OBITUARY

WILLIAM WERNER BOONE

William Werner Boone—Bill to all who knew him—died on 14 September 1983 at the age of 63. He was a regular and well-known visitor to the United Kingdom and became a member of the London Mathematical Society in 1979.

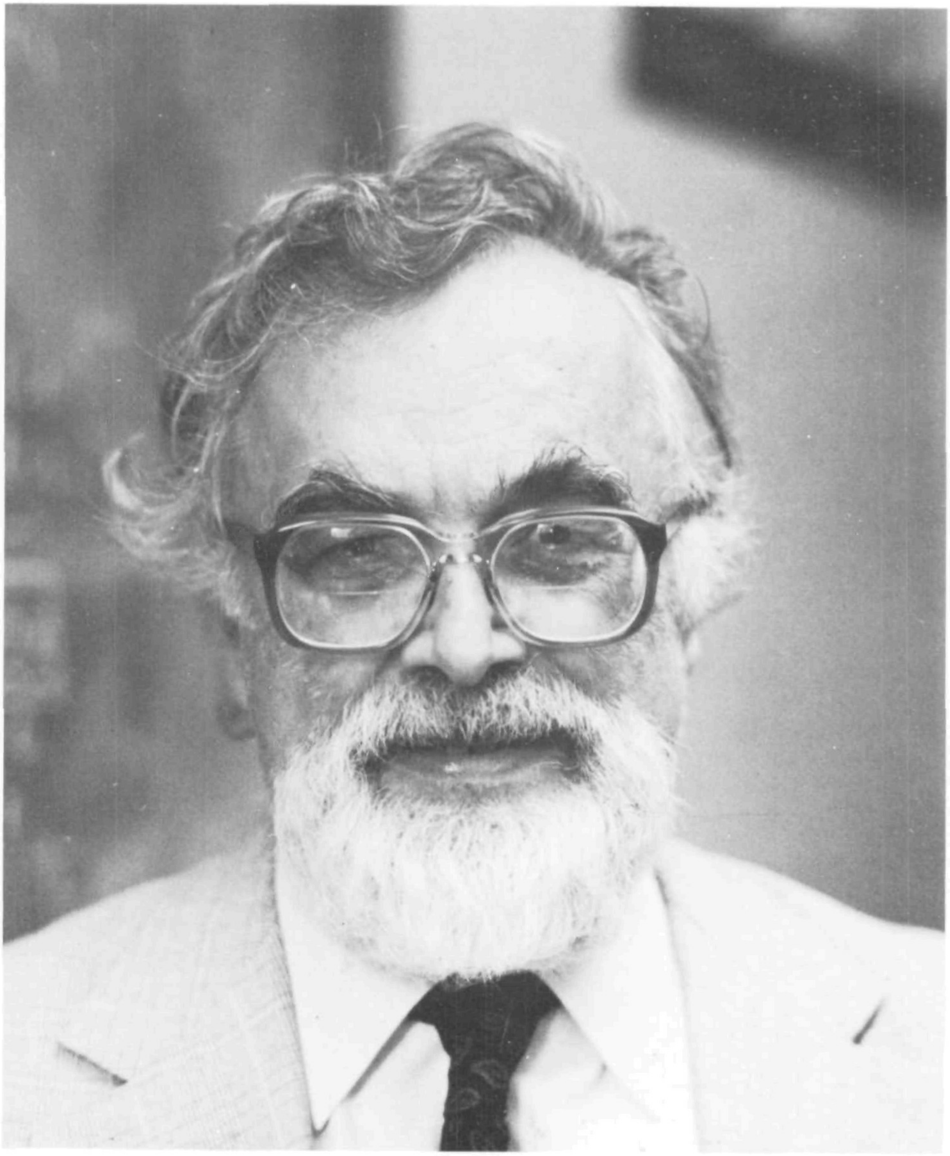
Bill was born on 16 January 1920 in Cincinnati, Ohio and was educated at the local Catholic grade and high schools. The family was not rich and financial pressure meant that Bill could not attend college directly upon leaving high school. Instead he trained as an accountant, supporting himself by working as a barman, and nursed dreams of becoming a writer. This was a serious ambition for a while and he attended classes in a writers' workshop and wrote a short story, set in London's Limehouse, called *Wang Fu Makes a Choice.* (Years later, while on a visit to Queen Mary College, he walked down to Limehouse to see how well it matched his imagined East End and was pleased to discover that there was at least a good Chinese restaurant there.)

