

GUIDO CASTELNUOVO

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Guido Castelnuovo, President of the Accademia Nazionale dei Lincei, and senior honorary member of the London Mathematical Society, died in Rome on April 27, 1952, in his eighty-seventh year. He will always be remembered as one of that distinguished band of geometers who began the great series of researches into geometry on an algebraic surface which is so closely identified with the Italian school of mathematics.

Castelnuovo was born in Venice on August 14, 1865, the son of Enrico Castelnuovo, director of the Scuola Superiore di Commercio, who also achieved some fame as an author. He attended the Liceo Foscarini in Venice, and later studied at the University of Padua, under the famous geometer Giuseppe Veronese. He graduated in 1886, and in the next year he was appointed assistant to Enrico d'Ovidio at Turin, a post he held until 1891. Corrado Segre was also a professor at Turin, and between him and the young Castelnuovo a close friendship developed, which had a great influence on the development of the young man's genius.

Before his appointment to Turin, Castelnuovo had already published a number of papers on geometry in the *Atti del R. Istituto Veneto*. The first, written in 1885, dealt with a problem in metrical geometry, and the others dealt with properties of rational curves, plane involutions, and some particular congruences of lines in space of four dimensions. But as soon as he moved to Turin the influence of Segre became obvious in his work. The two topics which at this time dominated the thoughts of the geometrical school at Turin were the geometry of hyperspace, to which Segre had contributed so much, and geometry on a curve as developed in the famous memoir of Brill and Nöther. Castelnuovo began to work on a fusion of these ideas, partly because this gave a suitable method of expressing the invariance of the Brill-Nöther ideas under birational transformations, and partly because it provided a powerful technique for solving problems in the theory of curves. The papers which Castelnuovo began to write on his arrival in Turin reveal his mastery of the technique, and his great ability in adapting it to solve many complex problems in the geometry on a curve, such as, for instance, the determination of the number of sets of $q+r$ points lying on a curve of genus p which are common to a g_n^q and a g_n^r on it. The early papers of this period were, perhaps, mainly devoted to the solution of difficult special problems, but soon Castelnuovo began to advance the bounds of geometrical discovery. His papers on linear systems of curves in a plane, which belong to the Turin period, while of considerable intrinsic importance, mark an essential step towards the work which he and his colleagues in Italy were to do later in initiating the researches on algebraic

surfaces which have had such a profound influence in all subsequent geometrical research, and in 1891 he wrote two papers in the *Rendiconti del R. Istituto Lombardo* (2), 24, on geometry on an algebraic surface, the first papers written in Italy on a subject in which Italian mathematicians were to make outstanding contributions for many years, indeed to the present time.

By 1891 Castelnuovo had published nineteen papers, a remarkable output by any standard for a man of twenty-six, but when their quality is taken into account one cannot be at all surprised that at this early age he was appointed to the chair of analytic and projective geometry at Rome, which he was to hold until he retired in 1935 on reaching the age limit. The leading mathematicians at Rome at the time of his appointment were Cremona, Cerruti, and Tonelli; and Beltrami was also appointed a professor in Rome in 1891. Castelnuovo soon made his mark in his new post; we shall refer to his scientific work below, but his concern with teaching was also very great, and shortly after his arrival he carried out extensive reforms in the teaching of geometry at the University of Rome. His reforms included the fusion of analytical and geometrical methods in teaching geometry; his well-known treatise on analytical geometry helped to modernize geometrical teaching over a wider area.

Castelnuovo's first years at Rome coincide with the beginning of that most intensive and fruitful period in the history of algebraic geometry during which the theory of the invariants of algebraic surfaces took shape in Italy. We have seen how Castelnuovo had already begun writing on this subject while at Turin, and much of this and later work arose out of discussions between him and Segre. About the same time, Federigo Enriques, six years Castelnuovo's junior, began to make his mark in the same field. He and Castelnuovo had many discussions on geometry in Rome, which resulted in a very successful collaboration lasting many years. Enriques went to Bologna in 1894, becoming a full professor two years later, but their association continued throughout the whole of Castelnuovo's productive period, though by the time Enriques returned to Rome as professor in 1922 Castelnuovo's work on algebraic surfaces was over.

The names of Castelnuovo and Enriques will always be linked in the history of the theory of surfaces. Each brought his own gifts to the partnership, and although the style and methods of thought of the two, as revealed by their separate papers, are very different (and, as I understand from those who knew both intimately, their personalities were very different) their partnership was one of the happiest examples of collaboration in mathematics; so much so that it is better to wonder at the perfect teamwork than to try to analyse the contributions of each to their joint papers. Castelnuovo's work is always marked with great breadth of vision, and brilliant clarity of thought. The intuitive grasp of a situation in geometry, and the extraordinary feel for the right method of approach to a problem,

which is so characteristic of this part of Italian mathematics, is evident in all he wrote. He strove continually for perfection, and was reluctant to publish anything which did not seem to come up to the high standard he demanded of himself; this in part accounts for the relatively short list of his published papers.

