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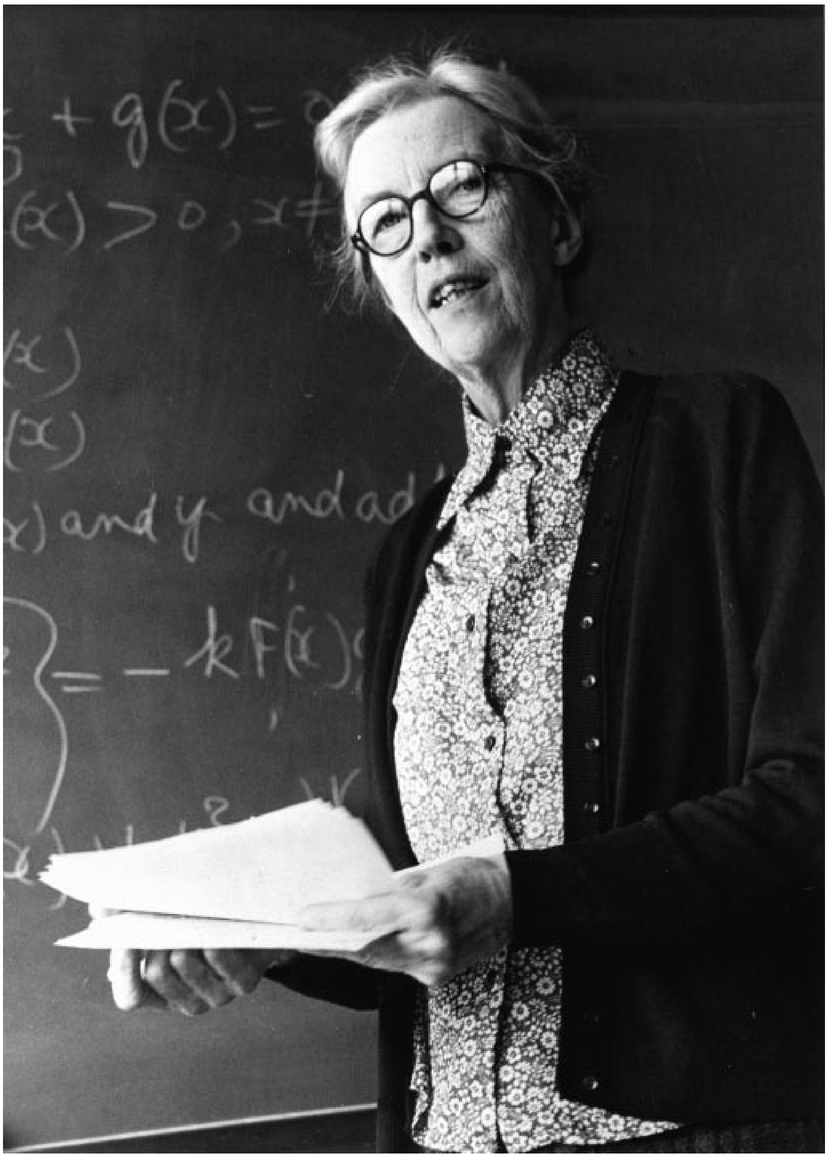
OBITUARY

MARY LUCY CARTWRIGHT (1900–1998)



Mary Cartwright with J. E. Littlewood FRS first observed the phenomena which developed into Chaos Theory. Thus she had a significant effect on the modern world. She was the only woman so far to be a president of the London Mathematical Society, one of the first to be a Fellow of the Royal Society, and the first woman to serve on its Council. She was born on 17 December 1900 and died on 3 April 1998.

Family background

Mary came from an old and distinguished Northamptonshire family going back to the Civil War and with a tradition of public service in the county. Her great-great-grandfather married Mary Catherine Desaguiliers whose grandfather, John Theophilus Desaguiliers FRS, was brought over from France (reputedly in a barrel) in 1685 by his father, a Huguenot pastor. John Theophilus became Curator of the Royal Society, wrote books on Physics and invented the Planetarium. His son, Thomas Desaguiliers FRS, was a general who invented a method of firing small shot from mortars. Another ancestor was the poet John Donne. A family tree proving this and handwritten by Dame Mary is contained in the archives of Girton College. Mary's father, William Digby Cartwright, was Rector of Aynho (a family living) after being Curate to his uncle, Frederick William Cartwright.

Her mother was Lucy Harriette Maud (née Bury). At one stage Mary's father was Curate to her mother's half-brother, Edward Bury, at New Ferry. From the time of the Civil War until 1847, the head of the family was usually a Member of Parliament.

