The Fundamental Theorem of the theory is the Parseval formula

$$M\{|F(t)|^2\} = \sum_{n=1}^{\infty} |a(\lambda_n)|^2.$$

The most important result in connection with the Fundamental Theorem is the theorem on uniform approximation to almost periodic functions by exponential polynomials with exponents belonging to the Fourier series. The general theory was developed for the case of functions of a real variable, and then, in the light of it, was developed the most beautiful theory of almost periodic functions of a complex variable. The theory was presented in the three large papers in *Acta Mathematica* 1924-26.

The creation of the theory of almost periodic functions of a real variable was a performance of extraordinary power, but it was not based on the most up-to-date methods, and the main results were soon simplified and improved. However, the theory of almost periodic functions of a complex variable remains up to now in the same perfect form in which it was given by Bohr.

The rest of Bohr's life was devoted to further developments of the theory, of which we may mention the work on systematic generalization of almost periodicity, and a most subtle and beautiful example of a function that is almost periodic on every vertical line of a strip and yet not almost periodic in the strip.

He remained active until shortly before his death, and took part in the International Congress of Mathematicians at Cambridge, Massachusetts, in September, 1950; he died soon after the New Year.

ELIE CARTAN

W. V. D. HODGE

Elie Joseph Cartan was born on April 9, 1869, at Dolomien (Dauphiné), the second of the four children of the local blacksmith. He attended the primary school at Dolomien, and then the Collège de Vienne (Isère) and the Lycée at Grenoble. After preparation at the Lycée Janson-de-Sailly in Paris, he gained admission to l'Ecole Normale Supérieure in Paris in 1888, and studied there for three years. He finally took his doctor's degree in Paris in 1894.

In the year in which he took his doctorate he was appointed maître de conférences in the Faculty of Science at Montpellier. Subsequently he held posts at Lyon and Nancy. In 1909 he returned to Paris as maître de conférences in the Faculty of Science, and three years later he was promoted professor, and remained there until he retired in 1940. From 1924 he held the chair of Géométrie Supérieure.

In 1903 Cartan married Mlle Marie-Louis Bianconi. Besides a daughter, there were three sons of the marriage—Henri, a mathematician who has made important contributions to numerous branches of mathematics and is now a professor in the Faculty of Science at Paris, Jean, whose brilliant promise as a composer was cut short by his premature death at the age of 25, and Louis, who was a professor of physics at the University of Poitiers, and was one of the first French workers in the field of electron optics. In the late war Louis became a member of the resistance movement; he was arrested by the Vichy authorities in 1942, and after fifteen months in captivity he was beheaded by the Germans. It was not until 1945 that Cartan could discover what fate had befallen his son. The tragic news, coming after more than two years of terrible anxiety, and the fact that a nephew of whom he was particularly fond had suffered a similar fate, was a crushing blow from which he never really recovered. He continued to turn out important mathematical papers, and in 1948 he was able to visit this country to be admitted as a foreign member of the Royal Society, but his vigorous youthful spirit had gone, and he died on May 6, 1951, not long after the death of his wife. His contributions to mathematical knowledge were recognized by many academies and universities throughout the world. He became an honorary member of the London Mathematical Society in 1939.

Cartan's thesis, published in 1894, was on the structure of finite continuous groups of transformations, and most of the ideas which directed all his subsequent work are to be found in it. Not that Cartan's work is in a narrow field of mathematics; quite the reverse. To follow the way in which the ideas which appear in the thesis develop and broaden in later years until vast tracts of mathematics come under their influence is one of the most impressive mathematical experiences the writer has ever enjoyed. The principal part of the thesis was devoted to the classification of simple Lie algebras over the complex field, and completed the work of Lie and Killing on this subject. In the next two or three years Cartan went on to make other major contributions to the theory of continuous groups.

About the year 1897 or 1898 Cartan turned his attention from Lie algebras to linear associative algebras, and wrote an important series of papers on this topic. One of his major contributions in this field was the first proof ever given of the theorem, now usually known as Wedderburn's theorem, on the structure of algebras over real and complex fields. Wedderburn gave his own proof ten years later, using methods more suited than Cartan's to the study of linear associative algebras; in his paper he made full acknowledgment of the fundamental importance of Cartan's work.

In 1899 Cartan began a great series of papers on Pfaffian forms, and in a sense it may be said that he continued this line of work for the rest of his life. Many of his great contributions to the general theory of Pfaffian forms and the theory of differential equations are contained in Goursat's famous book " Leçons sur le problème de Pfaff ", but the work was in effect continued in Cartan's fundamental work on such topics as contact transformations, invariant integrals and Hamiltonian dynamics, and in his great contributions to differential geometry.

Between 1904 and 1909 Cartan made substantial contributions to that most intractable part of the theory of groups, which deals with infinite continuous groups, that is, continuous groups of transformations which cannot be represented as depending on a finite number of parameters. He succeeded, for instance, in giving a complete classification of the primitive groups operating on n-dimensional space. Practically no progress has been made on this subject since Cartan finished working on it.

A mere catalogue of Cartan's work gives little idea of his immense powers, even though many of his results are now classical in mathematical literature. Nor is it possible, in this short notice, to give even a brief description of his methods, involving many complicated algebraic and geometric arguments, which would do justice to them. Fortunately, however, there is a volume of " Selecta ", published in 1939 (Gauthier-Villars) on the occasion of his scientific jubilee, to which Cartan himself has added a comprehensive description of his work, and the reader who seeks a broad view of his methods and achievements cannot do better than refer to this volume.

