which is impossible.

Now since $\sigma_2(z)$ is regular and schlicht in W, and the coefficient of z^2 is 2, it follows [2] that $\sigma_2(z) \equiv z(1-z)^{-2}$, $\sigma_1(z) \equiv z$, $\sigma(z) \equiv z$.

The above shows that if $\sigma(z) = z + O(z^2)$ is regular and schlicht in W, then *either* $\sigma(z) \equiv z$ *or* the values of $\sigma(z_1)\sigma(z_2)$ (for z_1 and z_2 in the open unit circle) cover the closed unit circle. For if $|\zeta| \leqslant 1$ then $\zeta^{-1}\sigma(\zeta z)$ is regular and schlicht in W, and *either* $\sigma(\zeta z) \equiv \zeta z$ *or* $\sigma(\zeta z_1)\sigma(\zeta z_2) = \zeta^2$ for some z_1 and z_2 of W.

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1. Littlewood, *Theory of Functions* (Oxford, 1944), p. 210.
2. ———, *op. cit.*, p. 212. The conclusion we require is not stated; but $|\sigma_2| = 2$ implies $k_n = 0$ (all n), and then $f(z) \equiv 1 - \frac{1}{2}\sigma_2 z$.

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HORATIO SCOTT CARSLAW

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Horatio Scott Carslaw was born at Helensburgh in 1870, being the fifth son of the Reverend W. H. Carslaw, D.D., and was educated at Glasgow Academy and Glasgow University, whence he graduated M.A. in 1891. He then entered Emmanuel College, Cambridge, as a scholar, and was fourth wrangler in the Mathematical Tripos of 1894. He was elected to a Fellowship of his College in 1899, and took his Sc.D. in 1905. In 1896 he was appointed Lecturer in Mathematics in Glasgow University and remained there until 1903, with the exception of a year spent in Italy and Germany where, in 1897, he was one of Sommerfeld's first students at Göttingen. In 1903 he became Professor of Mathematics in the University of Sydney where he remained until his retirement in 1935. Subsequently, he lived in the country at Burradoo until his death on November 11th, 1954. In 1907 he married Ethel Maude, daughter of Sir William Clarke, Bt., but she died within a year of their marriage.

Most of Carslaw's mathematical work may be regarded as built on foundations laid during his short stay at Göttingen. In 1894, Sommerfeld threw out the idea that the method of images in electricity and conduction of heat, which was known to fail for the wedge of angle $m\pi/n$ because the method of multiple reflections involved did not give a simple covering of the region, could be extended to this case by considering solutions in a

Riemann's space or surface. Subsequently, he used the method to study problems of diffraction as well as ordinary potential theory. This was followed by a period of intense mathematical activity in which many problems of diffraction were solved, partly in order to determine the possibilities of the method in the hope that it might lead to the solution of other outstanding problems such as that of diffraction by a circular aperture. Carslaw was one of the first to appreciate the value of Sommerfeld's methods, and in an early paper [**12**] applied them in a modified form to a number of problems in diffraction and conduction of heat. The modification was characteristic: Sommerfeld had made great play with the idea of the Riemann space, and he had arrived at the contour integrals in his solutions by considering those used in the theory of Bessel functions; Carslaw proceeded immediately to integrals of the required type, thereby gaining in directness but losing in generality. This was followed by a number of shorter papers ([**13**], [**15**], [**16**], [**17**], [**26**], [**37**]) on various problems in diffraction and conduction of heat. In these, there may be observed a general trend away from the use of the Riemann surface and towards the *ad hoc* construction of solutions appropriate to the regions involved. In particular, he had a great interest in Green's functions and the various methods of discussing them, including the use of integral equations ([**21**], [**22**], [**23**], [**24**], [**27**], [**34**], [**54**], [**56**], [**59**]). Finally, in this period, he began to take an interest in the use of contour integrals for the solution of problems in conduction of heat, and the first use of "Path P", which he later made fundamental, appears [**14**].

In 1903 he was appointed to the Chair of Pure and Applied Mathematics in the University of Sydney in succession to T. T. Gurney. In the early decades of this century Australia was fortunate in having in its Universities mathematicians of the calibre of Carslaw, J. H. Michell and Alexander McAulay: all these men to some extent sacrificed the mathematics they might have done to the building up of their subjects in this country. When Carslaw arrived in Sydney he had a staff of one to cope with perhaps fifty students in both pure and applied mathematics: by 1930 the staff had only grown to three. In addition to this, there was the very heavy and important burden of mathematics in the public examinations. The syllabus and the setting and marking of papers in these was virtually in the hands of the University and was a most important method of ensuring the correct development of school mathematics. Throughout the thirty years of his Professorship, Carslaw maintained tight control over the public examinations, setting many of the papers himself.