Two elder brothers, John and Nigel, were killed in the First World War. Mary's sister, Jane, married the Reverend Hugh Maclean and at one stage exhibited paintings and wood engravings in London, Oxford and Cheltenham, and her youngest brother, William Frederick Cartwright CBE, was Deputy Chairman of British Steel. The army, the church and parliament featured prominently in the family history.

Early years

Mary lived in Aynho (or Aynhoe, derived from Ayn, a spring of water, and hoe, meaning promontory) from 1900 until 1926, when her father died. Only three or four of the houses in the village had tap water and proper drainage. She roamed around the rectory and the grounds of Aynho Park which belonged to a cousin, William Cornwallis Cartwright, and later to William's son, Sir Fairfax William Cartwright, who was British Ambassador in Vienna until 1913. Mary had governesses until she was eleven.

For her eleventh birthday she was given a commonplace book with the motto 'Be good sweet maid and let who can be clever'. No doubt she was as cross about this as my wife was a generation later, when she received a pencil box with the same words. Mary went to Leamington High School from 1912 to 1915, and stayed Mondays to Fridays with her cousins, Dr and Mrs R. F. Bury.

She went as a boarder to Graveley Manor School, Boscombe, from 1915 to 1916, then to Godolphin School, Salisbury, from 1916 to 1919. There she had a remarkable self-taught Mathematics teacher, Miss Hancock, who really understood the fundamentals. She had learned by reading the Tutorial series of text books for the external London degree. Mary learned about the Calculus, Analytic Geometry and Uniform Convergence from her, but no Applied Mathematics. So she could not sit the scholarship examination at St Hugh's, but successfully took the entrance examination. She chose the College partly because her cousin, Cecily Ady, was a Fellow and Vice-Principal. Mary had qualified by passing the Senior Cambridge Local and completing coursework in Latin prose and Greek, which were then essential.

She went up to Oxford in October 1919 at the same time as a flood of men returning from the wars. In her memoir on Titchmarsh [**71**] Mary says: 'At J. W. Russell's first lecture the room was packed to the doors and Russell said "Ah, there's my clever pupil Mr Titchmarsh, he knows it all, he can go away" '. There were two other women doing Mathematics in her year at St Hugh's.

Mary got a Second in Moderations in June 1921, but only four men got Firsts. She was in two minds whether to switch to History. However, she persevered and got a First in Final Honours in 1923. On V. C. Morton's advice she read Whittaker and Watson's *Modern Analysis* and went to Hardy's evening classes. These went from 8.45 p.m. to about 11 p.m., and Mary had to get special permission to be out after 11.

In the four years after leaving Oxford she taught Mathematics at the Alice Ottley School, Worcester, and then at Wycombe Abbey School, Buckinghamshire, but the urge to do further mathematical work was strong and in 1927 she returned to work under G. H. Hardy. When Hardy went to America for a year, E. C. Titchmarsh took over as supervisor.

Her DPhil examiners in 1930 were J. E. Littlewood and W. L. Ferrar. She then moved to Cambridge as Yarrow Research Fellow at Girton College. This was followed by an appointment to a College Lectureship, which brought with it an official fellowship (then called staff fellowship). In 1935 she became a university lecturer in Mathematics, initially on a part-time basis. Between 1936 and 1949 she was Director of Studies for Mathematics at Girton College.

Brief comments on Cartwright as a teacher are to be found in the Girton Review, Lent 1969. Dr Kay Barker has written:

> Those of us whose studies she directed would probably all wish to say that she was a conscientious supervisor, but that to many of us she was a somewhat awe-inspiring figure. We knew of her as an eminent mathematician and this, together with her shyness and lack of small talk, made her supervision period seem rather daunting. Glimpses of her sense of humour did, however, indicate that behind this serious, scholarly, facade was a gentle and kindly human being.

Lady Jeffreys has written: 'From 1938–48 we shared the College teaching and she took a heavy load.'

A very able student said of her: 'If you ask a question and she doesn't know the answer, you work it out together.'

Later, as Mistress, she did an occasional hour, and someone who was struggling with Analysis was reputed to think well of the Mistress.

Research students, and College and University activities

Cartwright's research students included, apart from myself:

1. Lily Brown, now Lady Atiyah,
2. James Ejeilo, Professor and Vice-Chancellor for a time at Nsukka University, Nigeria,
3. Carl Linden, Senior Lecturer at University College, Swansea,
4. Elizabeth McHarg, Senior Lecturer at Glasgow University,
5. Sheila Scott Macintyre, Lecturer at Aberdeen University and later at Lexington, Kentucky,
6. Barbara Maitland, Lecturer at Liverpool University,
7. Marc Noble, Professor of Mathematics at the University of Canterbury,
8. Chike Obi, Professor at Lagos University,

9. Hilary Shuard, who did not take a DPhil but did become a prominent figure in the teacher-training world.

Mary was extremely conscientious as a supervisor and her comments on my early work were sometimes as long as what I had written. She paid particular attention to presentation and style. She pointed out that mathematical sentences had to obey the same rules of grammar as ordinary English sentences. At one stage she wrote: ‘your English is like that of the Italian tailor who said: “your trousers are now ready and should come down for a fitting”.’

