THOMAS MACFARLAND CHERRY

E. R. LOVE

Professor Sir Thomas Cherry, F.A.A.,† F.R.S., died at his home in Melbourne on 21st November, 1966, at the age of 68. He was widely known and highly respected as Australia's most distinguished mathematician and a leader in university affairs. He was associated with the University of Melbourne for most of his life, and latterly with La Trobe University as first chairman of its Academic Planning Board. He was a foundation member and a president of both the Australian Academy of Science and the Australian Mathematical Society. His greatest contributions to knowledge were probably made in the mathematics of air flow in trans-sonic flight, simultaneously with Lighthill in Britain; but he also made major contributions to global differential equation theory and general dynamics, and solved some difficult special problems in various branches of applied mathematics. He was a most distinguished teacher, amongst whose students are numbered two Fellows of the Royal Society and several professors in Australian and overseas universities. He was a man of wide interests and great ability, of keen insight and broad vision. He knew much more than he ever wrote, and his influence will live on in the minds of innumerable people with whom he worked.

He leaves a widow, Lady Olive Cherry, and a daughter, Jill, Mrs. J. D. Stowell of Newcastle, N.S.W. To both of them I accord my warmest thanks for much kind help in many matters connected with the writing of this article. I also thank his secretary, Miss Shirley Flinn, for her valuable assistance.

For help in preparing the research portion of this article I am grateful to W. A. Coppel, J. W. Craggs, D. Elliott, A. R. Jones and W. W. Wood. I hope that the account presented is both accurate and balanced. For any departure from these aims I must accept responsibility. I also thank C.S.I.R.O. (Australia) for permission to comment on some reports sponsored by that organization.

It is expected that articles describing Professor Cherry's life and work will also appear in the papers of The Royal Society of London and in the *Journal of the Australian Mathematical Society*. Articles of a more general nature about his life are also expected to appear in the papers of the Australian Academy of Science, in the *Australian Journal of Science* and in the *Australian Mathematics Teacher*.

Thomas MacFarland Cherry was born at Glen Iris, Melbourne, on 21st May, 1898, the second son of Thomas and Edith Cherry. His mother was a Melbourne graduate in classics, daughter of Dr. Gladman who was the first principal of the

† F.A.A. stands for : " Fellow of the Australian Academy of Science."

Melbourne Teachers' College. His father, Dr. Cherry, was a bacteriologist of distinction and a most versatile man, interested in matters ranging from classics to tuberculin-free milk. He held the Melbourne degrees of M.D., M.S.; he became State Director of Agriculture, and then in 1911 first Professor of Agriculture in the University of Melbourne. He served as a Major in the R.A.M.C. in Egypt in World War One, discovering the mechanism of dysentery there; and from that time on actively carried on cancer research until his death in 1945 at the age of 82. Dr. Cherry's father, Edward Cherry, had come from England to the Victorian gold fields in 1855, then built up a family business manufacturing " Cherry churns " and other dairy appliances in Gisborne, of which shire he became a councillor. His grandson's last resting-place is beside him at Gisborne, among the English trees which as a councillor he had been instrumental in planting.

In his secondary schooling, at Scotch College, Melbourne, young Tom Cherry came under the scholarly and kindly eye of W. S. Littlejohn, who in after years often remarked on his special excellence. It was a foregone conclusion that his school career would culminate, as it did in 1914, in his being a prefect and dux of the school.

At the University of Melbourne from 1915 to 1917 Cherry was taught by E. J. Nanson, Professor of Mathematics, and J. H. Michell, F.R.S., a distinguished worker in elasticity and hydrodynamics who in 1923 succeeded Nanson. Cherry esteemed these men highly, but it must have been his own peerless intellect and unerring insight which brought him to graduate two months before his twentieth birthday, with first class honours and the Dixson and Wyselaskie Scholarships. In Ormond College, where he resided, he had a special regard for D. K. Picken, the Master, and C. E. Weatherburn, the Tutor in Mathematics. Picken tirelessly emphasized fundamentals, especially the laws of algebra now called the properties of fields, and Weatherburn staunchly advocated vector methods which have now become so central in mathematics. Cherry retained a small but lifelong interest in Ormond College, and so it was that the first Meeting of the Australian Mathematical Society was held in residence in Ormond in 1956.

After a sojourn in the armed forces in 1918 Cherry began a medical course; but he abandoned it when his godfather, Sir John MacFarland, then Chancellor of the University of Melbourne, lent him £150 per annum for three years to go to Cambridge for more mathematics. After one year he was a Wrangler with B*, and Trinity College made him a Senior Scholar and then Isaac Newton Student. He went on to research in celestial mechanics and statistical mechanics, apparently under the supervision of H. F. Baker and (to some extent) of J. E. Littlewood. By 1924 he had published five substantial papers and received the Smith's Prize, the Senior 1851 Exhibition, and the then rare degree of Ph.D. In the same year his college elected him into a fellowship, setting the seal on an association which was to be lifelong: whenever he was in England he would make time to visit Trinity. In reference to his fellowship the Master of Trinity, Sir J. J. Thomson, wrote: " He had previously been granted the degree of Ph.D. by the University, a distinction which is only given to those who have done original work of great merit and promise. I am of the opinion that Dr. Cherry is one of the most promising of the younger mathematicians, and likely to rise to great eminence."

In the four years of his fellowship Cherry wrote several papers on Hamiltonian systems of differential equations. The twin founts of his inspiration were Poincaré's "Les Méthodes Nouvelles de la Mécanique Céleste" and Whittaker's "Analytical Dynamics". During this time he spent three terms as Associate Professor of Applied Mathematics in Manchester, deputizing for E. A. Milne, and one term in a similar position in Edinburgh deputizing for C. G. Darwin. His conduct under these responsibilities supplemented the high reputation he was building by his research, and his teaching was even compared with that of the illustrious Sir Horace Lamb. In Edinburgh he was associated with Sir Edmund Whittaker, who was very impressed not only by his contributions to analytical dynamics but also by his skill and originality as a teacher and his good relations with students and staff.

Cherry's Cambridge years were full of other activities too. He played tennis for Trinity, and was for one year captain of the team. He held office in the Trinity Mathematical Society, the Cambridge Mathematical Club and the $\nabla^2 V$ Club. His contemporaries remember especially his keenness for scouting and mountaineering. After extensive experience on British mountains he not only climbed the Matterhorn, but also traversed the Pyrenees from end to end without a guide in winter. As Scoutmaster and later as Commissioner he worked with boys and men of both town and gown. At least one of his Cambridge friends acknowledges Tom Cherry as the origin of his lifelong interest in scouting. And in Edinburgh it is still said that he took lodgings close to Waverley Station so that at week-ends he could reach the mountains more speedily.

On the retirement of his former teacher J. H. Michell from the Melbourne chair at the end of 1928, Cherry returned to Melbourne as Professor of Mathematics, Pure and Mixed. Such was his official title; and it described him with rare aptness. Few mathematicians since Poincaré have been so universally well-informed and able to speak authoritatively about most branches of mathematics and a great deal else besides. For the next 35 years he devoted himself without reserve to the welfare of his Department, inspiring staff and students alike; and his unerring judgment, his breadth of vision, and his tremendous capacity for work soon made him the unquestioned leader of mathematics in Victoria, and ultimately a leader of science in Australia.

