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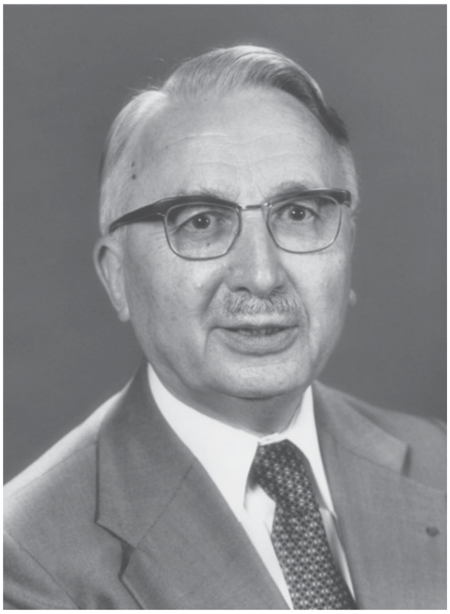
OBITUARY

Henri Cartan 1904–2008


Studios Harcourt

Henri Paul Cartan, Professor of Mathematics in Paris, was, for many of the younger generation, the symbol of the resurgence of French mathematics after World War II. He died in 2008 at the age of 104 years.

1. *Personal life*

Henri was the eldest son of the mathematician Elie Cartan (1869–1951), born in Dolomieu (Isère), and of his wife, Marie-Louise Bianconi, of Corsican origin.

Born at Nancy in 1904, he entered the École Normale Supérieure (ENS), 45 rue d'Ulm in 1923. It was there that he forged the friendships with mathematicians who were to play a major role in his life, beginning with André Weil, who had entered the ENS a year before; others included Jean Dieudonné, Jean Delsarte, René de Possel and Charles Ehresman. He left the ENS in 1926, supported by a grant until the completion of his thesis in 1928, and briefly became a teacher at the Lycée Malherbe de Caen. He was then appointed to positions at the University of Lille and subsequently the University of Strasbourg, where he taught from 1931 to 1939. The year 1935 was a particular high point of both his professional and his personal life: with his friends Weil, Dieudonné, de Possel and others, he founded the Bourbaki group, which he left only at the statutory age of 50 years; and he married the young and charming Nicole Weiss, daughter of one of his physics colleagues at Strasbourg University. This happy marriage, which lasted until his death (followed, six months later, by that of his wife), produced five children: Jean, Françoise, Étienne, Mireille and Suzanne.

In September 1939, at the beginning of the war, he moved to Clermont-Ferrand, where the University of Strasbourg had been evacuated. A year later he got a chair at the Sorbonne, where he was given the task of teaching the students of the ENS. This was a providential choice that allowed the 'normaliens' (and many others) to benefit for more than 25 years (1940–65) from his courses and seminars. In fact there was a two-year interruption when he returned to Strasbourg from 1945 to 1947 — alas for me, because I was then a student at the ENS and could not make his acquaintance until my final year.

He left the ENS in 1965 and, a few years later, to escape the internal disputes between the component parts (Paris VI and Paris VII) of the former Sorbonne, he accepted a chair at Orsay, where he taught until his retirement in 1975. A lecture theatre in the mathematics building has recently been named after him.

Further details on the life of Henri Cartan can be found in two interviews [**3**, **5**].

2. *Mathematical work*

Henri Cartan worked on many subjects but there was one to which he was particularly attached, and that was the theory of functions of several complex variables (which later became the theory of complex varieties and also 'analytic geometry'). I will begin with this.

His thesis ([**12**], no. 3) dealt with analytic functions of one variable, one of the most popular topics of the period in France. Cartan continued the work of André Bloch and Rolf Nevanlinna, studying in particular the properties of analytic curves in complex projective spaces of any dimension (for example curves not meeting a given family of hyperplanes). This sort of topic was highly fashionable at the time, but it became less so in later years (despite the work of Lars Ahlfors and H. and J. Weyl). It finally came back into the limelight thanks to the work of Shoshichi Kobayashi on hyperbolic manifolds (1970–80) (see [**1**]) and also to that of Paul Vojta (around 1980), who created an astonishing dictionary relating Nevanlinna invariants to the heights of rational points on algebraic varieties.