She also insisted that all statements had to be supported by full proofs or references. She did not approve of joint papers of students and their supervisors and my only joint paper with her is the Royal Society Memoir of Sir Edward Collingwood.

I have tried to follow her advice and to pass it on to my own research students.

In the years preceding and during the Second World War, Mary carried out a very full programme of teaching and research, and was also an active member of the College Council and its committees and of the Faculty Board of Mathematics. From 1940 to 1944 she was Commandant of the College Red Cross Detachment.

In 1947 Mary was elected a Fellow of the Royal Society. She served on the Council of the Society from 1955–1957, the first woman to do so. She was awarded the Sylvester Medal of the Society in 1964.

In 1948 she was pre-elected Mistress of Girton College. She spent part of 1949 as a consultant on US Navy mathematical projects at Princeton and Stanford Universities and in the summer of 1949 she took up office as Mistress in succession to Miss K. T. Butler. In 1959 she was appointed Reader in the Theory of Functions by Cambridge University. Although she is now probably best known for her work with Littlewood on Dynamics, the choice of title suggests that Function Theory continued to be her real love.

The demand on Mary as a member of many university committees was very heavy. She gave long service as Chairwoman of the Cambridge University Women’s Appointments Board and in the Education Syndicate of the University. She herself said that she felt that she had really arrived when she became one of the Septem Viri, a Cambridge University Court on all matters related to University staff. She was appointed for the period 1958–60, the first woman to be part of this body.

Professor Strathern has kindly sent me the following comment on the period when Mary Cartwright was Mistress of Girton College:

> Mary Cartwright has been Girton’s longest-serving Mistress. Two important events in the history of the College framed the period between 1949 and 1968. First, she came into office at the very moment when Girton and Newnham had finally been incorporated in the University and women were thus entitled to receive full University degrees. This meant several changes to the way College was run: the new Mistress was dealing with a new situation. Second, the last few years of her Mistress-ship, right up until the eve of Girton’s centenary in 1969, were marked by a vigorous campaign to increase the number of places available to women at Cambridge. An ambitious one hundred further rooms were planned at what is now Wolfson Court—but not until the building was practically under way did the news come that three of the men’s colleges (the first to do so) were opening up places to women. It so happened, then, that she retired at the moment when mixed colleges were on the horizon, although it was another eight years before Girton decided to admit men.
>
> The importance of her election in 1949 cannot be over-estimated. There was widespread interest in who was to be the new Mistress, and general

satisfaction when such a distinguished academic (it was soon after she had become a Fellow of the Royal Society) was elected—College, in its new status, had got off on the right footing. She in turn welcomed involvement in Faculty and University affairs, from ceremony (she declared she positively enjoyed being in processions) to administration (she served, and chaired, her Faculty Board and was on the University Council of the Senate for many years). In Girton she was known for the intimate detail with which she attended to all aspects of College affairs—'If the detail was right, the rest would follow'—and for her constant presence. College was her home, as indeed it was for many of the fellowship at the time. But more than that, in the words of one undergraduate, 'She was always there'. Fellows, staff and students could always find her when they needed. Yet she was a demon for work. She knew how to use the precious early morning hours. When asked by a research student how she managed to do so much, she replied, 'Well, you see, I don't go to chapel' [in the mornings; she was a regular attender on Sundays].

Not herself a great innovator, in College she was an active supporter of other people's innovations. Under her, for example, Girton science expanded, and Girton was a principal supporter of the new Department of Clinical Veterinary Medicine. She had a reputation for listening, and for always being ready to look ahead and respond to new needs. This last phrase is from Dr Helen Megaw's appreciation of her on retirement: in the context of the changes afoot in the 1960s, including greater student participation in college and university government, it was to Girton's great advantage that her 'alertness and sensitivity' was as marked at the end of her tenure as it had been at the beginning. As it was observed at the time, she had provided College with a long period of quiet, unassuming, clear-headed, and we may add shrewd, leadership.

Her portrait by Stanley Spencer hangs in the College Hall.