The third great master of this most productive period in the theory of surfaces, Francesco Severi (still happily with us) was considerably younger than Castelnuovo or Enriques, and only came on the scene in the early years of the twentieth century.

Although the two notes by Castelnuovo already referred to represent the first contributions by Italian geometers to the theory of algebraic surfaces, they deal with special problems—the first with surfaces containing an irrational pencil of curves, and the second with an inequality connecting some of the genera of an algebraic surface—and the first paper which makes systematic use of linear systems of curves on a surface to lay the foundations of a general theory of surfaces was written by Enriques in 1893. But Castelnuovo was quickly caught up in the general advance. It would take a volume to analyse the contributions of each member of the team, and in the space at my disposal I can only refer briefly to a few of Castelnuovo's major contributions.

Castelnuovo's main contributions to the general theory of surfaces were embodied in two long and fundamental memoirs, "Alcuni risultati sui sistemi lineari di curve appartenenti ad una superficie algebrica" [*Mem. Soc. Italiana detta dei XI* (3), 10 (1896)], and "Alcune proprietà fondamentali dei sistemi lineari di curve sopra una superficie algebrica" [*Annali di Mat.* (2), 25 (1897)], and a joint paper written with Enriques published in the *Annali di Matematica* in 1901. In these papers, Castelnuovo is concerned mainly with the problem of determining the deficiency of the characteristic series of a complete linear system of curves on a surface, and with the deficiency of the series cut on a curve of the system by the adjoint system. For both series, he established the result that the deficiency did not exceed the irregularity $p_g - p_n$, and from the result for the characteristic series he deduced the first proof of the Riemann-Roch theorem for surfaces. In the first paper quoted, the Riemann-Roch theorem is proved only for regular surfaces, but the proof for irregular surfaces is given in the second. Many years later, Castelnuovo explained that the general theorem was originally intended to appear in the 1896 paper, but the restriction to the case $p_g = p_n$ was made in the course of the proof-reading, owing to the author's doubt about one part of the proof. The general result was, in fact, announced in 1896 in a joint paper by Castelnuovo and Enriques in the *Mathematische Annalen*, which gives, without proofs, a masterly exposition of the whole theory of surfaces at the time of publication. Other proofs of the Riemann-Roch theorem for surfaces have been given subsequently, the proof with the widest validity being given in 1905 by F. Severi.

Another branch of the general theory of surfaces to which Castelnuovo made important contributions concerned the relation between the Italian theory of surfaces and the researches of the French school, headed by Picard and Humbert, into the theory of algebraic functions of two complex variables. The work of the French school had been greatly concerned with surfaces on which there exist exact simple algebraic integrals without singularities (Picard integrals of the first kind), and it soon became obvious that there was a close connection between these surfaces and surfaces for which $p_g > p_a$. Very intensive research was undertaken by the Italian school to elucidate the connection. The problem was first tackled by Severi in 1904, and the solution was given in the following year in the theorem that the number of independent Picard integrals of the first kind is exactly $p_g - p_a$, by Castelnuovo and Severi, simultaneously but independently.

Besides the general theory of surfaces on which I have touched, there are innumerable special problems of great importance relating to surfaces which were investigated at the same time, and some of the great triumphs in this field stand to the credit of Castelnuovo. I refer to one or two of these. It had been known for a long time that a curve was rational if its genus was zero, and it was natural to hope that a similar result would be true for surfaces. It was clear, however, from a knowledge of ruled surfaces, that the condition $p_g = 0$ would not be sufficient, but it was reasonable to believe that the condition $p_g = p_a = 0$ would be sufficient for the rationality of a surface. In 1894 Castelnuovo embarked on the task of trying to prove this; in the course of his lengthy and thorough investigation he was led to see, however, that in order to prove rationality he had to assume that the bigenus P was also zero. The question then arose whether there did in fact exist surfaces for which $p_g = p_a = 0$, $P > 0$; this question was put by Castelnuovo to Enriques, who constructed the first known example of such a surface, the sextic surface having the edges of a tetrahedron as double lines. The necessary and sufficient condition for the rationality of a surface was given in a paper published by Castelnuovo in 1896.

A problem in some ways related to this is to decide whether an involution of order n in a plane is necessarily rational. The corresponding result for the straight line is usually known as Lüroth's theorem, and its proof is not difficult. On the other hand, it is now known that there are involutions in three-dimensional space which are not rational; hence the case of the plane is somewhat critical. In 1893, Castelnuovo gave the first proof of the rationality of the plane involution; the proof now usually given of this result is a modified version of the original one, based on the condition for the rationality of a surface, and is also due to Castelnuovo.