Shortly after writing his thesis, his eyes were opened, by Weil, to the charms of functions of several complex variables. Cartan was definitely seduced by this new field. Between 1930 and 1940 he published many articles in collaboration with the German School (Heinrich Behnke and Peter Thullen), with whom he made great bonds of friendship that withstood World War II. A summary can be found in [**11**, §§ 2–5]. In particular:

- the introduction (in [**12**, no. 23]), with P. Thullen, of the notion of 'convexity' relative to a family of holomorphic functions; and
- the proof (in [**12**, no. 32]) that the group of automorphisms of a bounded domain of $\mathbf{C}$ is a real Lie group, and that the subgroup that fixes a point is compact and embeds in $\mathrm{GL}(n,\mathbf{C})$.

Starting in 1940 it was the 'Cousin problems' that attracted him most [**11**, § 6]. This involves the construction of functions whose local singularities (additive or multiplicative) are given. Is this possible, and if not what are the conditions that need to be met? The problem is only reasonable if one works in a domain of holomorphy, which is what Cartan assumes. He gets very close to his aim, thanks to a theorem on invertible holomorphic matrices [**12**, no. 35], but he lacked two auxiliary results (which he later interpreted as statements of 'coherence'). It was the Japanese mathematician K. Oka who proved the first of these two results. He published the proof and sent it to Cartan, who immediately saw how the same methods led to the second result [**12**, nos 36 and 38]. The first Cousin problem was thereby solved, at least for domains of holomorphy.

The second Cousin problem, in contrast, does not always have a solution. There are obstructions of a topological nature, which mean that the problem should have continuous solutions (a minimal requirement if one is searching for holomorphic solutions). How can one concretely exhibit these obstructions and moreover show that there are no others? I suppose (I never thought of asking him) that this was one of the reasons† that led Cartan to become interested in algebraic topology around 1945–50. There were some striking analogies — for those who could see them — between certain concepts introduced by Oka (the 'ideals of indeterminate domains') and the theory of sheaves, which was being created by Jean Leray. In his first seminars at the ENS (1948–51) Cartan took up Leray's theory in a slightly modified form that was easier to use. In the next seminar (1951/52) he reaped the fruits of his labours. He began by clarifying the notion of 'coherence', implicit in Oka's work, defined 'coherent analytic sheaves', and proved a vast generalization of the Cousin-type theorems: the famous 'Theorems A and B'.

The stronger statement is 'Theorem B', which says that the higher cohomology groups of a coherent analytic sheaf are zero; in other words that every reasonable problem (of additive type) has a solution (if the underlying manifold is a 'Stein manifold', the natural generalization of a domain of holomorphy).

Theorems A and B are very powerful tools. Cartan and I described several applications of them in a colloquium at Brussels in 1952; apparently these theorems made a strong impression on the participants because one of them (a German) said to his neighbour, 'The French have tanks (Panzers) while we have only bows and arrows' (see [**4**]). Indeed, the idea of applying the (algebro-topological) theory of sheaves to objects relevant to analysis (holomorphic functions) was a new idea; it was used later in many other situations (for example solutions of partial differential equations) and has now become standard.

Another original idea of Cartan (now also standard) was that, developed in the 1953/54 seminar, of defining a complex analytic space (possibly with singularities) as a topological space endowed with a sheaf of rings. For Cartan this sheaf was a sub-sheaf of the sheaf of continuous functions; Grauert–Remmert and Grothendieck showed a little later that it is better not to make such a hypothesis so as to allow nilpotent elements.

In subsequent years Cartan never lost interest in functions of several complex variables. He took great pleasure in expounding in Bourbaki seminars the works in this area of other mathematicians, notably those of Hirzebruch (exposé 84), of Grauert (exposé 115), of Douady (exposé 296) and of Ramis (exposé 354).