Travels and retirement

Mary was an intrepid traveller. In 1956 she was a member of the Royal Society delegation which visited the USSR as guests of the Academy of Sciences. After the delegation left she was the guest of Moscow University, visited the Polish Academy of Sciences at Warsaw and Cracow and returned to Moscow for the third All Union Mathematical Congress. She attended, as a delegate or by invitation, conferences at Helsinki, Vienna, Dresden and Naples, and in 1960 lectured at Bologna under the British Council exchange scheme. Her Nigerian research students arranged for Mary to be the guest of the University College of Ibadan at the independence celebrations at Lagos and Ibadan in 1960. The bonds of friendship created during these trips benefited others as well as herself. She fixed up a year in Brown University for me in 1949–50, and I found myself visiting Warsaw and Cracow a little after her. She certainly did a lot to create a more international outlook among British mathematicians.

In 1968 Mary retired from her Readership and from being Mistress of Girton. She spent most of the next three years travelling in American universities. In 1968–69 she was at Brown as the guest of Joe La Salle. Here she was given an honorary Doctorate and walked in the procession with Duke Ellington. There was a threat of protest by the military band against Henry Kissinger, who was also getting a degree.

Next autumn saw her at the Claremont Colleges and Case Western Reserve Universities, then three months as Royal Society Professor in Poland followed by a visit to the University of Wales. There were riots at Kent State University about the Vietnam War in the Lent term of 1969. Three students were shot by the National

Guard and the University was closed for a time. Mary spent a Christmas at Mexico City and asked to see the University. She says that she found an inscription 'happy repressions, long live the dictator (if my Spanish is good)'.

In 1971 Mary bought the flat in 38 Sherlock Close where she lived for the next twenty years. Here, although increasingly frail in body, she continued to do research. She came to stay with us in Goathland briefly in the summers of 1986 and 1988, combining the visit with one to her niece, Lucy. She told us 'that she preferred the country even with your rather peculiar washing arrangements' to our more comfortable house in York. We were very glad to have her, since she always had something interesting to say, whether it was about Russian icons or Petra or the things she had seen on her Hellenic cruises. Her historical work brought up some problems on value distribution in angles that Littlewood raised in his notebooks and that led to some papers by Yang Le and myself. Thus she provided me with problems until the time of my retirement.

In 1982 Mary was staying with her brother, Frederick, and his wife, Sally. When she came home from a walk she was pulled over by a dog and cracked her hip, having brittle bones.

After a week in Bridgend Hospital she wanted to go back to Cambridge, so, accompanied by Fred and Sally, she went back by helicopter ambulance which landed in the grounds of a school near the Evelyn Hospital. Mary had to read the map. She made a good recovery, with no permanent ill effects.

On her last Mediterranean cruise, Mary was accompanied by Phyllis Smart. Unfortunately, she had a fall in her cabin and spent all the time in a hospital in Athens.

In December 1994 she fell in her flat and was taken to Addenbrooke's Hospital with a fracture. From there she retired to Midfield Lodge Nursing Home, where she stayed until her death. She was beginning to lose her sight and hearing. But even at the age of 96 the television documentary 'Our Brilliant Careers' captured the sharp sparkle of her wit, as did the party to celebrate her fifty years in the Royal Society a year later.

Personality

Mary Cartwright was a complex person with interests in Art and History as well as Mathematics. She had a strong sense of duty towards her family and friends, looking after her sister Jane and her husband when they became frail and ill. When retired and in her eighties she still looked after 'the old lady upstairs'.

Mary had a gift of talking to children and adults of all ages. When she stayed with us in the 1950s she made complicated origami models for my children. Suddenly she said to me, 'This is not how I usually spend my Sunday mornings'. She liked operas and plays, and told fascinating tales about the places she had visited. She also had a sense of fun and I remember her pushing big balloons up and down during the Harvard Congress in 1950.

She planned her day efficiently, and this is how she could combine so much administration as well as research. She liked walking and found it a help to thought. Mary liked to arrange her life so as to cause the minimum of trouble, and made her arrangements carefully and in good time. When she felt she might become a burden to people in Sherlock Close, she decided to move to the nursing home at Midfield Lodge, where a member of the family would visit her at least once a month. She always liked things to be right, and at Midfield she instructed one of the maids on the right place to put the knives and forks.

Mary was a person who combined distinction of achievement with a notable lack of self-importance. Thus she left strict instructions that there were to be no eulogies at her memorial service. However, her considerable reserve put a certain distance between her and those who talked with her. While a good deal of her life was spent coping with other people's problems, if she had problems of her own she kept them to herself.

Research

For the purpose of describing Dame Mary's research I have divided her work into eight categories.

