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E. F. BAXTER, S. HOLGATE AND A. ERDÉLYI†

Joseph Burchnall was born in Whichford, Warwickshire in 1892. He gained an Open Exhibition to Christ Church from Boston Grammar School in 1911, and went to Oxford to read Mathematics in the same year. He was elected to the University Junior Mathematical Exhibition in 1913 and graduated the following year after a distinguished undergraduate career, that turned out in the event to be but the prelude to an even more distinguished one.

He joined the Army immediately in 1914, and served in France and Belgium throughout the European War, holding a commission in the Royal Artillery for four years. He was awarded the Military Cross after the heavy German offensive of March 1918, during which he was badly wounded while directing artillery operations and lost a leg as a result.

Probably the most remarkable thing about him was the mastery he showed over this handicap—in almost forty years of service to the University he was never heard to complain of its consequences, or even to comment on its existence. Indeed he gave the impression, both to his students and to his colleagues, that this was one of the co-ordinates that he chose to ignore. To see him cutting his lawn with hand shears was a revelation of his determined character. To witness his behaviour at a stormy meeting in the thirties was an unforgettable experience—others had spoken while seated amidst continuous interruption—Burchnall pushed back his chair with a clatter, rose on his one good leg, and was heard in silence.

He came to Durham in 1919 as Lecturer in Mathematics, was subsequently promoted to Reader, and succeeded to the Chair in 1939. His earlier publications, some of them jointly with T. W. Chaundy, were on differential equations, but his interests later crystallized in hypergeometric functions and Bessel functions, which he explained patiently and eruditely to his honours students. He wrote and published papers regularly on these subjects in various journals, and in 1953 was elected to a Fellowship of the Royal Society of Edinburgh. His teaching and his writing displayed equal clarity, and his interest in his subject was manifest and infectious, continually stimulating his students.

Much of his contribution to the University, however, was outside the lecture room. He is reported to have shown his eye for discerning and revealing the truth at his interview in 1919, and his outstanding capacity for academic and financial administration was soon recognised by his colleagues after he joined the staff—by 1925 he was Secretary to the Durham Colleges' Council, a part-time post held jointly with his lectureship. This was an influential position for anyone interested in University politics and administration, as he was; and he enjoyed it for some thirteen years, which was fortunate for Durham because the early thirties brought serious difficulties in the federal University, culminating in the Royal Commission of 1934. Most of the work with the Commission on Durham's behalf fell on his shoulders, and characteristically he did not shrink from it. He played a large part in the discussions on

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reconstitution, and although the Durham Council's recommendations were not all implemented, it was due to Burchnall and a small group of others that the Durham Colleges emerged from this chastening period with so much of their character intact. The Secretaryship became a full-time post in 1938, and he handed over its responsibilities, but the difficult foundation work had been done—there was a base from which the present healthy structure has been able to develop.

After he became Professor of Mathematics in 1939 he continued to be one of the leading statesmen, serving continuously on Court, Senate, Council and Academic Board, and on nearly all the important committees; and contributing, sometimes a little tersely, to all the major discussions. He never shirked an issue—he had little time for the devious, and less for those who would engineer it—but he saw further than most of his colleagues, and recognised that many problems would only be solved by what he called " the efflux of time ". His wisdom and counsel were invaluable and, punctuated by a sense of humour that was both dry and delightful, his personality imprinted itself on the immediate post-war era, when so much of what he had striven for began to materialise.

After the war, during which he had almost inevitably been in command of a section of the Home Guard, he saw the Department move from one room on Palace Green to quarters of almost unbelievable splendour in the West Building (typically seizing the opportunity to quip about the architecture at the opening ceremony) and found his faith in the future of Durham, and especially of its science, begin to reap its reward. He took the view, not universally held, that a Professor should administer his department in detail to the advantage of his colleagues and his students—he taught with the same care and lucidity that he had always shown—he was still held in affectionate awe by his students, and he cared for them as much as ever, but now he was able to let his concern for their welfare be seen publicly—and the memory of his generous hospitality, for example at the dinner after Examiners' meetings, is still cherished. No-one ever stayed away from his lectures—an unspoken tribute, not only to the quality of his teaching but also to his fundamental kindness, that must surely have given him considerable pleasure. When his services to education and the community were recognised by the award of the O.B.E. in 1956, his former students were as delighted as his countless friends, such was their esteem for him.