In the lecture room Cherry was terse, well-organized and often exciting. He developed his themes from the simplest starting points, and his teaching had a compelling common sense. Despite what he called the "vertical structure" of mathematics, he preferred "first principles" whenever practicable. At one time or another he taught every subject, pure and applied, at all levels in the undergraduate courses, and always with complete mastery. He would stand in for a sick member of staff with barely any notice, even when it involved three hours' consecutive lecturing. In the 1930s the stint was four courses simultaneously, for him and for his staff.

But he found this no burden, for he loved teaching and regarded it as his main responsibility. He was inclined to ascribe the success of his students to their undergraduate training at least as much as to his subsequent influence on them as research workers.

Cherry's attitude to his official responsibilities was ideal. He gave them all due attention, from homework marking to committees and from school examining to research; and he still found time to undertake extra tasks that colleagues in his own or other departments might ask of him. He seemed to regard nothing as a hindrance; everything was part of one great game which was supremely worth playing. Idleness was just not in his programme. He used to say, about teaching and administration, " the reward for work is—more work "; and about research, " genius is 10 per cent inspiration and 90 per cent perspiration ". As the years went on his own research became more and more a nocturnal affair, even though it was one of his greatest loves. He thought of research and teaching as complementary, not exclusive; both held top priority with him, and he encouraged his staff in both activities too. He once lectured to staff and senior students on the mechanism of research, and two memories of this stand out: " After hard thinking, ideas begin to come, perhaps at unexpected moments "; and, to the astonishment of some, " I'm happiest when I'm working ".

A year after leaving England he had made a brief return to marry Olive Ellen Wright, of Walkern, Hertfordshire. Scouts and Guides were among their common interests, but Tom and Olive were interested too in people of all kinds. Their hospitality was quite extraordinary. Whenever one visited them there would be someone staying with them; often a young man or woman making a start in a new country, and not infrequently a whole family. Sometimes the visitors were mathematicians, especially in later years, and Tom delighted in taking them to see the nearer mountains and the bush which he knew and loved so well. But more often they were young folk who could do with a little support and encouragement, and the Cherry home became for the time being their home. Besides this individual care that they gave so often though so discreetly, they were hospitable in a large but informal way, and a holiday with the Cherrys was often a community enterprise in which friends from various spheres enjoyed the stimulus of new company and climbed whatever mountains were within reach.

To promote informal contact between students and staff Cherry early founded the Melbourne University Mathematical Society. As he hoped, this soon came to be run mostly by students. Talks were given by visitors and staff members, and also by students themselves. Cherry's comments and questions at the end of a talk were always apt and well-informed, and often clarified things or led on to discussion. The meetings, for many years held in the evenings, were followed by informal conversation over tea and biscuits.

The Mathematical Association of Victoria soon made Cherry its president, an office he held from 1929 to 1934 and from 1946 to 1948. His real interest in the school teachers and their problems, and his hard work for the Association, gained not only their confidence but their enthusiasm and fullest allegiance, and eventually they made him one of their very few life members.

He was also on the University's Schools Board, and chairman of its Mathematics Standing Committee from 1929 to 1952, and was never far removed from syllabuses and question papers. (Such duties have been normal for many Australian professors until quite recently.) He was responsible for two major re-draftings of school syllabuses, for the suitability and correctness of about a dozen public examination question papers each year, and for the fairness with which they were marked. He did this mainly by supervising the work of teachers appointed to examinerships by the University. He also carried certain important overall responsibilities for the fairness of school examining in all subjects. He exercised these far-reaching powers with fair-mindedness and tolerance, and he regarded the welfare of the children as the thing of supreme importance.

Despite the vastness of his labours, Cherry actively kept up his interests in scouting, hiking and mountaineering. He instituted the Melbourne University Mountaineering Club, became its first President, and is remembered by "Cherry's Flake" as well as by generations of young men and women, mathematical and otherwise. As in Cambridge, he became a Scout Commissioner and Leader of the University Rovers, and for twenty years he also led the Rover Crew associated with his old school. These pursuits must have fostered in many the self-reliance and rugged independence that he himself possessed, but for him they were just hobbies and "relaxation".

It was only when one came to know him that one realized what great powers of leadership and force of character Cherry possessed. He had a mild manner, giving no impression of forcefulness; but he was never ruffled. His staff saw him as their adviser and, after a little while, friend; and junior staff came to regard him as the centre of their world. He helped them individually with his great reservoir of ideas, and educated them collectively by lectures on mathematical topics old and new, ranging from continued fractions to relativity and from integral equations to interplanetary flight. When Gödel's undecideability theorem appeared he gave it much attention and lectured on it even outside his department. His treatment of these topics was always illuminating, and often worthy of publication because of clarity or novelty. But he regarded his activities in these fields as little more than games compared with his researches in dynamical theory and compressible flow.

After his research achievements of the 1940s, Cherry became increasingly widely known. He became Sc.D. of Cambridge in 1950, received the Lyle Medal of the Australian National Research Council in 1951, and was made F.R.S. in 1954. He served the Australian Academy of Science, from its inception in 1954 until 1964, successively as council member, secretary and president; tasks which exacted tremendous toll of his time and energy. He was Foundation President of the Australian Mathematical Society from 1956 to 1958, President of ANZAAS Section A in 1958, and Foundation President of the Victorian Computer Society from 1961 to 1963.

Cherry was as superbly skilful at arithmetic as he was at mathematical analysis. He used to tell students that a little arithmetic "oils the works", and a numerical mathematics project was (and still is) included in courses. By 1950 he was spending vast amounts of time on the numerical stages of his research; this he saw simply as part of the task, and in some degree he enjoyed doing his own arithmetic and checking that of his helpers. But he could see what the future held, and he was reading increasingly about automatic computation. In 1956 the C.S.I.R.O. presented an electronic computer (from its Radiophysics Laboratory) to the University of Melbourne on extended loan. Cherry was the source of some of the modifications made when this computer was rebuilt in Melbourne by the staff of the newly formed Computation Laboratory, with the help of its original designer and builder, T. Pearcey. Called CSIRAC, it became the focus of his interests for several years. He became very knowledgeable about computation and supervised some of the developmental work, but wrote very little about it himself.

In these later years Cherry was honoured by other universities. On two occasions he visited the U.S.A. by invitation, in 1959 as a research associate at the California Institute of Technology, and in 1963 to deliver a lecture at a Symposium on Non-Linear Problems at the University of Delaware. Returning through London, he addressed the Royal Society. He also gave a few lectures of a more general character, on the occasions of honorary degrees and other such events. He was a past master at adapting his matter to suit his audience, and the manuscripts of his lectures, both mathematical and otherwise, are a mine of interesting material.

Cherry retired at the end of 1963. He became Professor Emeritus, and remained on in his former department as Senior Research Fellow during 1964. Late that year, as President of the Australian Academy of Science, he led a delegation of scientists to Peking at the invitation of the Academia Sinica. He was working with increasing intensity as chairman of the Academic Planning Board of La Trobe University, on whose Interim Council he had already served for some years. Nearly every evening he was busy at research until the small hours. The full life he had always lived continued unabated, and his worth was increasingly recognized. On the first day of 1965 he was made a Knight Bachelor.

Cherry still seemed much too active and robust to have retired; but not many weeks later he suffered a severe heart attack. With characteristic vigour he recovered rapidly and continued most of his activities. He spent the academic year 1965–66 at the University of Washington, Seattle, as a full-time staff member, giving regular courses of lectures. He was also working at problems in Hamiltonian systems, and discussing them with American leaders in differential equation theory. This led to a completed paper **[16]**, and to a sequel which he was writing later in 1966 after his return to Melbourne. He took up again his work for La Trobe University, which was to open in 1967. Unhappily he did not live to see the opening or to complete his last manuscript.