These represent just a few of Castelnuovo's many contributions to the theory of surfaces, but they will perhaps serve to indicate his position at the centre of a vast field of mathematical activity. Mention should also

be made of a number of invaluable descriptive articles on the theory of surfaces written jointly with Enriques. In addition to the one written in 1896 in the *Annalen*, there is a long appendix which they wrote to Vol. II of Picard and Simart's "Fonctions algébriques de deux variables", and the well-known Encyclopädie article. After 1906, Castelnuovo's contributions to algebraic geometry became rare—a paper in 1918 on the theory of curves, and one in 1921 on Abelian functions, are his only contributions after that date. He became interested in philosophical speculation, and he wrote two books, and a few papers, on the theory of probability and on relativity. It is interesting to note that his only paper published in this country is in the *Monthly Notices of the R. Astr. Soc.*, 91 (1931), and is on de Sitter's universe and the motion of nebulae.

Castelnuovo's retirement in 1935 coincided with his scientific jubilee, and the occasion was chosen to do him honour by presenting him with a gold medal. A volume of his selected papers was published, which contains a complete list of his published works (save for a book entitled "L'origini del calcolo infinitesimale nell'era moderna", written in 1938, and two short notes in the *Rend. Lincei* of 1949). In view of the frequency with which the name of Castelnuovo appears in the literature of algebraic geometry, it comes as quite a shock to find that he has written only about fifty papers in all, including those written jointly with Enriques. But one soon notices the absence of anything trivial, and nearly all his work is of outstanding importance. The "Memorie Scelte" make fascinating reading, both as mathematics and as prose, and they are greatly enriched by notes added by Castelnuovo himself, which do much to explain the origin of ideas developed in the papers, and their position in respect to the modern theory of surfaces.

The seventeen years which remained of Castelnuovo's life after his retirement were not years of quiet rest; indeed, they were full of activities which might have taxed the strength of a younger man. When Mussolini introduced racial discrimination into Italy in 1938, Castelnuovo organised a "Università segreta" for those excluded from the University by the new laws, and directed it until 1943. The degrees granted by his university were recognized by the Polytechnic School at Freiburg (Switzerland). During the period of the German occupation of Rome (1943-44), Castelnuovo was obliged to leave his home and go into hiding under an assumed name with friends. With the restoration of liberty in Italy, he was called on, in spite of his advanced age, to play his part in re-establishing the cultural life of Rome. He was elected President of the Accademia Nazionale dei Lincei, and took an active part in restoring the famous foundation to its former high position among the academies of the world. He was also one of the five persons elected a life-senator of Italy, and took his full share in the activities of the Senate of the new Republic of Italy. These public recognitions were as much a tribute to his nobility of character and the

integrity of his conduct throughout his long life as to his achievements as a mathematician.

In 1896 Castelnuovo married Elbina Enriques, the sister of his colleague and life-long friend. He leaves two sons and three daughters.

ARNOLD JOHANNES WILHELM SOMMERFELD

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Arnold Johannes Wilhelm Sommerfeld, Honorary Member of the Society, was born at Königsberg on 5 December, 1868, the son of a medical man, and was educated at the Gymnasium (where among his schoolfellows were Hermann Minkowski and Willy Wien) and University of that city. The University had long been famous for its Institute of Theoretical Physics, which had been founded by Franz Neumann (the veteran who, after having been left for dead on the battlefield of Ligny in 1815, died in his bed 80 years afterwards), and among Sommerfeld's teachers were Lindemann (who first proved the transcendence of π), Hurwitz, and Hilbert. In 1891 he obtained his doctor's degree: and in 1893 moved from Königsberg to Göttingen, where he came under the spell of Felix Klein.

Klein, who in the last years of the nineteenth century was the second greatest mathematician in the world (the first was Poincaré), was at this time much occupied in showing by example the value to dynamics, astronomy and physics of the new pure mathematics, especially the theory of functions of a complex variable. Sommerfeld, who became his assistant in 1894, was taken into partnership in a splendid piece of work of this character, namely the book *Ueber die Theorie des Kreisels*, of which the first two volumes were published in 1897-98*. But even before this, in 1896, Sommerfeld under Klein's influence had put forth some remarkable papers† on the mathematical theory of diffraction. He considered the two-dimensional problem (the light being emitted by a line parallel to the z -axis, and the screen being bounded by two parallels to the z -axis) and showed that it depended on integrating the equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + k^2 u = 0$$

on a two-sheeted Riemann surface. A rigorous solution was obtained in the form of a complex integral. The theory of the many-valued solutions of the partial differential equations of mathematical physics, which was created in these papers, was a very great advance.

* Leipzig, Teubner.

† *Math. Ann.*, 47, 317; *Proc. London Math. Soc.* (1), 28, 395.