Let us now change the subject slightly and turn to topology. I have already mentioned the expository talks, clarifying sheaf theory, in the seminars of 1948/49 and 1950/51. He had done something similar for fibre spaces in the seminar of 1949/50. Other results included the spectral sequence giving the cohomology of a Galois covering (with J. Leray), the method of 'killing homotopy groups' (with me) and the study of the real cohomology of principal fibre bundles of Lie groups (with Chevalley, Koszul and Weil). However, his most original contribution to topology was without doubt the long series of lectures in the 1954/55 seminar (reproduced in [**12**, no. 93], where he determined the homology of the Eilenberg–Mac Lane complexes ('which required great efforts', as he said in an interview in 1982 — I can readily believe it). This work would now be classified not as part of topology but as part of what is called 'homological algebra', a terminology introduced by Cartan and Eilenberg in their book with that title ([**7**], completed in 1953 but only published in 1956). A 'fundamental' book in the precise sense of that term, it collected scattered results and organized them in a systematic way, transforming them into an instrument of great power.

† Another reason may have been the translation by Weil of the Cousin problems in terms of holomorphic fibre bundles with additive structure group (for the first problem) and multiplicative structure group (for the second problem) — see [**12**, no. 39, § 5].

Cartan also worked on other subjects, which I will simply mention:

- classes of indefinitely differentiable real functions (with S. Mandelbrojt) [**12**, nos 63–68];
- general topology: introduction of the notion of a filter [**12**, nos 61 and 62] and construction of Haar measure [**12**, no. 69];
- potential theory [**12**, nos 70–75 and 84] (see the report by Deny [**2**]);
- harmonic analysis (with R. Godement) [**12**, no. 80];
- real analytic spaces (with F. Bruhat) [**12**, p. XVI; **12**, nos 45–46].

3. *Cartan's influence*

One cannot reduce the influence of Cartan to a mere list of the theorems he proved. He did much more than that. As I said at the outset, Cartan represented (both in France and abroad) the revival of mathematics in France after World War II. How did this come about? It is difficult to answer precisely. There were several factors, among which were the following.

- The large numbers of students whom he trained (in chronological order: Deny, Koszul, Godement, Thom, myself, Cerf, Douady, Karoubi, and others); he did not give them a research topic (believing, no doubt, that a mathematician who does not ask himself questions is not a real mathematician), but once they were launched he helped them to prove their results, to clarify them and to write them up properly. This took him on occasion much time (I am thinking in particular of a certain thesis in topology on which he — and I — spent many hours). But the pupil learnt much.
- Another reason for his influence: the Cartan Seminars. I have mentioned several above. There were 16 of them (from 1948 to 1964), and all except one (that of 1952/53) have been written up; a summary can be found in [**6**]. What made these seminars original and interesting was that they started from scratch and gave essentially complete proofs; despite this, at the end of a year (and some 20 lectures) they culminated in interesting and occasionally novel results. Many mathematicians, French and foreign, learnt their topology or their functions of several complex variables from these seminars.[†]
- Going beyond his own mathematics I should mention the efforts that Cartan made to improve relations between French and German mathematicians after World War II. He was also active, with L. Schwartz and M. Broué, in the 'Committee of Mathematicians' that came to the aid of mathematicians imprisoned for political reasons in various countries (notably in the USSR), for example L. Pliouchtch, A. Chtcharanski, A. Chikhanovitch and L. Massera.

4. *Distinctions*

Besides being a Foreign Member of the Royal Society, Henri Cartan was a member of the Académie des Sciences de Paris and Academies in Germany, Belgium, Denmark, Spain, Finland, Italy, Japan, Poland, Russia and Sweden. He was also an honorary member of the London Mathematical Society.

[†]The Cartan Seminars had a predecessor: the 'Séminaire Julia', organized between 1935 and 1938 by Weil, Chevalley, Cartan and others. Here also there was an annual theme (such as class fields, Hilbert spaces and the work of Elie Cartan) and the lectures were written up. And there was a successor: the impressive 'Séminaire de géométrie algébrique' of Alexander Grothendieck at the Institut des Hautes Études Scientifiques (1960–69), where the proofs were even more complete — if I may say so — and the results even more novel. Since 1970, mathematical seminars have multiplied, in France as elsewhere, but none, to my knowledge, has tried to follow the difficult model of Julia–Cartan–Grothendieck: they merely invite, week after week, a lecturer who presents (usually without proofs) his latest results, and then discusses them with specialists. This is not the same thing.

He had honorary degrees from ETH (Zürich), Athens, Cambridge, Münster, Oslo, Oxford, Saragossa, Stockholm and Sussex.