A great gathering of people from many walks of life assembled at the last, each with his own special memories of T.M.C. Among staff members it was agreed that "he didn't throw his weight around; he didn't have to". And many must have shared the thought expressed by a non-university man whom he had helped many years earlier, who said simply "He was my best friend".

Below I attempt to describe Cherry's published work under eight headings, including a miscellaneous section. The grouping is of course a little arbitrary, and some cross-threads connect several groups in the classification adopted. It would have been not unreasonable to group papers **17–20, 41** and **47** under a heading such as "analysis, untrammelled by applications", and papers **35, 44** and **45** under "natural philosophy"; but most of the papers contain both these extremes and much that lies between.

I have already acknowledged, on the first page of this article, the help of several colleagues in the Australian Mathematical Society in the descriptions of Cherry's research which follow. In particular, I am indebted to W. A. Coppel and A. R. Jones for some of the comment on differential equation theory; to D. Elliott for remarks on numerical aspects; to J. W. Craggs for the story of the hodograph method in compressible gas flow; and to W. W. Wood for observations on infinite linear systems of equations.

Ordinary Differential Equations

Cherry's earliest papers were about global behaviour of solutions of systems of non-linear differential equations, expressible vectorially in the form

$$\frac{d\mathbf{x}}{dt} = f(\mathbf{x}) \tag{1}$$

where $\mathbf{x}$ is unknown and f is a given analytic function from n-vectors to n-vectors. As a preliminary he discusses in [**1**] the analogous system of difference equations, establishing the general solution as an analytic function of certain arguments by use of a dominant series method. In [**2**] he presents "a new method of attack on a fundamental problem", avoiding the use of series which had bedevilled previous attempts. The background, here and elsewhere, is Poincaré's great treatise [**A**]. He sets out to show that, if the n-th order autonomous system (1) has a positive integral invariant and an n-dimensional compact invariant set Ω, then the solutions in Ω are quasi-periodic. Moreover the number of basic frequencies is less than n and, if the system is Hamiltonian, at most $\frac{1}{2}n$. Cherry admits that his investigation is open to criticism; but there is no doubt about the importance of the problem and the novelty of the method. Quite recently, further progress in this problem has been made by Moser [**B**, §7] and Cartwright [**C**].

In [3] Cherry constructs $n-1$ independent "integrals" of (1) near an ordinary point; an integral being a function of $\mathbf{x}$ which is constant along any solution curve. He shows further that in general there are no integrals developable in multiple power series about a singular point. Later, in [7], he began to exploit the exceptions to this, namely Hamiltonian systems. In [4] he gives his version of a proof of Poincaré's theorem on non-existence of "uniform" integrals (other than the energy integral) of Hamiltonian systems, but uses [3] to show, contrary to general opinion at that time, that there are integrals which fulfil all Poincaré's conditions except expansibility in powers of a certain parameter μ. In [5] he presents the view that the use of angular coordinates in the problem of three bodies leads to "small divisors" in the formal trigonometric series solutions, thereby introducing an artificial complication. However it seems that small divisors occur no matter what coordinates are used; and Arnol'd [Q] successfully uses angular coordinates.

It is clear that Cherry had a deep knowledge of this difficult subject, and that his powerful methods enabled him to open new lines of progress and even to constructively criticize Poincaré's work. It seems that he hoped that further study of integrals would lead to progress in the three body problem, and in [6] he made an extensive survey of global properties of solutions, as revealed by integrals, in eight particular systems of order 3 or 4. The last of these was reproduced by Whittaker [**D**, p. 412] and by Wintner [U, §136 *bis*]; it exhibits an unstable system which, when treated to the first order only, "becomes" stable. However, when Cherry returned to dynamical theory in later life, it was to problems akin to the quasi-periodicity of solutions, as in [2], rather than to integrals.

Hamiltonian Systems of Differential Equations

The problem in this group of papers is again to find the global behaviour of solutions, but equations (1) are now restricted to the Hamiltonian form, of order $2n$,

$$\frac{dx_r}{dt} = \frac{\partial H}{\partial y_r}, \quad \frac{dy_r}{dt} = -\frac{\partial H}{\partial x_r}, \quad (r = 1, 2, \ldots, n), \tag{2}$$

and considered at first near a singular point. Following the lead given by Whittaker's "adelphic integral" [E] and his own papers [3] and [4], Cherry constructs in [7] n analytic integrals of (2), and shows that there are no more than n independent such integrals. He fills an important gap left by Whittaker [E] concerning the vanishing of coefficients of critical terms, which Whittaker [**D**, p. 434] acknowledges. Birkhoff later gave an investigation [F] equivalent to [7], but not to [8].

An essential requirement in [7] is the absence of commensurability relations connecting the exponents $\lambda_1, \lambda_2, \ldots, \lambda_n$ at the singular point; that is, the absence of linear dependences

$$A_1 \lambda_1 + A_2 \lambda_2 + \ldots + A_n \lambda_n = 0 \tag{3}$$

in which the A_r are integers not all zero. In [8] Cherry extends his theory, showing that if there are exactly p linearly independent relations of commensurability (3) then there are exactly $n-p$ analytic integrals.

Among the great complications in [7] and [8], a major difficulty is that the formal series solutions which arise in constructing the integrals are probably divergent, although convergent if rearranged as multiple power series in $x_1, \ldots, x_n, y_1, \ldots, y_n$. This unsatisfactory feature leads on to [10] and [11], to which [9] is a preliminary.

In [9] Cherry justifies the assumption he makes in [7], [8] and [10] that the terms of lowest degree in the Hamiltonian H may be taken as $\lambda_1 x_1 y_1 + \lambda_2 x_2 y_2 + \ldots + \lambda_n x_n y_n$. This is essentially the congruence reduction of a real symmetric matrix by a linear contact transformation. He also prepares the ground for [11] by a similar reduction for the case in which H involves t and is periodic in it.

In [10] formal solutions of (2) near an equilibrium point are constructed, in the form of multiple power series, when there is no commensurability relation (3). As in [7] the difficulty of small divisors again comes in; and Siegel [G] showed, much later, that convergence is in a sense exceptional. Cherry, intuitively aware of this, nevertheless gave a proof of convergence in the case $n = 2$ and λ_1/λ_2 unreal. Many years later Moser [H] found, and filled, a serious gap in Cherry's proof.

Periodic solutions of (2) are studied in [11]; they have the advantage that the formal series specifying them can in certain cases be proved convergent. As in [7] and [8] Cherry is seeking to avoid certain obscurities in Poincaré's work. Here he aims to find *all* periodic solutions of *one* Hamiltonian system, whereas Poincaré worked with a Hamiltonian involving a small parameter μ and his theory furnished, for a given non-zero μ, only those periodic solutions whose period is not too large. The gigantic scale of the work, and the formidable nature of the problem, are indicated by the fact that, despite its eighty-odd pages the memoir limits some substantial parts of the discussion to an outline.

Cherry's investigation of periodic solutions goes further than that of Birkhoff in [F], in that commensurable exponents are considered in [11]. Again, Birkhoff starts by fixing the energy constant at a value chosen to reduce the order of the system; so he obtains only a cross-section of the families of periodic solutions considered in [11], and does not obtain the details of their branching. In this connexion Whittaker [D, p. 396] mentions with surprise Cherry's paradoxical conclusion [11, p. 216] that " for an arbitrary Hamiltonian system the periodic solutions are in general ' singular ' ". Indeed, Whittaker was evidently so impressed by Cherry's work that he ended his book with a footnote [D, p. 449] which seems almost to say that the continuation of the subject would be found in Cherry's papers.