He had many outside interests—for example, he was a founder member of Durham Rotary Club, a Freemason of long standing, and Chairman of Barnard Castle School Governors. The list is long and varied; but he still found time, in lighter mood, to be President of the Staff Cricket Club, and no figurehead either—that would have been out of character. Indeed his behaviour in that office was a cameo of his personality—he stood alertly as umpire for hours in spite of his disability—he granted appeals against the University Secretary and the Chief Constable apparently with identical equanimity—and smiled a little at the efforts of an ageing team to keep pace with undergraduates, but it was the whimsical smile of a man who had the world in perspective—he knew what was happening, just as he knew what he wanted to happen to the University.

When he retired in 1959, his dreams about Durham really had begun to come true. The dark days of the thirties, when extinction was on the horizon if not actually on the cards, seemed a long way off. The Division was flourishing and vigorous, and was clearly close to the point at which the federal University could be replaced by the two separate ones we now know. That it was so close could be attributed to the work of many people at many different stages, but none of their work would have been possible without the effort he made before the war, when even survival was not guaranteed, and a wavering in determination might have endangered it. In that respect his contribution was unique, and will always remain so. It is said that " cometh the hour, cometh the man "—Durham was indeed fortunate that when its bleak hour came, such a man was already there. In 1975 he died peacefully, in an armchair by the fire, reading a book.

J. L. BURCHNALL'S MATHEMATICAL PUBLICATIONS

Nearly one half of Burchnall's publications consists of joint papers with T. W. Chaundy, results of a collaboration which stretched over a period of some twenty years. Most (but not all) of the joint work, as well as much of the independent work of Burchnall, involves differential equations and formal properties of differential operators. A differential operator P is a polynomial in $D = d/dx$ with coefficients which are functions of x. In **[1]**, Burchnall and Chaundy describe the general problem of commuting differential operators and obtain special families of such operators. In [3], two commuting operators P and Q are subjected to a constraint $f(P, Q) = 0$, where f is a polynomial with constant coefficients. There arises here a connection with the uniformisation of algebraic curves and (as H. F. Baker pointed out in a paper immediately following [3]) also with Abelian functions. Special circumstances prevail in the presence of multiple points of the algebraic curve, and in [8] the special example $P^n = Q^m$ is studied. Much later Burchnall showed in [20] that many of the results on the then newly introduced Bessel polynomials can be obtained from [8] and he investigated for these polynomials generating functions and properties of the zeros.

Another offshoot of the investigation of commutative differential operators is [5] in which Burchnall and Chaundy considered a sequence of polynomials y_r, $r = 0, 1, 2, \ldots$, where

$$y_0(x) = 1, y_1(x) = x+a_1, \text{ and } y_{r+1} = y_{r-1} \int y_r^p \, y_{r-1}^{-2} \, dx$$

with a fixed positive integer p. They showed that y_r satisfies a linear differential equation of the second order whose coefficients depend on y_{r-1}, and for $p = 2$ they obtained an explicit expression for y_r as a determinant.

In much of the work on differential equations $\delta = x d/dx$ (rather than D) is the basic operator of differentiation. δ is the natural choice with differential equations satisfied by hypergeometric functions. In [6] and [7] differential equations $f(\delta)y = x^m g(\delta)y$ solvable in terms of Bessel or Legendre functions are studied and in [10], δ is used in establishing relations between products of hypergeometric functions. But δ also plays a role in the symbolic representation [9] of the Laplace transform of a function $f(x)$ in the form $x^{-1}\Gamma(1-\delta)f(1/x)$ and in the corresponding symbolic representation of Fourier sine and cosine transforms. Operators of the form $\Gamma(\delta+h)$ appear also in [17] where they are used to give new proofs of some famous hypergeometric identities and to obtain new identities for hypergeometric functions. Investigating in [18] the differential equation satisfied by a well poised ${}_3F_2$, Burchnall transformed from the variable x to $-(x-1)^2/(4x)$ and used this transformation to obtain a complete set of solutions in terms of ${}_3F_2$.