1. *Summability* [**1**, **3**, **16**, **17**]. This group of early papers shows the width of Cartwright's interest. In [**1**] she identifies the half plane of summability of a Dirichlet series and this work is extended and deepened in [**3**]. In [**16**] the authors show that if a function

$$f(z) = \sum_{0}^{\infty} a_n z^n, \quad |z| < 1,$$

tends to a limit as $z \to 1$, and is not too large in an angle then f is Césaro or Hölder summable to some index. In [**17**], similar results are obtained under the assumption that the coefficients a_n are restricted.

2. *Cluster sets, limits and asymptotic values* [**24**, **28**–**30**, **34**, **43**, **50**, **64**, **67**, **77**, **91**]. Suppose that a function $f(z)$ is meromorphic, that is, analytic except for poles, when z lies in a domain D. How does $f(z)$ behave as z approaches either an individual point ξ of the frontier of D (behaviour in the small) or the frontier ∂D as a whole (behaviour in the large)?

The simplest case occurs when ∂D consists of a single point ξ which we may take to be the point at ∞ and then D is the open plane. The situation is similar when ξ is an isolated point of ∂D, so that f has an isolated essential singularity at ξ. In this case the following results hold.

(a) $f(z_n)$ approaches an arbitrary value as $z_n \to \xi$ through a suitable sequence (Weierstrass' theorem) and more strongly.

(b) $f(z)$ assumes every value with at most two exceptions infinitely often in every neighbourhood of ξ (Picard's theorem).

(c) If w is an omitted value, that is, one not assumed by $f(z)$ in some neighbourhood of ξ, then

$$f(z) \to w$$

as $z \to \xi$ along a suitable path Γ (Iversen's theorem). In this case w is called an asymptotic value at ξ and Γ is a corresponding asymptotic path.

In this area of work the author, in some cases with Collingwood, explores the question of how these three theorems can be extended to other domains D and non-isolated singularities ξ.

If the point ξ is an isolated point of the boundary ∂D then ξ is an isolated essential singularity and (a), (b), (c) above apply. The thinner ∂D is at ξ, the closer to this behaviour is the function f. Thus for instance the author proves in [**30**] that if ∂D has logarithmic capacity zero near ξ and w is an omitted value, then there exists a path Γ ending at a point ξ_0 of ∂D arbitrarily close to ξ, such that

$$f(z) \to w$$

as $z \to \xi_0$ along Γ.

In [**29**] she proves that if ∂D has linear measure zero and omits w near ξ then the set of asymptotic values near ξ either contains w or it contains a set A whose projection on a certain line through w includes a whole segment with endpoint w.

A systematic exposition of results obtained until then by Cartwright and Collingwood is contained in [**50**]. The results of Cartwright also feature prominently in the book by Collingwood and Lohwater ⟨**5**⟩.

In [**64**] the authors prove that if f is meromorphic in the unit disk and has radial limits on a subset of an arc A which has second category, that is, is not a countable union of non-dense sets, then the set of corresponding radial limit values has positive linear measure unless f is constant. This significantly extends an earlier theorem of Privalov and has applications to conformal mapping.

In the papers [**24**], [**67**] and [**77**] the author is concerned with generalisations of a theorem of Montel. If f is bounded and analytic in a strip

$$S : z = x + iy, \quad c < y < d,$$

and $f \to w$ as $z \to \infty$ on a curve Γ in S, then

$$f \to w \text{ uniformly as } z \to \infty \text{ for } c + \varepsilon < y < d - \varepsilon.$$

For instance in [**24**] she shows that Γ can be replaced by a suitably thick set. In [**77**] she considers when an extension to harmonic functions f is possible.

3. *Level curves of integral and meromorphic functions* [**33**, **35**]. Let f be an entire or meromorphic function in the plane and let Γ be a curve on which $|f|$ is constant. Such a curve is called a level curve. Γ may be closed or open; in the latter case Γ goes to ∞ in both directions.

In [**33**] the author gives a simple proof of a result of Valiron which gives conditions under which two functions may have a common closed level curve Γ. At the same time she extends Valiron's result to meromorphic functions. In [**35**] she obtains some results in the much more difficult case when Γ is open.

This latter work was later taken up and extended by Brannan, Fuchs, Hayman and Kuran ⟨**2**⟩.

4. *Functions in the unit disk* [**2**, **9**, **15**, **20**, **26**, **57**]. In the papers [**15**] and [**20**] the author discusses relations between the growth of a harmonic function u and its conjugate v, so that $f = u + iv$ is analytic in the unit disk $\Delta : |z| < 1$.

If u is positive or bounded below we have Carathéodory's classical inequality

$$|f| < C(1 - r)^{-1}, \tag{1}$$

for $|z| = r$ where C is a constant.