Topological Dynamics

Ten years later Cherry wrote again [12] on systems of ordinary non-linear differential equations, in something like the same vein as in [2]. Topological ideas occur frequently in his earlier differential equation work, but here there is a more formalized topological approach, evidently due to the appearance of Birkhoff's book [F] meanwhile. The leading idea in [12] is to classify trajectories by relating them to their α- and ω-limit sets; and whereas Hadamard [J] and Birkhoff had recognized the importance of trajectories which are contained in both these limit sets, Cherry explores all the possible relations between a trajectory and its limit sets. Besides several additions to Birkhoff's theory, he shows that Birkhoff's classification of trajectories is not exhaustive, and gives examples to show that all the logical possibilities can actually occur.

A further example is given in [**13**], where Cherry constructs an analytic third order autonomous system with recurrent solutions of discontinuous type. The interesting point is that the trajectories have this pathological character whereas the differential equations are analytic; in fact the right sides of (1) are polynomials. Another discussion of such a phenomenon has since been given by Levinson [**K**].

The importance of this pioneering work by Birkhoff and Cherry can be seen in the subsequent growth of the subject, as witnessed, for instance, by the books of Gottschalk and Hedlund [**L**] and Nemytskii and Stepanov [**M**].

Later Dynamical Studies

Cherry's presidential address to the newly-formed Australian Mathematical Society in 1956 was published [**14**] as the opening paper in the Society's *Journal*. In it he recalls ideas and examples from [**12**] and [**13**], and the "small divisor problem" mentioned in [**7**], as a springboard for new thoughts on pathological global behaviour of the kind discussed in [**13**]. He uses the example of a rigid pendulum with pivot forced to vibrate vertically (step-wise, not harmonically) to demonstrate the possibility of existence of an integral F which "has continuations along its level surface $F = 0$ which are essentially singular at all points of this surface". He also discusses trajectories which are everywhere dense on an energy-hypersurface, called "transitive" or "quasi-ergodic" trajectories, with special reference to Artin's example [**N**]; there the well-known modular figure was used to exhibit transitive geodesics, and in particular to show that "the hyperbolic billiard table permits transitive shots".

The paper goes on to recent progress on small divisor problems. One item is Siegel's *tour de force* [**O**] on the convergence "in general" of formal series solutions of (1). Another is Moser's proof [**H**] of their convergence in the Hamiltonian case, when there are $\frac{1}{2}n$ relations of commensurability and the remaining exponents have unreal ratios. Finally Cherry gives support for his conjecture that when these remaining exponents are commensurable there may be some simpler way of expressing the solutions, perhaps by asymptotic expansions. The work of Arnol'd [**Q**], which had not then been translated, is not mentioned.

In [**15**] Cherry presses the view that the small divisor problem may be illuminated by studies of the analogous problem of iteration of analytic functions, that is, of sequences $\{z_n\}$ satisfying

$$z_{n+1} = f(z_n) \quad (n = 1, 2, 3, \ldots) \tag{4}$$

where f is regular at 0 and $f(0) = 0$. The "singular" case occurs when the attempt to reduce (4) to the linear form $w_{n+1} = \lambda w_n$ by formal substitution of

$$z = w + c_2 w^2 + c_3 w^3 + \ldots \tag{5}$$

leads to values of c_n for which (5) is divergent for all $w \neq 0$; and the object is to find under what conditions this case can arise. When $\lambda = e^{2\pi i \alpha}$ and α is real and irrational (5) is a small divisor series, but the singular case cannot occur unless α is "highly" transcendental. In the converse direction, assuming that f is a rational function

Cherry reaches conditions which secure that the iteration is singular, albeit through a labyrinth of detail much of which is only sketched.* This is an advance on the analogous dynamical problem, but he maintains that it is the global knowledge of f as a rational function that makes this advance possible.

In [**16**], his last completed paper, Cherry adds to the meagre stock of real analytic Hamiltonian systems which are known to be non-integrable, that is, whose only globally single-valued integral is the energy integral. Poincaré [A] had, by an incomplete argument, made it seem probable that Hamiltonian systems are in general non-integrable, although Moser [P] and Arnol'd [Q] have reduced this probability somewhat. Artin [N] exhibited actual non-integrable analytic systems, by showing that the geodesics on certain closed surfaces of negative curvature have a topological pattern which precludes integrability. The non-integrable system Cherry discusses has an additional feature, one of physical interest, namely a periodic solution with first order stability. The non-integrability is demonstrated by discussing two " asymptotic surfaces ", generated by trajectories which approach a certain unstable periodic solution as $t \to \pm\infty$; it is shown that the analytic continuations of these surfaces intersect but are not coincident, and that the consequent pattern of trajectories is topologically inconsistent with the existence of an adelphic integral. Cherry points out that Birkhoff [**R**, p. 460] abandoned his attempt to construct an analytic non-integrable system by modifying a C^{∞} non-integrable system. He also mentions allied work by Morse [S] and by Smale [T].

In his last months Cherry was writing a sequel to [**16**] entitled " Forced oscillations of a rigid pendulum ". He started from the same Hamiltonian system as in [**16**], which is effectively equivalent to any one of

$$\ddot{x} = g(t)\sin x, \quad \tfrac{1}{2}\dot{x} = F(t)\sin\tfrac{1}{2}x, \quad \dot{F} = (g-F^2)\cos\tfrac{1}{2}x, \tag{6}$$

F being defined from the given function g. In [**16**] the solutions asymptotic to the unstable periodic solution $x = 0$ (the upward vertical position) were studied over the half-revolution to $x = \pi$, but now he proposed to consider the solutions in general. The key to [**16**] was a function $f(t)$ defined by (compare the last of (6))

$$\dot{f} = g - f^2 \tag{7}$$

and expressible in terms of Jacobian elliptic functions. Instead of (7), the distinctive first step in the new investigation appears to be a " transformation to osculating variables ", namely

$$\sin\tfrac{1}{2}x = \mathrm{dn}(Kv, k), \quad \cos\tfrac{1}{2}x = -k\,\mathrm{sn}\,(Kv, k), \quad F = k\sqrt{g}\,\mathrm{cd}\,(Kv, k), \tag{8}$$

where K is the real quarter period of the elliptic functions. Much work follows, but apparently without reaching the conclusions he sought.

* Fuller details of this may possibly be written in his notebooks; it is likely that he studied this subject deeply over many years.

Fourier-type Expansion Theorems

During World War Two Cherry gave much attention to specific technological problems in electromagnetic and elastic contexts, and this greatly changed the character of much of his subsequent work. He was now solving partial differential equations by series or integral transformations, and the complications of existing expansion theorems spurred him to seek improvements. From here on his work was mostly classical analysis, especially complex function theory, carried on with remarkable ingenuity and complete rigour, in an applied context.

The theorems in **[17]** centre around the Hankel transform, arrived at by a method analogous to Cauchy's treatment of Fourier series. This method uses the solution $y = F(x, w)$, satisfying certain boundary conditions, of

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + (w^2 x^2 - v^2) y = wx^2 f(x), \tag{9}$$

where f is the function to be transformed. This solution $F(x, w)$ is expressed, by the method of variation of parameters, as an integral transform of f involving Bessel functions. The inverse transform is

$$f(x) = -\frac{1}{\pi i} \int_{ic-\infty}^{ic+\infty} F(x, w)\, dw, \tag{10}$$

as might be expected by deforming the path of integration to a large semi-circular contour and guessing from (9) that $F(x, w) \sim f(x)/w$ when w is large. Various boundary conditions with (9) give rise to various integral expansion theorems, including new ones as well as those of Hankel, Weber, Dini and Watson.

