2 = abab^2, \dots$  are explicitly given words.

- The largest *known* finite quotient of  $G$  is  $B_0$ .
- The relators  $w_i^{12}$  are all *essential* (if any one is omitted, the resulting group will have finite homomorphic images of exponent 24).

The following questions concerning  $G$  are of interest:

- Is  $B_0$  the largest finite quotient of  $G$ ?
- Does the least number of words  $w_i$  in  $(*)$  that define a group with no finite quotients bigger than  $B_0$  equal 18?
- Is  $G$  finite?

A positive answer to the last question would imply that  $B = B_0$



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- Start with  $G = \langle a, b \mid 1 = a^2 = b^3 = r^{12}, \forall r \in R \rangle$ , where  $R = \emptyset$ , and with a known quotient  $G_0$  of  $G$  of period 12, say  $G_0 = \mathbb{Z}_6$ .
- Try “Solvable Quotient” to find a bigger quotient  $G_1$  of  $G$  of order  $2^e |G_0|$  successively for  $e = 1, 2, \dots$  until either  $e \leq \max\_e$  or  $G_1$  is found.
- If period of  $G_1$  is 12 then replace  $G_0 := G_1$  and start over.
- Otherwise, find  $w \in L$  such that  $w^{12} \neq 1$  in  $G_1$ .
- Add  $w$  to  $R$ , and start with a new  $G$ .

$G_0$  is still a quotient of the new  $G$ , but  $G_1$  no longer is

Replace  $2^e$  with  $3^e$ , then back to  $2^e$ , etc.

A more formalized version of the algorithm:

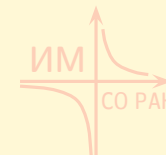


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Input:
  l := 10; // Maximal length in the alphabet {s, t} of elements in L
  d0 := 6; // Known order of a (2,3;12)-quotient G0 of G
  R:=[]; // Found relators
Multiplier:
  p:=2 or 3; e:=1; // Searching for a new quotient of G0 of order d0 * p^e
  max_e:=10; max_t:=100 h; max_m:=16 G; // Maximal exponent e, time, memory
Start:
  G := < a,b | 1 = a^2 = b^3 = w^12, w in R >;
Quotient:
  d1 := d0 * p^e; // New order of a quotient to search for
  G1 := SolvableQuotient(G, d1); // Invoking "Solvable Quotient" routine
  If time > max_t Or memory > max_m Then -> Output;
  If G1 = fail Then { e := e+1; If e > max_e Then -> Output;
                    Else -> Quotient; }
  If period(G1) = 12 Then { G0 := G1; d0 := d1; e := 1; -> Quotient; }
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Search:
  find w in L : w(a,b)^12 <> 1 in G1;
  If found Then { add w to R; e := 1; -> Start; }
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  return G0;

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- GAP: found  $G_0$  of order  $2^{24} \cdot 3^7$ , but exceeded time for  $2|G_0|$  and  $3|G_0|$
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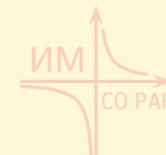


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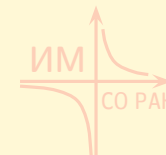


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Maximality of  $G_0$  of order  $2^{66} \cdot 3^7$ .

Suppose there is a bigger  $(2, 3; 12)$ -group  $E$ .

$$1 \rightarrow V \rightarrow E \rightarrow G_0 \rightarrow 1$$

- $V$  is elementary abelian  $p$ -group with  $p = 2, 3$ .
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Restrictions on  $V$ :

$p = 3$ :  $x \in G_0, |x| = 3 \Rightarrow x^2 + x + 1 = 0$  on  $V$  (*quadratic action*)

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$p = 2$ :  $O_2(G_0) \leq \text{Ker } \mathcal{X}$ , where  $\mathcal{X}$  is a representation corresp. to  $V$

$$|O_2(G_0)| = 2^{62}, \quad |G/O_2(G_0)| = 2^4 \cdot 3^7$$

$p$	2					3		
$V$	$V_1^{(2)}$	$V_2^{(2)}$	$V_3^{(2)}$	$V_4^{(2)}$	$V_5^{(2)}$	$V_1^{(3)}$	$V_2^{(3)}$	$V_3^{(3)}$
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Let  $B = B(2, 3; 12)$ ,  $K = \mathbb{F}_3$ , and  $W = KB/(x^8 + x^4 + 1 \mid x \in B)$

- $W$  is a free cyclic  $KB$ -module with quadratic action of elts. of order 3.

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An analog was proved by A. S. Mamontov without computer help.

- There are 3 irreducible quotients of  $W$ .



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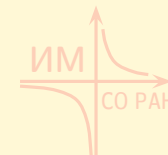
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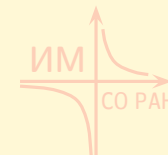
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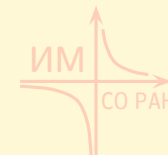
Let  $B = B(2, 3; 12)$ ,  $K = \mathbb{F}_3$ , and  $W = KB/(x^8 + x^4 + 1 \mid x \in B)$

- $W$  is a free cyclic  $KB$ -module with quadratic action of elts. of order 3.

Lemma.  $\dim W = 16$ .

An analog was proved by A. S. Mamontov without computer help.

- There are 3 irreducible quotients of  $W$ .





$p = 2$ :  $O_2(G_0) \leq \text{Ker } \mathcal{X}$ , where  $\mathcal{X}$  is a representation corresp. to  $V$   
 $|O_2(G_0)| = 2^{62}$ ,  $|G/O_2(G_0)| = 2^4 \cdot 3^7$

$p$	2					3		
$V$	$V_1^{(2)}$	$V_2^{(2)}$	$V_3^{(2)}$	$V_4^{(2)}$	$V_5^{(2)}$	$V_1^{(3)}$	$V_2^{(3)}$	$V_3^{(3)}$
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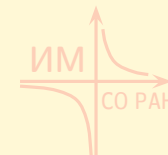
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Suppose  $V$  is fixed. Check, for every extension

$$1 \rightarrow V \rightarrow E \rightarrow G_0 \rightarrow 1,$$

- if  $E$  has period 12 (  $V \rtimes G_0$  automatically does )
- if  $E$  is 2, 3-generated

Extensions  $E$  are parameterized by elements of  $H^2(G_0, V)$ .

For small  $H^2(G_0, V)$ , one could use exhaustive search

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