This varied background left Bill with a wide range of experience and literary interests and he always retained a fascination with the dramatic aspects of life. One book he much admired was Thomas Wolfe's *Of Time and the River* whose hero, afflicted with a Faustian thirst for knowledge, moves from the rural South of the United States to Boston and then to England and France, and there is a superficial parallel to this in Bill's all-inquisitive progress from the Mid-West to sophisticated Princeton and subsequent regular visits to Europe. However, Bill never lost the directness and openness that one thinks of as characteristic of the provincial America of his youth, and his vigorous personality and broad Mid-West accent left an abiding impression on all who knew him.

Eventually Bill decided that mathematics was his first love and he studied part-time at the University of Cincinnati, graduating in 1945 with an A.B. degree. He was not called up for active military service but in the latter years of the war did do technical work for the armed services. (This period left him with the view, unusual in an academic, of the American Marine Sergeant as the ultimate personification of practical intelligence.) In 1945 Bill began graduate study at Princeton, working as an Instructor at Princeton and then at Rutgers. This was a happy time for Bill and in 1949 he married Eileen Herweh. With his literary background it was perhaps natural that Bill should gravitate towards mathematical logic. Then, as more than twenty-five years later when the writer was a graduate student at Princeton, the dominant figure there in logic was A. Church who lectured on the propositional and functional calculus with remorseless lucidity, and whose exhaustive thoroughness and precision Bill admired all his life. In 1936 Church **[R3]** had proved that there was no decision procedure for the first order functional calculus and the first such 'unsolvability' result not directly in foundations came in 1947 when E. L. Post **[R9]** in New York and A. A. Markov **[R6]** in Moscow each constructed a finitely presented semigroup with unsolvable word problem. A natural next step was to try to do the same for groups and Bill was bold enough, encouraged by Post, to tackle this. There was at the time considerable interest

Received 3 December, 1984.

Bull. London Math. Soc., 17 (1985), 168–174



WILLIAM WERNER BOONE 1920–1983

(*Facing p.* 168)

in the question and B. H. Neumann has related how, in the same week in Manchester, he and A. M. Turing announced contradictory solutions. Both had to withdraw though Turing did move towards a negative solution by giving [**R11**] a finitely presented cancellation semigroup with unsolvable word problem. This paper of Turing was to provide the crucial idea for the final step in Bill's construction of a finitely presented group with unsolvable word problem.

Bill obtained his doctorate from Princeton in April 1952. He had not managed to carry out his project to its completion and had to remain content at this stage with a weaker result—the construction of a finite group presentation such that there is no way to decide if an element lies in the subsemigroup generated by a fixed finite set. Already in 1950 Bill had taken a position as Assistant Professor at Catholic University, Washington and during 1952–54 prepared his thesis for publication as [**2**, I–IV].

Although by this time Bill was fully conversant with Turing's paper [**R11**], having reviewed it for the Journal of Symbolic Logic, and in his own words 'knew instantly', once he had properly understood Turing's work, that the idea of 'phase-change' contained there would enable him to obtain a group with unsolvable word problem, teaching duties meant that he was not able to turn his attention fully to the problem until the period 1954–56 which he spent at the Institute for Advanced Study in Princeton.

In later years Bill regretted a little that it had taken him so long to complete his proof, but it was not in his nature to rush things and whenever he had teaching or other commitments these would absorb much of his time and he would not be able to devote himself to the process of slowly becoming immersed in his research. Bill's strength as a mathematician was his ability to master, painstakingly and thoroughly, a mass of intricate combinatorial detail and this strength was severely sapped by routine distractions.

However, the two years at the Institute gave him the freedom and leisure he needed to bring his project to fruition. They also brought him something he prized above all else in his professional career, the friendship of Kurt Gödel. Bill's admiration for Gödel bordered on veneration and he was rewarded by the interest and encouragement Gödel gave him. Indeed Gödel, on a friendly basis, actually refereed the long paper [**6**] and Bill was fascinated by Gödel's mastery of it.