In France he was awarded the Gold Medal of the Centre national de la recherche scientifique in 1976 and was Commandeur des palmes académiques, Grand Officier de l'Ordre national du mérite, and Commandeur de la Légion d'honneur.

He was President of the French Mathematical Society (1950). Internationally he received the Wolf Prize in 1980 and the Heinz R. Pagels Human Rights of Scientists Award in 1989. He was President of the International Mathematical Union (1966–70) and President (and subsequently honorary president) of the Federalist European Movement (1974–85).

This memoir was translated by Sir Michael Atiyah, who has, at the invitation of Professor Serre, added the following personal note on Henri Cartan.

> I got to know Cartan mainly through serving with him on the Executive Committee of the International Mathematical Union. This gave me a chance to see him operate on the international scene. By then he was an elder statesman and he looked the part, always impeccably dressed in a style that one associates with earlier periods. But this formality hid a charming and friendly personality, and the 25 years that separated us were no barrier to our friendship.
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> I first met Cartan at a conference in Mexico in 1956 and although he was then over 50 years old I was struck by his restless intellectual energy. During the lectures his eyes were alarmingly alert; he seemed to be on the verge of springing from his seat, with impatience at the slow pace of the lecturer. But he was never aggressive or rude, just interested and enthusiastic. I can only imagine what a live-wire he would have been 20 years earlier.
>
> I was very pleased when in 1973 Oxford gave him an honorary degree. The public orator in his (Latin) speech referred (as Serre has done) to the important role played by Cartan in maintaining links with German mathematicians after 1945. He also mentioned the fact that a younger brother of Henri, a talented composer who died young, had one of his compositions played in a prewar concert in the same theatre (the Sheldonian) where Henri received his honorary degree.
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> My last memory of Cartan is of his attending a lecture of mine in Paris when he was at a very advanced age and seriously infirm. It was a touching symbol, both of his friendship and of his dedication to mathematics.

Acknowledgement. The photograph is by Studios Harcourt and was supplied by Mme Suzanne Cartan. This obituary is published with kind permission by the Royal Society. It appeared previously in *Biographical Memoirs of Fellows of the Royal Society* 55 (2009) 37–44.

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Publications of Henri Cartan

Books

7. (with S. Eilenberg) *Homological algebra* (Princeton University Press, 1956). (Also translated into Russian.)
8. *Théorie élémentaire des fonctions analytiques* (Hermann, Paris, 1961). (Also translated into English, German, Japanese, Russian and Spanish.)
9. *Calcul différentiel* (Hermann, Paris, 1967). (Also translated into English and Russian.)
10. *Formes différentielles* (Hermann, Paris, 1967). (Also translated into English and Russian.)
11. 'Brève analyse des travaux', Notice written for the Académie des Sciences, reproduced in [**12**], ix–xxiv.
12. *Oeuvres—Collected Works* (eds R. Remmert and J.-P. Serre, 3 volumes; Springer, Berlin, 1979).

Seminars

13. *Séminaires de l'École Normale Supérieure* [called 'Cartan Seminars'] (W. A. Benjamin, New York):
1948/49 *Topologie algébrique*;
1949/50 *Espaces fibrés et homotopie*;
1950/51 *Cohomologie des groupes, suites spectrales, faisceaux*;
1951/52 *Fonctions analytiques de plusieurs variables complexes*;
1952/53 *Groupes d'homotopie* [not written up];
1953/54 *Fonctions automorphes et espaces analytiques*;
1954/55 *Algèbres d'Eilenberg–Mac Lane et homotopie*;
1955/56 (with C. Chevalley) *Géométrie algébrique*;
1956/57 *Quelques questions de Topologie*;
1957/58 (with R. Godement and I. Satake) *Fonctions automorphes*;
1958/59 *Invariant de Hopf et opérations cohomologiques secondaires*;
1959/60 (with J. C. Moore) *Périodicité des groupes d'homotopie stables des groupes classiques, d'après Bott*;
1960/61 (with A. Grothendieck) *Familles d'espaces complexes et fondements de la géométrie analytique*;
1961/62 *Topologie différentielle*;
1962/63 *Topologie différentielle*;
1963/64 (with L. Schwartz) *Théorème d'Atiyah–Singer sur l'indice d'un opérateur différentiel elliptique.*

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