The author shows that the same conclusion follows from

$$v > -K(1 - r)^{-\alpha} \quad \text{where } 0 \leqslant \alpha < 1.$$

This result was extended by Hayman and Korenblum ⟨**7**⟩ who showed that a condition of the form

$$v > -k(r), \quad |z| = r,$$

implies (1) if and only if

$$\int_0^1 \sqrt{\frac{k(r)}{1-r}} dr < \infty.$$

In [**20**] Cartwright explores similarly the relations between the means of u and v or f. However, the star among this group of papers is [**26**].

A function $f = \sum_0^\infty a_n z^n$ is said to be univalent in Δ if the equation $f = w$ never has more than one root in Δ. For such functions the inequality

$$|f - a_0| \leqslant |a_1| \frac{r}{(1-r)^2}, \quad |z| = r,$$

was classical. It was natural to ask whether one could obtain corresponding results for p-valent functions, that is, those for which the equation $f(z) = w$ never has more than p roots. The conjecture was that in this case

$$|f| < C(1-r)^{-2p}, \tag{2}$$

where C is a constant depending on the function. The example $f(z) = \{z/(1-z)^2\}^p$ shows that this would be sharp.

Some partial results were obtained earlier but the full result (2) was proved in [**20**] by Cartwright, 'by methods differing fundamentally from those of previous writers on the subject' as she rightly says in her paper. She used the new method of conformal mapping which Ahlfors introduced in his thesis to solve an entirely different problem, namely to show that an entire function of order ρ can have at most 2ρ asymptotic values. Here Cartwright's expertise on entire functions played a key role. She also used the Ahlfors distortion theorem to show that if the equation $f(z) = w_n$ has at most p roots for a sequence w_n, such that

$$w_n \to \infty \quad \text{and} \quad \left|\frac{w_{n+1}}{w_n}\right| \to 1 \quad \text{as } n \to \infty$$

then

$$|f| = O(1-r)^{-2(p+1)+\varepsilon} \tag{3}$$

for every positive ε. These results were taken up by a number of writers. Spencer ⟨**13**⟩ generalised (1) to mean p-valent functions. Biernacki ⟨**3**⟩ obtained the analogous bounds

$$|a_n| = O(n^{2p-1})$$

for the coefficients a_n of p-valent functions. Baernstein and Rochberg ⟨**1**⟩ obtained the corresponding bounds to (3) for the coefficients

$$|a_n| = O(n^{-2p-1-e}) \tag{4}$$

under the hypotheses that led to (3). I was able ⟨**6**⟩ to get rid of ε in (3) under the more stringent assumption

$$\sum_1^\infty \left\{\log\left|\frac{w_{n+1}}{w_n}\right|\right\}^2 < \infty$$

but the question of extending (4) similarly remains open. With this paper the author essentially created a new field. It was almost the only paper quoted by Littlewood in his book ⟨**11**⟩ and led me to ask Mary Cartwright to become my research supervisor.

5. *Entire functions* [**4–8**, **10–14**, **18**, **21–23**, **25**, **27**, **31**, **32**, **56**, **70**]. This is the principal area in which the author worked in the 1930s, and she established a considerable reputation here. Much of the work is concerned with entire functions $f(z)$ of exponential type, that is, those for which

$$\log|f(z)| < A|z| \tag{5}$$

for all large z, where A is some constant. Of particular interest is the behaviour of the zeros of f under various additional assumptions. For instance the author proves [**21**, **25**] that if in addition

$$\int_{-\infty}^{+\infty} \frac{\log^+|f(x)|dx}{1+x^2} < \infty \tag{6}$$

then the number $n(r, f)$ of zeros of f in $|z| < r$ satisfies

$$n(r, f) \sim Br$$

where B is another constant, and if $z_n = r_n e^{i\theta_n}$ are these zeros then

$$\sum \frac{|\sin\theta_n|}{r_n} < \infty.$$

Thus most of the zeros lie near the real axis. Priority is hard to establish, since for instance N. Levinson ⟨**10**⟩ was proving similar results at much the same time. Russian mathematicians and others (see for example Paul Koosis ⟨**8**⟩) call the class of functions satisfying (5) and (6) the 'Cartwright class'.

Perhaps the finest, and certainly the most quoted result of Cartwright in this area is the following.

Theorem ([**27**]). *If $f(z)$ satisfies* (1) *with $A < \pi$ and further $|f(n)| < M$ for all integers n, then*

$$|f(x)| < kM$$

for real x, where the constant k depends only on A.

The theorem has applications in signal processing. It fails for $A = \pi$, as the example $f(z) = \sin(\pi z)$ shows.