Cherry was especially pleased with the directness and power of this method. It is not surprising that it was also being exploited, on a more general level, by Titchmarsh at the same time. His book **[a]** appeared during the five years that Cherry's manuscript languished in the hands of printers struggling with wartime difficulties.

A similar method is used in **[18]** to obtain an expansion in terms of the parabolic cylinder function $D_v(z)$, where $z = xe^{\pm i\pi/4}$, $v = -\frac{1}{2} + i\mu$ and x and μ are real. It yields a companion to a well-known expansion in Hermite polynomials, useful in studying wave propagation in the presence of parabolic boundaries. The formula obtained had also been found by Magnus **[b]** under more restrictive conditions; and a related special result had been given by Erdélyi **[c]**.

Asymptotic Expansions

These papers were auxiliary to the calculation of trans-sonic gas flows achieved in the later 1940s. They provide usable approximations to functions whose character changes radically in the region of interest, which in the gas flow situation extends from subsonic to supersonic speeds. A typical instance is the Bessel function $J_v(vz)$, which can be said to have a "transition point" at $z = 1$: it changes from monotonic to oscillating when z is real and increases through $z = 1$, and this change is compressed into any small neighbourhood of $z = 1$ when v is made sufficiently large. Cherry discusses what he calls "uniform asymptotic formulae" for $J_v(vz)$ in **[19]**.

This paper, substantial as it is, turns out to be little more than an illustrative sketch to the next.

In **[20]** a full systematic theory of "uniform asymptotic formulae" is given for functions $F_\nu(z)$ satisfying a differential equation

$$\frac{d^2 y}{dz^2}+y\{-\nu^2 f(z)+g(z, \nu^{-2})\} = 0 \tag{11}$$

where f and g are known analytic functions. The problem of approximating $F_\nu(z)$ by elementary functions, when ν is large, *uniformly on a compact set* Z *independent of* ν, had been solved in particular cases by Debye and other earlier workers. Their treatments all supposed that Z contained no zero of $f(z)$; this corresponds in the gas flow context to wholly subsonic, or wholly supersonic, flow. In the case where Z contains a simple zero of $f(z)$, corresponding to gas flow which is subsonic in some regions and supersonic in others, it seems to be necessary to abandon elementary functions as the approximants, and to use some standard function satisfying another equation of the form of (11); the Airy function in particular. Watson **[d]** and Langer **[e, f]** succeeded in obtaining first approximations of this kind. Cherry obtains such approximations to arbitrarily high order, and his results can be expressed as an asymptotic expansion.

In **[21]** Cherry brings this work into the gas flow context, which involves hypergeometric functions with parameters all dependent on one large parameter ν. These functions $\chi_\nu(\tau)$, $\psi_\nu(\tau)$ are expressed by asymptotic formulae, both of the Debye and of the uniform kind, the latter involving Bessel functions of argument νt where t is a none-too-simple function of τ. All these formulae are developed to the terms in ν^{-4}, giving high accuracy over the physically significant range; and the coefficients involved are tabulated for an adequate set of values of τ. However, these tables need to be supplemented by a table of Bessel functions, and **[22]** removes this need by tabulating $\chi_\nu(\tau)$, and several associated functions, as functions of both ν and τ.

Paper **[23]** was also intended for numerical use in the gas flow context, but other developments apparently made it unnecessary. Although written soon after **[20]** it is quite different. Aiming to sum a slowly convergent power series $\sum C_r t^r$, Cherry replaces C_r by $c_r f(r)$ where $f(z)$ is analytic in the half-plane $|\arg z| < \frac{1}{2}\pi$ and has an asymptotic expansion for large z therein. The remainder after n terms is then estimated by use of Taylor's theorem with remainder, actually in two different ways. Such techniques go back to Euler, as is well known; Cherry illustrates the effectiveness of this relatively elaborate one by the far-from-trivial instance of the Kapteyn series $\sum x^r J_r(ry)$.

This and other papers by Cherry are model contributions to numerical analysis; the problem is reduced as far as possible by analytical methods before numerical work is begun. He habitually did a great deal of his own arithmetic, even after helpers and desk machines became available; and usually he used only Crelle's or Peters's "Multiplication Tables". (These great tomes extend the 12×12 tables that used to be learnt at primary school to 999×999.)

Flow of Gases

The earliest of these papers appeared in 1947, before any of Cherry's work on asymptotic expansions had been developed. Thus some raw material for the latter papers appears in the former, for instance an asymptotic formula found by the method of steepest descents. The contrast highlights the originality and sophistication of the long processing which led to the papers on uniform asymptotic formulae. What is more, Cherry's work on flow of gases led to knowledge which was significant even beyond the frontiers of mathematics and science; the development of the hodograph method to elucidate plane flow of a compressible fluid ranks as one of the milestones in the progress of aeronautics.

It had been discovered in 1904 by Chaplygin [**g**] that, although the equations of compressible flows are non-linear, the hodograph equation, giving the Legendre potential in terms of the magnitude and direction of the velocity, is linear; and not only linear but separable. Moreover the directional factor in a separable solution is merely trigonometric, and the speed factor no worse than hypergeometric.

For many years these facts remained unused; but with the advent of jet-propelled aircraft interest was quickened. Most approximate methods of calculating compressible flows seemed to break down when the velocity approached that of sound. Could it be that Chaplygin's hodograph transformation held the key to calculating trans-sonic flows? Ringleb and Tollmein in Germany, Bers and Bergman in America, Goldstein, Lighthill and Craggs in England, and Cherry in Australia, all became interested, at about the same time, in the search for shock-free compressible flows by this means.

It was soon apparent that there were two main difficulties. One was the behaviour of the hypergeometric functions. The other, pinpointed by Tollmein, was as follows. Since the velocity may have equal values at different points, and certainly does in symmetrical flows, the position coordinates x, y are many-valued functions of the velocity $qe^{i\theta}$; and for flow past an aerofoil they have a branch point at q_∞, the velocity at infinity. So a disentangling of the branches was needed before a hodograph solution could be much use.

Lighthill [**h**] and Cherry [**24** and **25**] almost simultaneously made the analytical continuations needed for the aerofoil case. Some hint of the difficulty can be seen in the fact that the hodograph solutions for x and y as functions of q and θ are given by series of the form of (12) below (τ being a certain function of q). Lighthill proposed a simplified method of constructing wind-tunnel flows [**i**], and this was adopted and improved by Cherry [**28**]. Both men attacked the problems of analysis still involved, although from this point on the main work was Cherry's [**30–33**]. His immense experience in classical analysis led him to devise intricate transformations which elucidated the branching of the functions and made the calculation of exact trans-sonic flows a practical possibility.

In [**24**] a family of exact plane compressible flows past a roughly circular cylinder is found, flows in which the circulation is zero and the speed at infinity subsonic. The shape of the cylinder depends on infinitely many disposable parameters in the family; discussion of it is deferred until [26]. The main task achieved in [24] is the continuation of the hodograph solutions round the branch point at the velocity at infinity. There is some discussion of an alternative method depending on a transformation of the hodograph equation which removes certain singularities.

The almost simultaneous work of Tsien and Kuo [j] is discussed in an appendix. Cherry remarks that their analytical continuation rests on an unproved assumption, and that in their chief example this assumption is in fact false. Lighthill [h] makes similar comments.