Bill's proof of the unsolvability of the word problem for groups finally appeared in 1957 [**2**, V, VI] having previously been incorporated in a December 1956 report to the Fulbright Commission.

In the meantime, though, P. S. Novikov [**R8**] had also published a proof. Since Novikov's article appeared in 1955, it is clear that in the end Bill was not first past this particular post. Though he certainly possessed the competitive urge that spurs on many mathematicians, this did not in itself disturb Bill unduly. What did disturb him more was an announcement by Novikov in 1952 of the unsolvability of the word problem for groups. For when Bill tried to compare the proof in [**R8**] with the announcement [**R7**] he was unable to understand how the two were connected and when he later actually met Novikov he understood him to say that one of his (i.e. Novikov's) students had actually solved the word problem for the group given in [**R7**]. Their conversation was in German which Bill spoke, but not at that time fluently, and it is conceivable that they misunderstood each other. But Bill would never really accept this possibility and, though he liked Novikov and bore him no personal grudge, Bill never entirely lost the feeling that this announcement, appearing so much earlier than his work, had diminished the value of what he had done.

In 1956 Bill began the first and longest of his many mathematical trips to Europe. Oslo, in 1956–57, visiting T. Skolem, was followed in 1957–58 by Münster, Manchester and Oxford. This was the most carefree time in Bill's career—though he was, by temperament, a man never entirely free from worries—for he had a major result behind him and time to think. Also the variety and diversity of Europe and its languages and cultures made a deep impression on him and, at a less elevated level, he acquired a taste for German beer and for the English habit of ending the working day by discussing mathematics in a nearby pub. He was proud, too, of the fact that he learnt some Norwegian and was extraordinarily pleased when someone described his occupation as 'peripatetic scholar'. Bill was gregarious in the extreme and made many friends among European mathematicians, notably G. Hasenjaeger in Germany and J. L. Britton, G. Higman and B. H. Neumann in England, to name only a few. He also lectured in many universities and a characteristic 'Boone story' was recalled, twenty-seven years later, by one who had heard him lecture in Dundee in 1957. In his lecture Bill needed a diagram so large and complicated that it required the whole blackboard including the part not normally visible because of the enormous workbench across the front of the room. Bill was not one to let such matters stand in his way and, so the story goes, the entire janitorial force of the then Queen's College, Dundee was mustered to remove the offending object.

His main mathematical preoccupation during this period was the refinement and improvement of his unsolvability proof and this finally appeared in 1959 in the long paper [6]. He also, in Manchester, wrote a paper [7] with G. Baumslag and B. H. Neumann applying the unsolvability of the word problem to show that many other decision problems in groups were unsolvable. Baumslag has recalled how Bill, who had laboured for almost a decade to prove and polish one theorem, felt there was something almost indecent about getting a new result every day of the week.

In the autumn of 1958 Bill was appointed Associate Professor, and in 1960 Professor, at the University of Illinois, Urbana and was based there for the rest of his life. He devoted much effort to building a considerable school in logic and group theory and at one time or another such distinguished mathematicians as K. I. Appel, C. G. Jockusch, J. J. Rotman, P. E. Schupp and G. Takeuti figured on his NSF contract. Over the years he had a number of research students, including W. E. Singletary, A. Ihrig Yasuhara, C. F. Miller III, G. S. Sacerdote and the writer.

The early 1960s saw more substantial results from Bill when he showed in [**10**] and [**11**] that arbitrary degrees of unsolvability could be represented by word problems for semigroups and groups. The latter followed on from work of J. L. Britton who, during a visit to Urbana in 1961, had succeeded by taking a properly group-theoretical approach in decisively shortening and simplifying Bill's argument in [**6**]. These two papers were completed during a two-year visit in 1964–66 to the Institute in Princeton. Some idea of Bill's thoroughness and of the lavish post-Sputnik funding of mathematics in America can be gleaned from the fact that, through his NSF contract, he not only paid the writer, then a graduate student, to help him check the printer's proofs of [**10**] and [**11**] by reading aloud to one another, but he paid two graduate students in Urbana to do exactly the same thing. Two more substantial papers, [**13**] and [**14**], emerged from these Institute years, the second a joint paper with W. Haken and V. Poenaru on decision problems in topology. Bill was candid enough to admit that he was not master of all the topological delicacies.