The discovery of the general theory of relativity in 1916 turned the attention of many mathematicians, including Cartan, to the consideration of the general concept of geometry, and nearly all Cartan's work from this time onwards is devoted to the development of a general theory of differential geometry. While it would be extremely rash to say that any particular part of Cartan's work was his most important, it is perhaps allowable to express the opinion that his work on differential geometry, spread over the period from 1917 to 1949, is most relevant to work which is going on at the present time. It forms a most vital contribution to modern mathematics, and it is indeed remarkable that it was begun when he was nearly fifty, and carried on until he was eighty—a most striking exception to Hardy's dictum that mathematics is a young man's game.

For nearly fifty years, ideas concerning the foundations of geometry had been dominated by Klein's Erlanger Program, and the concepts put forward in Riemann's Inaugural Dissertation had been largely ignored, except in the analytical work of Ricci and his followers. The impact of general relativity forced mathematicians to realize that Klein's concept of geometry was not entirely adequate, and the problem of formulating a more general concept was attacked with vigour. A period of great activity began (the first paper on the subject was written by Weyl in 1918), and different schools of thought began to develop. The most radical reaction was along the lines of what is sometimes called the Princeton school, who regarded geometry as the study of a " geometric object ", such as a tensor or projective connection, a point of view first formulated explicitly by Veblen at the

Bologna International Congress in 1928. The Europeans, such as Weyl, Schouten and Cartan, while sympathetic to these ideas, were less violent in their reactions.

Cartan's approach to the problem is along lines which proceed smoothly from the classical ideas. In fact, an invaluable tool which he uses to investigate the differential properties of spaces, the "repère mobile" (or "moving frame"), is just a generalization of the moving axis of classical geometry. Briefly, his view of the principal axes at points of a curve C in Euclidean 3-space can be described as follows. In the 3-space we call a set of three mutually orthogonal unit vectors at a point a frame. The set of all frames in the space represents the group of Euclidean displacements, in the sense that every frame can be obtained in one, and only one, way from a given frame F_0 by a displacement, and every displacement transforms a frame into a frame. By assigning (in a continuous manner) a frame to each point of C, we define a mapping h of C into the group manifold of Euclidean displacements. The geometry of the group manifold is determined by a set of Pfaffian forms $\omega_1, \ldots, \omega_r$ (r being the dimension of the manifold). The mapping h determines a mapping h^* of the forms ω_i into forms on C. The forms $h^* \omega_i$ determine the variation of the frames assigned to the points of C as we move along the curve, expressing the variation in terms of the parameters of the frames at a point. By special choice of the frames assigned to points of C, some of these parameters can be given constant values. It is possible to assign the frames so that the number of parameters remaining is a minimum. If C and C' are two curves of the 3-space and frames are assigned in this way to the points of each, a necessary and sufficient condition that C may be transformed into C' by a Euclidean displacement is that a correspondence can be set up between the points of C and C' so that at corresponding points the differential parameters are equal. The choice of the principal axes at the points of C as the assigned frames is the simplest method of reducing the number of variable parameters to a minimum.

Now, instead of the 3-space consider a differential space X, and instead of C consider any analytic sub-manifold V of X. In place of the groups of displacements operating on the 3-space, we consider any Lie group G of transformations which operate transitively on X. Then the same process as we have described above can be applied to X, V, G. It is only necessary to supply X with a suitable set of frames from which to start, and this is easily done. Thus Cartan is able to describe the geometry of any space operated on transitively by a Lie group of transformations. The geometries thus obtained are Kleinian, that is, they fall within the scope of the Erlanger Program.

The process can then be generalized. For instance, in a Riemannian space X of n dimensions it is possible to define a frame as a set of n mutually orthogonal unit contravariant vectors at a point of the space. The set of all frames at all points of X defines a space B which is a fibre space over the base space X, the projection of any frame (that is, point of B) being the origin of the frame. A process similar to that described above can be applied, in which B takes the place of the group manifold. But since B is not a group manifold, in general, the forms attached to B do not satisfy the equations of Maurer-Cartan, and the amount by which they fail to satisfy these equations leads to a measure of the curvature of X. Working on these lines, Cartan was able to give great generality to the study of differential geometry.

The portion of Cartan's work on differential geometry which we have described deals with the local geometry of a space, but Cartan was able to make most valuable contributions to problems of differential geometry in the large. This has now become one of the most exciting fields of modern geometrical research, and it is becoming increasingly clear that Cartan's methods, involving the use of exterior forms, are ideally suited to this kind of work, so that his methods, which were for long only followed in France, are now becoming recognized universally and widely adopted.

Cartan's researches led him to recognize the importance in geometry of the spaces known as symmetric spaces. These are spaces whose curvature tensor has zero covariant derivative. The work in this field began, in 1925, in two papers written jointly with Schouten, on the geometry of semi-simple groups. It has important applications to group theory, and the properties of symmetric Riemannian spaces go very deep. Much research has been done in this field, and results of the greatest elegance have been obtained. In fact, symmetric spaces seem to be the spaces in which many of the global properties of differential manifolds attain their most perfect form.

He was attracted by the generalization in differential geometry originating in the ideas of Finsler and made important contributions in this field. He also introduced a new kind of geometry, based on the notion of area. He made each of these topics the subject of a book which has had a great influence on the development of formal differential geometry.

The foregoing remarks on a few scattered items of Cartan's contributions cannot give any idea of the importance of his work to modern mathematics. But perhaps they may help to convey something of the impression that Cartan makes on the writer; that of a great mathematical genius taking in the scene in a broad survey, and picking out the essentials, so that with a master-stroke he goes straight to the heart of a problem. His knowledge of innumerable special cases, and his mastery of intricate argument, enabled him to advance his subject by giant strides, and make a lasting mark on the vast range of mathematical endeavour. By his death, the world has indeed lost one of the great architects of modern mathematics.†

† In the preparation of this notice, I have had the advantage of using the longer notice written for the Royal Society by Prof. J. H. C. Whitehead.