Another group of papers is concerned with hypergeometric functions of two variables. Operators such as

$$\nabla(h) = \frac{\Gamma(h)\Gamma(\delta+\delta'+h)}{\Gamma(\delta+h)\Gamma(\delta'+h)}$$

($\delta' = y\,\partial/\partial y$) were used in **[13]** and **[15]** to derive

$$F_2(\alpha, \beta, \beta', \gamma, \gamma'; x, y) = \nabla(\alpha)\Gamma(\alpha, \beta; \gamma; x)\Gamma(\alpha, \beta'; \gamma'; y)$$

and other similar elegant formulae, and to obtain series expansions and integral representations. In **[16]** Burchnall studied the ordinary differential equation satisfied by $F(px, qx)$ when F is a hypergeometric function of two variables, paying particular attention to those equations whose solutions can be identified in terms of known functions.

G. N. Watson and W. N. Bailey discovered relations connecting Appell's function F_4 and products of Gaussian hypergeometric functions. Burchnall **[12]** proved these relations by an ingenious transformation of the system of partial differential equations satisfied by F_4 with parameters subject to the condition $\gamma+\gamma' = \alpha+\beta+1$. He (with Larkin) also proved in **[19]** theorems on generalised hypergeometric series from the differential equations they satisfy, and he gave in **[14]** a new proof for the expansion of a product of Hermite polynomials in a series of such polynomials.

In his last three published papers, **[21]** to **[23]**, Burchnall uses the theory of quantics and their invariants to obtain elegant results involving the classical orthogonal polynomials and some of their generalisations. An interesting sample is the following result on Legendre polynomials P_r. Let $\phi[a_r]$ be a homogeneous isobaric invariant or semi-invariant of weight w for

$$f_n(X) = \sum_{r=0}^{n} \binom{n}{r} a_r X^{n-r}.$$

Then $\phi[P_r(x)]$ vanishes if w is odd and is a constant multiple of $(x^2-1)^{w/2}$ if w is even. Turán's identity

$$P_r(x)^2 > P_{r+1}(x)\,P_{r-1}(x)\;(-1 < x < 1)$$

and other inequalities can be deduced from this work.

It remains only to mention three publications not in the main line of Burchnall's research. **[2]** is an early paper in which functions discontinuous at all rationals are constructed by a process (described by E. Kamke as a "pretty process') depending on relating real numbers to infinite sequences (decimal representations, continued fractions, Cantor's quasi-exponential series, etc.). In another paper, **[11]**, solutions in integers of

$$\sum_{r=1}^{n} x_r^{\,k} = \sum_{r=1}^{n} y_r^{\,k},\; k = 1, \ldots, n-1$$

are considered. The "squares" mentioned in the title are formed by the $x_r - y_s$. The product of entries in each row (column) is the same and the ratio of row product to column product is $(-1)^{n-1}$. In any two rows or columns the algebraic difference of corresponding entries is constant for that pair of rows or columns. It is shown that the construction of such squares is equivalent to Tarry's problem, and solutions are obtained for values of n up to 8.

Lastly, [4] is the substance of an address to a conference of secondary school-teachers in which Burchnall gives an introduction to what nowadays would be called functional analysis. He concludes with a peroration in which he describes the theories presented as the " natural expression of the analytic spirit of an age which, alike in its religion, philosophy, politics and physics, is constantly digging itself up by the roots to inspect its processes of growth ", but he adds that " there is in the thought of today a synthetic and perhaps more significant trend ".

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