Cartwright put together her thoughts on entire functions in the *Cambridge Tract* [**56**]. It is typical of her that the above beautiful theorem is barely mentioned. On the other hand, P. Boas ⟨**4**⟩ quotes ten of the eighteen papers written by Cartwright before his book came out and proves many of her results. These results also feature prominently in B. Ya Levin ⟨**9**⟩.

6. *Ordinary differential equations* [**19**, **37–42**, **45**, **46**, **48**, **49**, **51**, **53**, **55**, **58**, **60**, **61**, **63**, **66**, **68**, **69**, **72**, **75**, **76**, **78–81**, **84**, **85**, **87–90**, **93**, **94**, **101**, **102**]. Cartwright wrote thirty-eight papers in this area and it includes perhaps her most significant work. In the *Times* obituary her joint work with Littlewood was rightly described as ground-breaking work, making an important contribution to chaos theory. Caroline Series in her *Guardian* obituary wrote:

'Smale embarked on a detailed study of these papers and eventually arrived at his masterly abstraction of the phenomenon causing the strange behaviour. The abstraction was the horseshoe map, the driving mechanism and corner stone of the whole modern theory of chaos'. Smale (see for example ⟨**12**, p. 149⟩) could locate the horseshoe in the Van der Pol equation. Cartwright and Littlewood's fine structures were typical manifestations of what is now known as 'the butterfly effect'. The paper [**37**] with Littlewood is Cartwright's most frequently quoted paper, with seventy-five references in the citation index.

In January 1938 the Radio Research Board of the Department of Scientific and Industrial Research issued a memorandum requesting the really expert guidance of pure mathematicians with certain types of non-linear differential equations involved in the technique of radio engineering. Littlewood and Cartwright in particular became interested in an equation of Van der Pol:

$$\ddot{x} - k(1 - x^2)x + x = 0 \tag{7}$$

or

$$\ddot{x} - k(1 - x^2)\dot{x} + x = bk\cos\lambda t, \tag{8}$$

which arose in connection with thermionic valves. They succeeded in showing [**49**] that when k is large all solutions of (7) except the trivial solution $x = 0$ converge to a periodic solution whose amplitude tends to 2 as $k \to \infty$.

Most of the early research of Cartwright and Littlewood concentrated on equations such as (8) with two or more stable solutions. Their results appeared in [**38**].

Cartwright, in a letter dated 31 August 1993 to Freeman Dyson, wrote:

> Littlewood was always changing his mind and very slow. So I asked him to let me write up, or rather complete, sundry bits that we had done earlier. Littlewood said 'Yes', but on condition that I did not call it a joint paper, but said 'based on joint work', and so I did, except that I made him let me call a fixed point theorem ([**47**]) joint. He agreed, provided that someone approved by him read it. H. D. Ursell did.

Littlewood wrote:

> We had a long, highly fruitful and harmonious collaboration. Miss Cartwright is the only woman in my life to whom I have written twice in one day. If I may for once suppress my well-known modesty I will say that the joint papers are good.

She also described their collaboration:

> We communicate mainly by letter or postcard or in brief encounters out of doors, occasionally by long walks or telephone but hardly ever sitting indoors, seldom in his room, never in mine, never on a blackboard. Several important points were settled at accidental meetings out of doors. I remember drawing curves with my finger on the stone at the back of the Guildhall and meeting near Girton College when he was returning or starting one of his favourite walks from the bus at the corner.

Professor Wilfred Kaplan has kindly supplied the following more detailed assessment:

> In the short paper [**89**] Cartwright tells of being inspired to work on nonlinear oscillations by a January 1938 memorandum of a Radio Research Board of the British government. This led to several papers with J. E. Littlewood, of which the first [**37**] considered forced oscillations associated with the famous Van der Pol equation, a nonlinear second order equation. The paper studies the Poincaré map T associated with the equation, a powerful tool exploited in many other Cartwright papers (and widely used in research on dynamical systems). For the particular case considered here, the authors show that for certain sufficiently small choices of the amplitude of the sinusoidal forcing term

the map T has a pathological invariant set of zero area separating the plane; this result realizes by a concrete differential equation a phenomenon shown by G. D. Birkhoff to occur for certain analytic transformations of the plane.

In a sequel [**38**], the same authors modify the equation of the first paper in significant ways and show the existence of a single periodic trajectory which is approached by all trajectories as the 'time' parameter t becomes infinite. In a third paper [**41**] the problem of the first paper is considered again and it is shown that, for suitable parameter values, a famous approximate solution using 'slowly varying' functions is indeed a good approximation of the (unique) periodic solution.