The level of mathematical teaching in 1903, both in the Universities and the schools, was very low. Carslaw's method of coping with the situation was characteristic: he investigated conditions, worked for improvement, and wrote books to fill the gaps. In particular, he played a leading part—against strong opposition in the University—in the reform in 1913 of the system of public examinations in New South Wales, and he was for many years a member of the Board of Secondary School Studies. The state of mathematics at the time, as well as the trouble and interest that he devoted to the subject, may be seen from his report, *The Teaching of Mathematics in Australia* [25], written for the International Commission on the Teaching of Mathematics. In addition to this he wrote a number of notes on problems of teaching, both for journals and for the Department of Public Instruction. He fully realized that the proper training of teachers and their retaining an active interest in mathematics after leaving the University are essential to the development of mathematics, and to foster these he founded a branch of the Mathematical Association of which he was President for many years. Of his school text books, the *Plane Trigonometry* [3] keeps its place as a standard work.

It was probably from this context of school mathematics (as well as his fondness for languages and skill as a translator) that his interest in non-Euclidean geometry developed: he was concerned about the difficulties in the teaching of elementary geometry which arise from the Euclidean theory of parallels, and repeatedly stressed the desirability of teachers' knowing something of the background. In 1912 his translation of Bonola's *Non-Euclidean Geometry* [4] appeared, followed in 1916 by his own book on the subject [6]. He was a frequent contributor to the *Mathematical Gazette* on topics of general interest.

In University Mathematics the story is similar: a discontinuity in the mathematical syllabus being apparent between 1903 and 1904, while in the latter year the first edition of his *Introduction to the Infinitesimal Calculus* [2] appeared. This was described as "Notes for the use of students of Science and Engineering" and consisted of the bare bones of the subject which the lecturer could cover as he desired.

Carslaw's most important contribution to mathematical literature was his *Introduction to the Theory of Fourier's Series and Integrals and the Mathematical Theory of the Conduction of Heat* [1] which appeared in 1906. The history of this is interesting: Carslaw had for some years contemplated the writing of a Treatise on Conduction of Heat—in doing this he found it necessary to provide a rigorous background for the use of Fourier Series. It is usually considered that the development of rigour in modern English pure mathematics dates from Hardy's *Course of Pure Mathematics*; in fact, Carslaw's work, which many teachers prefer, antedates both this and Bromwich's *Theory of Infinite Series* by two years. In its second edition it was split into two parts, *Fourier's Series and Integrals* [7] and *Introduction to the Mathematical Theory of the Conduction of Heat in Solids* [8]. The Fourier's Series contains what probably still is the best English introduction to the theory of functions of a real variable; it would probably have had an even wider influence if this introduction had not been coupled with Conduction of Heat in the first edition and, even when the latter was split off in 1921, with Fourier Series. It illustrates very vividly Carslaw's most important characteristic as a mathematician, his love of the " good treatment ": for him, mathematics had to be crystal clear in exposition and, subject to its assumptions, rigorous. This love of the good treatment led him to endless experiments in exposition: when he had finally decided on the best treatment he wrote very fluently. Sometimes he was never satisfied, as with the treatment of the Lebesgue integral in the third edition of " Fourier's Series ". Apart from his book, he wrote little on Fourier Series beyond a series of papers on the Gibbs phenomenon ([**33**], [**43**], [**44**], [**45**], [**51**]).

The subject of Conduction of Heat had attracted the great French authors Fourier, Lamé, Poisson and Boussinesq, but Carslaw's was the first treatise on the subject in English. In it he covered the classical problems, in addition to developing fully his own method of solution by the use of contour integrals. He wrote little about Conduction of Heat beyond his book and the papers referred to earlier, and in the latter the interest is rather in the development of methods for the solution of the partial differential equations of mathematical physics, but he had an encyclopaediac knowledge of the literature of the subject. In 1947 a new version [**10**] of the Conduction of Heat was produced in collaboration with the present writer, in this the contour integral method was abandoned in favour of the Laplace transformation.

Carslaw's final contribution to mathematics, made mostly after his retirement in 1935, was his work on the Laplace transformation. In connection with his work on Conduction of Heat he had read Heaviside's challenging contributions to the subject, and when Bromwich in 1916 showed that many of Heaviside's results could be justified by the use of contour integration, it was pointed out ([**38**], [**39**], [**47**]) that Bromwich's contours* were equivalent (for problems in Conduction of Heat) to those which Carslaw had been using since 1903 and which he made fundamental in *Conduction of Heat* in 1921. Mathematically there was little difference between the two treatments—in Bromwich's hands the operational method was simply an automatic means of constructing the integrands of the contour integrals, but Carslaw tended to lay much more stress than other writers on the fact that the solutions were tentative and must be verified subsequently.