The return to teaching and other duties, and a growing family, were henceforth to make large demands on Bill's time and energy and, with the notable exception of his joint paper [**19**] with G. Higman giving a kind of algebraic characterisation of groups with solvable word problem, his further research was more low-key. In the 1970s, partly in collaboration with Higman and a research student V. Moore, he devoted much time to trying to develop a theory of recursive functions acting on groups but was never wholly satisfied with the outcome. However, he certainly never lost his appetite for thinking about mathematics and I recall vividly the flash of inspiration that led to the joint paper [**15**].

In his later years Bill was also an active and tireless promoter of the field of word and decision problems. The most tangible manifestations of this were the two Word Problems conferences he organised, in Irvine, California in 1969 and in Oxford in 1976, the latter a marathon five-week workshop—for Bill was never one to do things by halves—during the hottest summer England had seen for many years. The corresponding proceedings [**22**] and [**23**] were highly influential and helped to ensure that word problems in one form or another will continue to be an area of great interest and activity.

Further sabbaticals were spent in Oxford in 1972–73 and 1978–79, the Boone–Higman theorem emerging from the first of these following a discussion in the *Lamb and Flag*, next to the Mathematical Institute, after a seminar. A minor pleasure also associated with his visit in 1972–73 was that his two sons were taught mathematics at Magdalen College School by F. Garside who had solved the conjugacy problem for the braid groups. Bill loved Oxford, and was very happy there; so it was fitting that Higman, at the meeting in his honour on the occasion of his retirement, should dedicate the lecture he gave to Bill's memory.

As a young man Bill was thin and fit but he put on weight in his middle years and perhaps this in combination with his intense nature caused him to have a major heart attack in the spring of 1971. However, he recovered well and enjoyed good health throughout the 1970s. In December 1981 he became seriously ill with cancer of the pancreas but, after surgery, recovered and came to Oxford in the summer of 1982, looking well and sporting a white beard. By the spring of 1983, though, something was wrong again but eventually after further surgery he was deemed to be cured. Still in hospital, he was making good progress when he died suddenly of a heart attack.

He is survived by his wife Eileen, and two sons William, a graduate student in seismology at the University of Wisconsin, and Theodore, a student at Columbia Law School.

Mathematical work

An indication of Bill's main mathematical work has been given above but some more detailed comments should be made. Without doubt his outstanding achievement was his construction of a finite group presentation with unsolvable word problem, completed in [**2**, VI] and then refined in [**6**]. The following is an outline of simplified version of [**6**].

Bill's approach was to try to transform an unsolvable problem in a semigroup into the unsolvability of the word problem in a group. For this he began with a semigroup presentation T, with a distinguished generator q, such that there was no way to decide of an arbitrary word W whether $W =_T q$. The first stage of the construction transforms T into a group presentation H in which each relation $A_i = B_i$, $1 \leqslant i \leqslant n$, of T is replaced by a relation $l_i A_i r_i = B_i$, each pair of new symbols l_i and r_i being thought of as a pair of operators. By adding further symbols and relations that carefully control the movement of the operator symbols, one obtains the conclusion that for any (semigroup) word W of T,

(A) $$W \underset{T}{=} q \Leftrightarrow W \underset{H}{=} LqR, \text{ for some 'operator words' } L \text{ and } R.$$

From (A) follows the main result of Bill's thesis: for one cannot decide of an element of H whether it lies in the subsemigroup generated by q, together with the operator symbols and their inverses. Although there were some precedents to go by, notably the work of A. A. Mal'cev **[R5]** on embedding semigroups in groups, Bill was breaking new ground here and the skill with which he chose relations so that the introduction of inverse symbols did not cause too much interference make this intermediate result already a remarkable achievement.

The second stage is to transform (A) into a condition not involving operator symbols. Bill was stuck for some time at this point but finally reading Turing's paper **[R11]** gave him the key.

The trick is to introduce two 'separation symbols' t and k that commute with the appropriate operators and also satisfy the 'phase-change' relation $k(q^{-1}tq) = (q^{-1}tq)k$. Then (A) yields

(B) $$W \underset{T}{=} q \Leftrightarrow k(W^{-1}tW) \underset{H}{=} (W^{-1}tW)k$$

and the unsolvability of the word problem follows.