In a number of other papers [**40**, **45**, **46**, **49**, **88**, **90**, **94**, **102**] Cartwright (in several cases with a co-author) obtains simiilar results for related second order nonlinear differential equations. These results include conditions that solutions be ultimately bounded in the phase plane and asymptotic formulas associated with the original Van der Pol equation. In the two papers [**60**] and [**55**] Lyapunov functions are applied to some nonlinear equations of third and fourth order to show that, under appropriate hypotheses, solutions approach the origin of phase space as t becomes infinite.

In a series of papers published in the 1960s, Cartwright embarked in a new direction: the determination of conditions which produce almost periodic solutions of nonlinear differential equations (see for example [**63**, **66**, **68**, **69**, **76**, **78**]). In the first of these papers a second order equation with periodic forcing term is considered and the almost periodic solutions are related to minimal sets under the Poincaré map; in the second paper a purely topological form of the relationship is obtained. The third paper strengthens the results of the first two. The last three papers extend the theory to flows on n-dimensional sets.

Cumulatively the research in these papers provides a wealth of detailed information about a large class of problems and demonstrates how hard analysis and a certain amount of topology can be applied to give insight into the behavior of very complicated systems.

7. *Topology* [**44**, **47**, **59**, **74**]. Cartwright and Littlewood's most significant work in this area was motivated by their study of differential equations. I would like to quote the following result [**47**, p. 3].

THEOREM A. *If τ is a* (1,1) *continuous and orientation preserving transformation of the whole plane onto itself which leaves a bounded continuum I invariant, and if the complement $C(I)$ of I is a single simply-connected domain, then I contains a fixed point.*

This theorem is well adapted to applications in differential equations. We note that I need not be a cell as in Brouwer's fixed point theorem but on the other hand the transformation τ needs to be defined not just on I but in the whole closed plane.

8. *History, biography and general* [**36**, **52**, **54**, **62**, **65**, **71**, **73**, **82**, **83**, **86**, **92**, **95**–**100**, **103**]. Cartwright was one of the last great classical analysts and it naturally fell to her to write obituaries. She brought out well the quality of her subjects as mathematicians and as administrators. She took immense trouble over this work. Reading her biography of Titchmarsh [**71**] or Hadamard [**73**] one is struck by her mastery of all the different branches of research these men engaged in although they were in many cases far removed from her own interest. In describing the life of her collaborator Collingwood [**86**] she brings out how Collingwood's famous name enhanced his ability to serve the community. She wrote:

> His was a life of service, for the most part voluntary service and enjoyed to the full. Although some may envy him for having been able to avoid the grilling

> work for schedules A and B of Part II of the Mathematical Tripos and yet have the opportunity to do research and for escaping the drudgery of years of undergraduate teaching, there is no doubt that mathematics, science and the whole country would have been the poorer without the service that his unusual circumstances enabled him to give to them. It does not seem possible that modern conditions will ever produce his like again.

From my own conversations with her I feel that Cartwright's admiration for Collingwood as an aristocrat and public servant were not quite matched by her regard for him as a mathematician. She was fully aware that here Collingwood could not compare with Titchmarsh, let alone Hadamard. On the personal level her approach to people was factual. She did not delve into their souls any more than she delved into her own. Whether she or her subjects had romantic episodes in their lives is something we shall never know.

Mary continued her research and her writings on general subjects into her 80's. Her long life and personal knowledge of the people in question make her discussions [**97**, **98**, **100**] of the manuscripts of Hardy, Littlewood, M. Riesz and Titchmarsh fascinating and important historical documents.

Honours

Mary Cartwright's achievements were increasingly recognised from the 1930s onwards. She received honorary doctorates from the Universities of Edinburgh, Leeds, Hull, Wales, Oxford and Brown University, Providence, RI, USA. She received the Sylvester Medal of the Royal Society in 1969 and served on the Council of the Society from 1955 to 1957. She was President of the London Mathematical Society from 1961 to 1963 and received its highest honour, the de Morgan Medal, in 1968. She was an honorary FIMA and FRSE. Her interest in education was recognised by her Presidency of the Mathematical Association in 1952. She was made a Dame Commander of the Order of the British Empire in 1969. After Queen Margrethe of Denmark (then Crown Princess) was a student at Girton, Dame Mary was made a Commander of the Order of the Dannebrog.

Finally I would like to quote from the excellent biography by Caroline Series in the Guardian:

> I was once present at a heated debate among some very eminent mathematicians about which women would have been worth a chair at one of the top US mathematics departments. Emmy Noether? Yes. Sonya Kovalevskaya? Perhaps. The only other name on the table was Cartwright's.
>
> Generations of other women have been inspired by Mary Cartwright's achievements.

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