In [25] Cherry extends the work of [24] to the case of flow with circulation, with two stagnation points on the cylinder. Solutions are obtained by generalizing those for incompressible flow. This involves an infinite set of linear equations of the second kind with kernel in l^2 (to use wording analogous to that of integral equation theory). The Fredholm theorems for compact operators are applicable, and Cherry solves the equations explicitly by comparing them with a partial fraction expansion for a certain hypergeometric function.

In [26] three of the solutions obtained in [24] are exhibited diagrammatically. They are obtained by heavy numerical work which is presented in [27]. The shapes of the corresponding cylinders are obtained, and also some neighbouring stream lines. The Mach number is subsonic at infinity, but is well into the supersonic range near the cylinder. Debye's asymptotic formulae are used for the hypergeometric functions, and the slowly convergent series are summed by sound but crude methods. The subtleties of [19–23] had not yet been born.

Paper [28], among many others, exhibits Cherry's astonishing intuition, as well as his quite extraordinary virtuosity in analysis. The aim is to find symmetrical plane nozzle flows; for these the Legendre potential $\Omega(\tau, \theta)$ is three-valued in certain regions and one-valued in others, as Lighthill [i] had shown. The hodograph equation gives

$$\Omega(\tau, \theta) = \sum A_\nu \chi_\nu(\tau)\, e^{i\nu\theta}, \tag{12}$$

where $\chi_\nu(\tau)$ is a hypergeometric function and A_ν are disposable constants. From these requirements an appropriate Ω is *guessed*, knowing that the sum of the Kapteyn series

$$K(t, \theta) = 1 + 2 \sum J_n(nt) \cos n\theta \tag{13}$$

has branch points of the kind required for Ω, and that there is an asymptotic resemblance between $\chi_\nu(\tau)$ and $(e\delta/\nu)^\nu\, \Gamma(\nu+1)\, J_\nu(\nu t)$ if t is a suitable function of τ. What is more, Cherry makes the analogy between (12) and (13) lead to the exact details of the analytical continuation of Ω.

Paper [29] identifies analytically a form of hodograph solution obtained by Bergman with Chaplygin's original form (12).

Perhaps [30] is the culmination of these triumphs, even if it appears to make Cherry's uniform asymptotic formulae less necessary. The idea here is that of "uniformizing" a relation between variables by expressing them as one-valued functions of an auxiliary variable or parameter ϕ. For hodograph solutions whose branch points are sufficiently restricted, a transformation from (q, θ) to (q, ϕ) is produced which makes the solutions one-valued, and expressible by a single series, over the whole region of interest. Moreover this series is rapidly convergent. By this method Cherry shows how to construct nozzle flows with prescribed axial velocity, and also flows round a cylinder of aerofoil shape, with blunt leading edge and cusped trailing edge. An important ingredient, which he calls the "principal solution", stands in the same relation to the Chaplygin solutions $\chi_\nu(\tau)\, e^{i\nu\theta}$ as does the generating function of Legendre polynomials to the harmonic functions $r^n P_n(\cos\theta)$.

In [31] a survey is made of hodograph solutions which might usefully be superposed on the principal solution to obtain nozzle flows, with a view to supersonic wind tunnel design. Several such flows are calculated and exhibited graphically. In continuation of this, [32] contains further discussion of nozzles for which the supersonic flow is ultimately uniform. Finally [33] is a semi-expository article in which these same topics are considered. In these papers the weight is towards practical considerations, from engineering and computational viewpoints. By this time Cherry's centre of attention was in fact becoming computation.

Miscellaneous Topics

A typescript report [36], on the magnetron, presents a theory of interpenetrating streams of electrons in an evacuated cavity, subject to less drastic approximations than had been customary; for instance the relativity effect on the mass of an electron is allowed for, and there is no restriction to steady state. However it is assumed that electrons leave the cathode with zero velocity, and it is proved that their motion is then derivable from a potential; a proof of this under more stringent conditions was given independently by Ferraro [k]. An integral resembling the pressure equation in irrotational hydrodynamics is obtained, and the flow is also shown to admit formulation as a variational problem. Singular surfaces or fronts are admitted, and boundary conditions across a front are formulated. Much of the report is concerned with solving the equations near a front, whether stationary or moving.

In [37] the detection of aircraft by a radar station is discussed, assessing the probability of non-detection on account of the rotating beam used and the blind areas due to interference between direct beams and indirect beams reflected by land or sea. In [38] the opposite problem of detection of a radar station by aircraft is discussed. The mathematics involved in these papers is elementary, but the practical complications and details are numerous and call repeatedly for sound judgment.

By contrast [39] handles a formidable physical problem which involves deep and elaborate mathematics, as well as judgment, to correctly find even the order of magnitude of the quantities involved. This herculean paper is one of the few in which the mathematics is not fully rigorous; indeed, some drastic approximations are made, under the dominant consideration of reaching physical conclusions. The problem is concerned with the flow and temperature of a viscous liquid, nitroglycerine in particular, which is squeezed out from between a fixed flat anvil and a parallel flat-faced hammer.

The simplest theory would treat the liquid motion as slow and the viscosity as constant, and would neglect conduction of heat and deformation of the surfaces. That theory gives fantastically high pressures and temperatures, which experiments reject, and the three main parts of the paper are concerned with what modifications in those assumptions are needed. Part I uses some exact solutions of the Navier-Stokes equations, and perturbations therefrom, to arrive at an approximate motion which is stable, for which the inertia terms are negligible, and to variations in which the maximum pressure and temperature are insensitive. Part II examines the consequent deformation of the anvil if it is assumed elastic, and uses heavy successive approximations and much arithmetic to assess the modifications necessary in the preceding motion. In a typical instance this introduction of elasticity reduces the pressure- and temperature-rises to one hundredth. Part III introduces conduction and convection but omits elasticity, and shows that their effect is to reduce the pressure- and temperature-rises to roughly one-half. This part is made to depend on a study of the eigenfunctions of

$$y'' - (x-\lambda)\, y = 0, \tag{14}$$

including some approximate formulae for the higher eigenfunctions. This attention to what is essentially Airy's equation foreshadows Cherry's important later work on asymptotic expansions [**19** and **20**].

If [**39**] displays Cherry as an applied mathematician of great power and resourcefulness, [**41**] shows him delighting in delicacies seemingly quite detached from the physical world. Paper [**41**] is just pure mathematics; but it is related to the nozzle flow theory in [**28**] and the Kapteyn series (13), and it is full of the function $J_\nu(\nu x)$ which haunts Cherry's papers from [**19**] to [**28**]. Its starting point is the formulae, dating from Lagrange and Bessel,

$$1 + 2 \sum_1^\infty J_n(nx) \cos n\theta = \frac{1}{1 - x\cos\xi^*}, \tag{15}$$

$$\theta + 2 \sum_1^\infty J_n(nx) \frac{\sin n\theta}{n} = \xi^*; \tag{16}$$

where θ is real, $0 < x < 1$, and $\xi = \xi^*(\theta, x)$ is the (then unique) real root of Kepler's equation

$$\xi - x \sin \xi = \theta. \tag{17}$$

Cherry obtains formulae analogous to (15) and (16) for the unreal roots ξ of (17), and for the power series in $e^{-i\theta}$ corresponding to the left sides. Typical features of the work are the occurrence of Fourier-type integrals such as in

$$\frac{1}{1 - x\cos\xi(\theta)} = -\int_0^\infty e^{-i\nu\theta} J_\nu(\nu x)\, d\nu, \tag{18}$$

and the use of analytic continuation and of deformation of contours of integration. There are " second proofs " also, and it seems as though Cherry is presenting in collected form some by-products of his gas-flow theory.