We know now how to give relatively compact proofs of (A) and (B) using the normal form theorem for HNN extensions. For what Bill had done, albeit unwittingly, was to construct a sequence of presentations each of which was, essentially, an HNN extension of its predecessor. However, like almost all those interested in decision problems at the time, Bill had been trained as a logician and thought of presentations as axiom systems for producing 'proofs' of equality by successive substitutions. Consequently his arguments all involve complicated inductions on the elimination of unwanted inverse symbols in 'proofs' and it was a considerable *tour de force* to push all these arguments through.

Almost all his remaining papers are applications or elaborations of the work in **[2]** and **[6]**. The two most notable are the pair **[10]** and **[11]** on degrees of unsolvability and word problems, **[10]** for semigroups and **[11]** for groups. In **[10]** the task is to translate the halting problem for a Turing machine into the word problem for a semigroup in such a way that unsolvability enters only for words derived from initial configurations of the Turing machine. The way in which relations are introduced to ensure that words derived from 'garbage configurations' of the machine do not cause problems is especially impressive. Interestingly, in this paper too, a significant idea is based on one of Turing in **[R11]**. In the second **[11]** of the two there was, for once, no need to invent a new presentation. Instead what was required was a thorough analysis of the presentation in **[6]** to show that the only way unsolvability entered was exactly that specifically built in. Also **[11]** is interesting in that the argument is almost entirely group-theoretical in character, revealing that Bill had fully grasped the effectiveness of the methods introduced by Britton in **[R2]**.

The papers **[13]** and **[14]** form a pair as well. In **[13]** the work of S. I. Adjan **[R1]** and M. O. Rabin **[R10]** is refined to show, for example, that degrees of unsolvability can be represented by the problem of recognising when a group is trivial, or of recognising when two groups are isomorphic. In contrast to **[10]** and **[11]** where the passage from simple unsolvability to a degree result requires much hard work, here all that is needed is a clear understanding of, say, the argument of [**R10**] and a couple of simple tricks. With the help of his two topologist coauthors, these results are translated into analogous results on decision problems in topology, the transition of course being via the fundamental group.

Latterly Bill's best-known work was his paper [**19**] with Higman. While much of the depth of the result arises from the use of Higman's embedding theorem [**R4**], the unexpectedness of the connection between embeddability in a simple group and having a solvable word problem was arresting and the paper has continued to attract attention. The long unpublished manuscript [**21**] contains the results of his efforts to develop recursive function theory over a group.

We omit comment on the remaining publications which are primarily in the nature of occasional pieces, or are just announcements of results.

Bibliography of W. W. Boone

1. 'Several simple unsolvable problems related to the word problem', Dissertation, Princeton, N.J., 1952.

2. 'Certain simple, unsolvable problems of group theory' I–VI, Indag. Math. 16 (1954) 231–237; 16 (1954) 492–497; 17 (1955) 252–256; 17 (1955) 571–577; 19 (1957) 22–27; 19 (1957) 227–232.

3. 'The equivalence of the word problem and Magnus' extended word problem', *Bull. Amer. Math. Soc.* 63 (1956) 148.

4. 'Analysis of Turing's "The word problem in semigroups with cancellation"', *Ann. of Math.* 67 (1958) 195–202.

5. 'The word problem', *Proc. Nat. Acad. Sci. U.S.A.* 44 (1958) 1061–1065.

6. 'The word problem', *Ann. of Math.* 70 (1959) 207–265.

7. (With G. Baumslag and B. H. Neumann) 'Some unsolvable problems about elements and subgroups of groups', *Math. Scand.* 7 (1959) 191–201.

8. 'Partial results regarding word problems and recursively enumerable degrees of unsolvability', *Bull. Amer. Math. Soc.* 68 (1962) 616–623.

9. 'Finitely presented group whose word problem has the same degree as that of an arbitrarily given Thue system (an application of methods of Britton)', *Proc. Nat. Acad. Sci. U.S.A.* 53 (1965) 265–269.

10. 'Word problems and recursively enumerable degrees of unsolvability. A first paper on Thue systems', *Ann. of Math.* 83 (1966) 520–571.

11. 'Word problems and recursively enumerable degrees of unsolvability. A sequel on finitely presented groups', *Ann. of Math.* 84 (1966) 49–84.