The puzzle of the operational and Laplace transformation methods, namely that simple manipulative operations lead to a solution but that it is difficult to state general conditions under which the solution so obtained

* Bromwich, *Proc. Cambridge Phil. Soc.*, 2 (1921), 411.

may be regarded as rigorous, has baffled many writers and resulted in a tendency to verify only simple cases and, on the strength of this, to assert results for more difficult ones—Bromwich, in his 1916 paper, which is frequently stated to have justified the operational methods, indicated quite clearly the difficulties involved in an adequate discussion of continuous systems and did not even complete the theory of systems of ordinary linear differential equations. At the level of engineering mathematics, many books and articles employed a purely intuitive approach. Carslaw, from 1928 onwards, continually sought the " good treatment " of the subject which would be both elementary and rigorous. " There is no room for mystery in mathematics ", he wrote, " if we can be clear, let us be so ". Certainly, his simple, direct approach did much to clear the subject of the mystery which he so much disliked. He was sometimes accused of pedantry in his attitude towards operational methods: this was not so—he merely asked that mathematicians who stated results should be able to prove them.

His hope always was that the " inversion theorem " could be established under sufficiently general conditions to make the justification of many solutions automatic: this is apparent from his earlier papers on the subject ([**52**], [**53**]. The writer had the privilege of sharing this last ten years' work with him, during which a number of papers ([**54**]-[**59**]) and a book [**9**] on the subject were written. The standpoint of these, again, is a little different and represents a return to the *a posteriori* verification used in his contour integral methods which can be carried out simply for large classes of problems in conduction of heat.

In all, Carslaw wrote some seventy papers on various topics, of which only the more important have been referred to above. Among his many activities was his interest in progressive rates of income taxation in which, as he put it, each pound pays a little more than the one before it. He was a keen advocate of this system and enjoyed pointing out to successive Treasurers the deficiencies of the tax schedules in their Budgets. For many years it was his practice to analyse these schedules ([**30**], [**35**], [**36**], [**49**], [**50**], [**60-66**]) and this regular discussion probably had its effect in producing simple and equitable rates.

Carslaw enjoyed contact with students at all levels, and generations of them, good and bad, have been stimulated by his lectures—for he was something of a showman—and have profited by his interest in their careers. He will be remembered, too, at his old College, where he invariably spent his Sabbatical leaves, and in whose life he took the fullest part. This continuing Cambridge connection was of the greatest value to his students, the best of whom he encouraged to go to Cambridge on graduation to take the Mathematical Tripos. Also, in the comparative isolation of Sydney or Burradoo, he kept in touch with many of his old mathematical friends, being a copious letter writer with something of the tang of that other Horace of the Eighteenth Century. He will be remembered by all who knew him as a man of great charm and kindness.

I am indebted to many friends of Professor Carslaw for information about him, and to Professor T. G. Room for material for the bibliography.

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ALBERT EINSTEIN

G. TEMPLE.

Albert Einstein was born on 14 March, 1879, at Ulm in Württemberg, Germany, the son of Hermann Einstein and his wife, Pauline, *née* Koch. His education was somewhat irregular, owing chiefly to changes in domicile brought about by unsatisfactory circumstances in his father's business. Between the ages of 10 and 15 he attended the Luitpold Gymnasium at Munich. Later for a year he became a pupil of the cantonal school at Aarau in Switzerland, and at the age of 17 he entered the Technische Hochschule at Zurich, each successive move involving some discontinuity in the curriculum. By 1901 Einstein had completed his studies at Zurich Polytechnic School and had taken the step of becoming a Swiss citizen. He then obtained a position in the Patent Office in Berne, and married a lady, Mileva Maric, who had been a student with him at Zurich. She was a native of Hungary, of Serbian race, and nominally of the Greek Orthodox religion. Two sons, Hans Albert and Eduard, were born to them on 14 May, 1904, and 26 July, 1910, respectively.

Einstein made his first contribution to theoretical physics in 1901, but it was in the year 1905 that he made his name with three remarkable papers, each of which has had a profound influence.

The papers of 1905 made such an impression that Einstein was widely recognized as a man worthy of exalted academic position, and after becoming a *Privatdozent* at the University of Berne he was, in 1909, appointed "Professor Extraordinary" of Theoretical Physics in the University of Zurich, and in the autumn of the following year, Professor of Theoretical Physics in the German University of Prague. This was one of the two parts, German and Czech, into which the ancient University