In [**42**], with a former research student, Cherry considers the formulation of compressible flow problems as calculus of variations problems, with the aim of harnessing the Rayleigh–Ritz method to compressible flow. They begin with plane subsonic flow in a bounded region, using two integrals of Bateman [**1**]; one of these is to be maximized, the other minimized, and the extreme values are to be equal, for the actual flow. The authors extend their methods to an infinite stream past a cylindrical obstacle, without circulation, buttressing some work by Wang [**m**]; and they go on to mention an extension to trans-sonic flow through a hyperbolic nozzle. They do not attack questions of existence and uniqueness of solutions to the variational problem; consequently the work lacks the great pure mathematical structure that characterizes Cherry's earlier papers on compressible flow.

Paper [**43**] was presented to a conference on numerical analysis, and exhibits the pre-computational analysis of the conduction-convection problem discussed in the third part of [**39**]. Emphasis is placed on judgment, particularly in regard to finding a partial differential equation which is both a reasonable physical approximation and one that might admit substantial mathematical reduction before numerical treatment becomes necessary.

Paper [**45**], presented to a summer research institute, is a speculative discussion directed towards an understanding of turbulent flow; Cherry thought it would need much work before it could approach this aim. The leading idea comes from Poincaré's " principle of exchange of stabilities at a point of bifurcation ", which might nowadays be called a conjecture. It receives some support from Taylor's work on the stability of flow between rotating cylinders. Cherry thought that this work might be extended to reveal, at larger Reynolds numbers, many more families of steady motions branching from the original one, and from one another; and that the continual breakdown through instability of many of these would present a situation resembling turbulence. He seeks to illustrate this by viscous liquid flow between parallel walls and between inclined walls, but points out certain inadequacies in each case.

Contrasting sharply with this, paper [**47**] is wholly pure mathematics. It is a by-product of years of study of elastic plate problems, in which the aim was to solve the plane biharmonic equation using eigenfunction expansions, and on which Cherry himself published nothing. One such treatment of a clamped rectangular plate he held to be in error. Continual probings served to deepen his knowledge of existence and uniqueness theory of infinite linear systems of equations, and he read much pure mathematics, ranging from F. Riesz and Hellinger-and-Toeplitz to Muskhelishvili, in his quest. Late in his career such a system was handled by Wood [**n**] by relating it to a Wiener-Hopf integral equation, and Cherry, working with a more powerful but allied technique, soon after produced this brilliant and comprehensive treatment of such systems.

The given system of equations

$$x_m + \sum_{n=N}^{\infty} \frac{1}{m} k\left(\frac{n}{m}\right) x_n = g_m \quad (m = N, N+1, \ldots), \tag{19}$$

where (x_n) is unknown, is first converted into an equivalent integral equation by a series transform, supposing that the kernel k is expressible as an inverse Mellin transform

$$k(z) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} K(s) z^{-s} ds \tag{20}$$

for suitable K. The integral equation so obtained is akin to a singular one, but has an extra complication which makes it necessary to develop solution theory independently of standard work. Cherry reduces it to a Fredholm equation of second kind, although with infinite range, and establishes the applicability of iteration when N is sufficiently large. Some asymptotic estimates for x_n when n is large are also given, and some extensions.

One cannot leave the subject of Cherry's research without mentioning various topics to which he gave much attention at various times with little or no publication. Among these are the elastic plate problems just mentioned; relativity, viscous flow, schlicht functions, Gödel's theorem, and computation. He was tremendously industrious and worked far into the night, and his manuscripts show how he returned persistently to some problems which it seems he did not solve to his satisfaction. He was often consulted by research institutes, and by their staff members, many of whom were his former students. His great knowledge and incisive mind gave inspiration and leadership in subjects as far apart as foundations of geometry and interplanetary flight, and he might have become an authority on any of them if circumstances had permitted him.

Publications by T. M. Cherry

Ordinary Differential Equations

1. " On the solution of difference equations ", *Proc. Cambridge Philos. Soc.*, 21 (1923), 711–729.
2. " On the form of the solution of the equations of dynamics ", *Trans. Cambridge Philos. Soc.*, 23 (1924), 43–70.
3. " Integrals of systems of ordinary differential equations ", *Proc. Cambridge Philos. Soc.*, 22 (1924), 273–281.
4. " On Poincaré's theorem of the non-existence of uniform integrals ", *Proc. Cambridge Philos. Soc.*, 22 (1924), 287–294.
5. " Note on the employment of angular variables in celestial mechanics ", *M. N. Roy. Astronomical Soc.*, 84 (1924), 729–731.
6. " Some examples of trajectories defined by differential equations of a generalized dynamical type ", *Trans. Cambridge Philos. Soc.*, 23 (1925), 169–200.

Hamiltonian Systems of Differential Equations

7. " On integrals developable about a singular point of a Hamiltonian system of differential equations ", (Part I), *Proc. Cambridge Philos. Soc.*, 22 (1924), 325–349.
8. " On integrals developable about a singular point of a Hamiltonian system of differential equations ", (Part II), *Proc. Cambridge Philos. Soc.*, 22 (1925), 510–533.

9. " On the transformation of Hamiltonian systems of linear differential equations with constant or periodic coefficients ", *Proc. London Math. Soc.* (2), 26 (1926), 211–230.

10. " On the solution of Hamiltonian systems of differential equations in the neighbourhood of a singular point ", *Proc. London Math. Soc.* (2), 27 (1927), 151–170.

11. " On periodic solutions of Hamiltonian systems of differential equations ", *Philos. Trans. Roy. Soc. London Ser. A*, 227 (1928), 137–221.

Topological Dynamics

12. " Topological properties of the solutions of ordinary differential equations ", *Amer. J. Math.*, 59 (1937), 957–982.

13. "Analytical quasi-periodic curves of discontinuous type on a torus ", *Proc. London Math. Soc.* (2), 44 (1938), 175–215.

Later Dynamical Studies

14. " The pathology of differential equations ", *J. Australian Math. Soc.*, 1 (1959), 1–16.

15. "A singular case of iteration of analytic functions: a contribution to the small divisor problem ", *Non-linear Problems of Engineering*, ed. W. F. Ames (Academic Press, 1964), 29–50.

16. "Asymptotic solutions of analytic Hamiltonian systems", *J. Differential Equations*, 4 (1968), 142–159.

Fourier-type Expansion Theorems

17. " On expansions in eigen functions, particularly in Bessel functions ", *Proc. London Math. Soc.* (2), 51 (1948), 14–45.

18. " Expansions in terms of parabolic cylinder functions ", *Edin. Math. Proc.* (2), 8 (1948), 50–65.

Asymptotic Expansions

19. " Uniform asymptotic expansions ", *J. London Math. Soc.*, 24 (1949), 121–130.

20. " Uniform asymptotic formulae for functions with transition points ", *Trans. Amer. Math. Soc.*, 68 (1950), 224–257.

21. "Asymptotic expansions for the hypergeometric functions occurring in gas flow theory ", *Proc. Roy. Soc. Ser. A*, 202 (1950), 507–522.

22. " Tables and approximate formulae for hypergeometric functions, of high order, occurring in gas flow theory ", *Proc. Roy. Soc. Ser. A*, 217 (1953), 222–234.

23. " Summation of slowly convergent series ", *Proc. Cambridge Philos. Soc.*, 46 (1950), 436–449.