12. (With H. Rogers, Jr.) 'On a problem of J. H. C. Whitehead and a problem of Alonzo Church', *Math. Scand.* 19 (1966) 185–192.

13. 'Decision problems about algebraic and logical systems as a whole and recursively enumerable degrees of unsolvability', *Math. Logic* (*Colloquium, Hannover 1966*) (North-Holland, Amsterdam, 1968) pp. 13–33.

14. (With W. Haken, V. Poenaru) 'On recursively unsolvable problems in topology and their classification', *Math. Logic* (*Colloquium, Hannover 1966*) (North-Holland, Amsterdam, 1968) pp. 37–74.

15. (With D. J. Collins and Ju. V. Matijasevič) 'Embeddings into semigroups with only a few defining relations', *Proceedings of the Second Scandinavian Logic Symposium* (*Oslo 1970*) (Studies in Logic and the Foundations of Mathematics 63, North-Holland, Amsterdam, 1971) pp. 27–40.

16. 'Between logic and group theory', *Proceedings of the Second International Conference on the Theory of Groups* (*Canberra 1974*) (Lecture Notes in Math. 372, Springer, Berlin, 1974) pp. 90–102.

17. 'Word problems and recursively enumerable degrees of unsolvability. An emendation', *Ann. of Math.* 94 (1971) 389–391.

18. (With D. J. Collins) 'Embeddings into groups with only a few defining relations', *J. Austral. Math. Soc.* 18 (1974) 1–7.

19. (With G. Higman) 'An algebraic characterisation of groups with solvable word problem', *J. Austral. Math. Soc.* 18 (1974) 41–53.

20. (With G. Higman) 'An algebraic classification of groups with soluble order problem', *Logic Colloquium '73* (*Bristol 1973*) (Studies in Logic and the Foundations of Mathematics 80, North-Holland, Amsterdam, 1975) pp. 53–54.

21. (With G. Higman and V. Moore) 'Recursive function theory on an abstract group', Manuscript, 1982.

22. (Editor, with F. B. Cannonito and R. C. Lyndon) *Word problems: Decision problems and the Burnside*

problem in group theory (Studies in Logic and the Foundations of Mathematics 71, North-Holland, Amsterdam, 1973).
23. (Editor, with S. I. Adjan and G. Higman) *Word problems II: the Oxford book* (Studies in Logic and the Foundations of Mathematics 95, North-Holland, Amsterdam, 1980).

References

R1. S. I. Adjan, 'Algorithmic unsolvability of problems of recognition of certain properties of groups', *Dokl. Akad. Nauk SSSR* 103 (1955) 533–535 (Russian).
R2. J. L. Britton, 'The word problem', *Ann. of Math.* 77 (1963) 16–32.
R3. A. Church, 'A note on the Entscheidungsproblem', *J. Symb. Logic* 1 (1936) 40–41.
R4. G. Higman, 'Subgroups of finitely presented groups', *Proc. Royal Soc. London, Series A* 262 (1961) 455–475.
R5. A. I. Mal'cev, 'Über die Einbettung von assoziativen Systemen in Gruppen', *Mat. Sbornik* 6 (1939) 331–336; 8 (1940) 251–264 (Russian).
R6. A. A. Markov, 'The impossibility of certain algorithms in the theory of associative systems' I, *Dokl. Akad. Nauk SSSR* 55 (1947) 583–586 (Russian).
R7. P. S. Novikov, 'On the algorithmic unsolvability of the problem of identity', *Dokl. Akad. Nauk SSSR* 85 (1952) 709–712 (Russian).
R8. P. S. Novikov, 'On the algorithmic unsolvability of the word problem in group theory', *Trudy Mat. Inst. Steklow* 44 (1955) 1–143 (Russian).
R9. E. L. Post, 'Recursive unsolvability of a problem of Thue', *J. Symb. Logic* 12 (1947) 1–11.
R10. M. O. Rabin, 'Recursive unsolvability of group theoretic problems', *Ann. of Math.* 67 (1958) 172–194.
R11. A. M. Turing, 'The word problem in semigroups with cancellation', *Ann. of Math.* 52 (1950) 491–505.

School of Mathematical Sciences
Queen Mary College
Mile End Road
London E1 4NS

D. J. Collins