Flow of Gases

24. " Flow of a compressible fluid about a cylinder ", (Part I), *Proc. Roy. Soc. Ser. A*, 192 (1947), 45–79.

25. " Flow of a compressible fluid about a cylinder ", (Part II: Flow with circulation), *Proc. Roy. Soc. Ser. A*, 196 (1949), 1–31.

26. " Numerical solutions for transonic flow ", *Proc. Roy. Soc. Ser. A*, 196 (1949), 32–36.

27. " Numerical solutions for compressible flow past a cylinder ", *C.S.I.R. (Australia) Div. of Aeronautics Rep. A* 48 (1949), 1–25.

28. " Exact solutions for flow of a perfect gas in a two-dimensional Laval nozzle ", *Proc. Roy. Soc. Ser. A*, 203 (1950), 551–571.

29. " Relation between Bergman's and Chaplygin's methods of solving the hodograph equation ", *Quart. Appl. Math.*, 9 (1951), 92–94.

30. "A transformation of the hodograph equation and the determination of certain fluid motions ", *Philos. Trans. Roy. Soc. London Ser. A*, 245 (1953), 583–624.

31. " Some nozzle flows found by the hodograph method ", (Part I), *J. Australian Math. Soc.*, 1 (1959), 80–94.

32. " Some nozzle flows found by the hodograph method ", (Part II), *J. Australian Math. Soc.*, 1 (1960), 357–367.
33. " Trans-sonic nozzle flows found by the hodograph method ", *Partial Differential Equations and Continuum Mechanics*, ed. R. E. Langer (University of Wisconsin Press, 1961), 217–232.

Miscellaneous Topics

34. " The value of inoculation—a statistical inquiry ", pages 89–103 of *Influenza and Maritime Quarantine in Australia* by J. H. L. Cumpston (Commonwealth of Australia, 1919).
35. " Newton's Principia in 1687 and 1937 ": a lecture (Melbourne University Press, 1937), 7–28.
36. " General theory of the magnetron ", *C.S.I.R.* (*Australia*) *Radiophysics Lab.*, Rep. MUM 1 (1943), 1-24.
37. " Probability of detection of aircraft by radio direction finding ", *C.S.I.R.* (*Australia*) *Radiophysics Lab.*, Rep. MUM 2 (1943), 1–28.
38. " Probability of detecting a radar station from an aircraft ", *C.S.I.R.* (*Australia*) *Radiophysics Lab.*, Rep. MUM 3 (1944), 1-16.
39. " Flow and generation of heat in compressed films of viscous liquid ", *C.S.I.R.* (*Australia*) *Div. of Lubricants and Bearings, Explosives Rep.* 8 (1945), 79pp.
40. "A lesson on number ", *Australian Math. Teacher*, 3 (1947), 33–41, 65–74.
41. " On Kepler's equation ", *Proc. Cambridge Philos. Soc.*, 51 (1955), 81–91.
42. (With P. E. Lush) " The variational method in hydrodynamics ", *Quart J. Mech. Appl. Math.*, 9 (1956), 6–21.
43. " Numerical solution of a problem in forced convection ", Proc. W.R.E. Computing Conference, Weapons Research Establishment, Adelaide (1957), 115–1 to 115–13.
44. "A mathematician looks at physical theory: presidential address to ANZAAS Section A", *Australian J. Sci.*, 21 (1958), 17–27.
45. " Steady motions related to problems of hydrodynamic stability ", *Australian Math. Soc. Summer Res. Inst.*, Rep. II (1961), IV–1 to IV–8.
46. (With B. H. Neumann) " Felix Adalbert Behrend ", *J. Australian Math. Soc.*, 4 (1964), 264–270.
47. " Infinite linear systems with homogeneous kernel of degree -1". *J. Australian Math. Soc.*, 5 (1965), 129–168.

References

A. Poincaré, H. *Les Méthodes Nouvelles de la Mécanique Céleste* (Paris, 1892–1899).
B. Moser, J. K. *SIAM Review*, 8 (1966), 145–172.
C. Cartwright, M. L. *Proc. London Math. Soc.* (3), 17 (1967), 355–380.
D. Whittaker, E. T. *Analytical Dynamics* (C.U.P.) 4th edit. (1937).
E. Whittaker, E. T. *Proc. Roy. Soc. Edinburgh*, 37 (1916), 95–116.
F. Birkhoff, G. D. *Dynamical Systems* (New York, 1927).
G. Siegel, C. L. *Math. Ann.*, 128 (1954), 144–170.
H. Moser, J. K. *Comm. Pure Appl. Math.*, 11 (1958), 257–271.
I. Krein, M. G. and Jakubovic, V. A. *Proc. Intern. Sympos. on Non-Linear Vibrations*, Vol. 1 (Kiev, 1963), 277–305.
J. Hadamard, J. *J. Math. Pures Appl.*, 3 (1897), 331–387.
K. Levinson, N. *Ann. of Math.*, 50 (1949), 127–153.
L. Gottschalk, W. H. and Hedlund, G. A. *Topological Dynamics* (Providence, 1955).
M. Nemytskii, V. V. and Stepanoff, V. V. *Qualitative Theory of Differential Equations* (Princeton, 1960).
N. Artin, E. *Abh. Math. Sem. Univ. Hamburg*, 3 (1923), 170–175.
O. Siegel, C. L. *Nachr. Akad. Wiss. Göttingen, Math.-Phys. Kl.* IIa (1952), 21–30.
P. Moser, J. K. *Nachr. Akad. Wiss. Göttingen, Math.-Phys. Kl.* II (1962), 1–20.

Q. Arnol'd, V. I. *Russian Math. Surveys*, 18 (1963), 85–191.
R. Birkhoff, G. D. *Coll. Papers*, 2 (1950), 333, 453–460, 530.
S. Morse, H. M. *Trans. Amer. Math. Soc.*, 22 (1921), 84–100.
T. Smale, S. in *Differential and Combinatorial Topology* (Princeton, 1965).
U. Wintner, A. *Analytical Foundations of Celestial Mechanics* (Princeton, 1947).
a. Titchmarsh, E. C. *Eigenfunction Expansions* (Oxford, 1946).
b. Magnus, W. *Jahresbericht Deutsch. Math. Verein.*, 50 (1940), 140–161.
c. Erdélyi, A. *Proc. Roy. Soc. Edinburgh*, 61 (1941), 61–70.
d. Watson, G. N. *Proc. Cambridge Philos. Soc.*, 19 (1918), 96–110.
e. Langer, R. E. *Trans. Amer. Math. Soc.*, 33 (1931), 23–64.
f. Langer, R. E. *Trans. Amer. Math. Soc.*, 34 (1932), 447–480.
g. Chaplygin, S. A. *Ann. Sci. Moscow Imp. Univ.* (*Math. Phys. Sec.*), 21, (1904), 1.
h. Lighthill, M. J. *Proc. Roy. Soc. Ser. A*, 191 (1947), 352–369.
i. Lighthill, M. J. *Proc. Roy. Soc. Ser. A*, 191 (1947), 323–341.
j. Tsien, H. S. and Kuo, Y. H. NACA Tech. Note 995 (Washington, 1946).
k. Ferraro, V. C. A. *Proc. London Math. Soc.* (2), 49 (1945), 77–98.
l. Bateman, H. *Proc. Nat. Acad. Sci.*, 16 (1930), 816–825.
m. Wang, C. T. *J. Aeronautical Sci.*, 15 (1948), 675–685.
n. Wood, W. W. *Proc. Cambridge Philos. Soc.*, 61 (1965), 781–794.