GRACE CHISHOLM YOUNG

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Grace Chisholm Young was a true pioneer; she was one of the very few women mathematicians of her generation to achieve an international reputation, and the repercussions of her enthusiasm and her ideas on the mathematical world extend far beyond her own individual achievements. Something of her life and work has already been told in the account of her husband, W. H. Young, published in this Journal*; but that is by no means the whole story, and I shall try to avoid repetition.

Grace Chisholm was born on 15 March, 1868, and her early education was at home; she took the Senior Cambridge examination in December, 1885. In April, 1889, she entered Girton College as Sir Francis Goldsmid Scholar; and she was a wrangler in Part I of the Mathematical Tripos in 1892. Immediately after the Tripos she and I. M. Maddison went to Oxford, and sat for the Final Honours School of Mathematics, obtaining a first and a second class respectively. I believe that they were the first women to sit for the Final Honours School of Mathematics, and that they did it to refute a suggestion from one of their coaches that it was more difficult for a woman to obtain a first at Oxford than at Cambridge. Their names do not appear in the lists in the Oxford University Calendar, probably because they took the examination by some unofficial arrangement; but two or three years later the names of women, chiefly students of Royal Holloway College, become quite usual in the Oxford Finals list, and often in the first class.

Miss Chisholm then proceeded to take Part II of the Mathematical Tripos, which was a most unusual thing for a woman to do in those days. According to the "Vita" in her Ph.D. dissertation she was taught chiefly by Berry, Richmond, and W. H. Young, her future husband. She also attended lectures by Smith and Webb, and later (when reading for Part II presumably) she attended lectures by Forsyth, Darwin and Cayley. After Part II, since there was no possibility of a Smith's Prize or Fellowship for a woman, she was advised to go to Göttingen. By whom I have failed to discover, but it would be most interesting to know in the light of her subsequent career and its influence on Cambridge pure mathematics. Berry and Forsyth wrote to Klein asking him to admit her to lectures, and it seems likely that it was the latter· but it is also

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* 17 (1942), 218–237.

worth observing that her Ph.D. dissertation is dedicated to her father, Henry Williams Chisholm, "von welchem sie frühzeitig gelernt hat Deutschland und die mathematische Wissenschaft hochzuschätzen".

Klein replied* that the decision did not rest with him but with the Faculty in general, and ultimately with the Ministry in Berlin. He made no promises, but advised her to come to Göttingen early in October. Klein returned from Chicago a few days after she arrived, and she seems to have been immensely impressed by his height, his long capable-looking hands, and above all by his smile. Two American ladies had also just arrived on the same errand, independently of Miss Chisholm and of each other. Miss Winston, a graduate of the University of Wisconsin, former Fellow of Bryn Mawr, had been at Chicago for a year; she went to the same lectures as Miss Chisholm at Göttingen. Miss Maltby, of Wellesley College and the Boston Institute of Technology, was an experimental physicist and worked under Nernst on conductivity. She was an M.A. and B.Sc., and held a travelling fellowship from the Institute.

These three, following Klein's advice, made an application to the Minister of Education to be admitted to lectures. Klein told them that it would be hopeless to ask for permission to matriculate, but he arranged for them to be admitted unofficially to lectures until the reply came. On the first day they went to the Room of the Mathematical Models before eleven, so as to avoid the crowd of students wandering about during the quarter of an hour's grace between lectures; and at 11.15 they followed Klein into the lecture room. They seemed to have been needlessly apprehensive about the attitude of men students towards the presence of women in the auditorium, and found themselves made welcome. By the third lecture official permission for them had come, but only as exceptional cases. Miss Chisholm wrote to the Girton Mathematical Club: "There are lectures given here by University professors outside the University itself to women, and there are about thirty women who go to these lectures; naturally some of these would like to be admitted to the University and allowed to go to any lectures they please. In a German University there is none of that organization of colleges, and tutors, and coaches, and examinations which makes it easy at Cambridge to systematize and control the studies of the students, and this makes the question of the admission of women here one of much greater difficulty than the corresponding one at home. Prof. Klein's attitude is this, he will not countenance the

* Much of what follows about Göttingen is taken from a letter from Miss Chisholm to the *Girton Review* published in March, 1894. I follow her terminology as regards American colleges and universities.

admission of any woman who has not already done good work, and can bring him proof of the same in the form of degrees or their equivalent, or letters from professors of standing; and, further, he will not take any steps till he has assured himself by a personal interview of the solidity of her claims. Prof. Klein's view is moderate. There are members of the Faculty here who are more eagerly in favour of the admission of women, and others who disapprove altogether. But the chief difficulty is in Berlin. Were not Hanover reduced to the condition of a province of Prussia, a condition very much disliked by a strong party here, I should have very little doubt of the success of the cause in a few years".

Miss Chisholm and Miss Winston went to Klein's and Weber's seminars. She wrote: "The latter simply gives us problems to solve, and one of the students works them out on the board at the next meeting. The problems are interesting enough to solve, but Prof. Weber always seems to choose out the dullest person to work out the problems on the board, and it is none the less wearisome because the function takes place at 8 o'clock in the morning, and necessitates breakfast at 7.30.

"Prof. Klein's seminary is quite different; it takes place every Wednesday at 11 o'clock, and lasts about two hours, and the members make 'Vortrag's on their special subjects on different Wednesdays. The students who have been here some time, and some of the new students who have come from other Universities, have already got their special subjects; for the others, Prof. Klein has always suggestions as to special lines of work which they might take up, generally in connection with the lectures. Miss Winston made her Vortrag on the last Wednesday before the Christmas holidays. It would be nervous work in any case to make a Vortrag before an audience of about a dozen men, half of whom are Doctors, and one Prof. Klein; but the strain is considerably increased by having to speak German. There are about a dozen of us in our lectures; we are a motley crew: five are Americans, one a Swiss-French, one a Hungarian, and one an Italian. This leaves a very small residuum of German blood".

The position of women must have improved quite soon. For Miss Chisholm obtained her Ph.D. degree "magna cum laude" in 1896 for a dissertation on the algebraic groups of spherical trigonometry, a subject evidently suggested by Klein. He was so much interested in the problem that he discussed it at length*, and her treatment of it, twelve years later in *Elementary mathematics from an advanced standpoint*, where he referred†

* See *Elementary mathematics from an advanced standpoint, arithmetic, algebra, analysis*, translated by Hedrick and Noble (London, 1932), 177–180.

† *Loc. cit.* 180.

to her as the first woman in Prussia to pass the normal examination for the doctor's degree. Things had moved a long way since 1874, when Sonja Kowalewski* took the degree at Göttingen *in absentia*, having been refused permission to attend Weierstrass' lectures.

In 1896 Grace Chisholm married W. H. Young, and from this point the story of her life and work is closely bound up with that of her husband's career, which has already been told†. Until his marriage he had done practically no research, although he was by then about thirty-three; but a year later he gave up most of his coaching and examining, and they went to live abroad and do research. There is a strong impression, in spite of Mrs. Young's statement‡ that he suggested the move, that the idea of it came from her; the impression is based partly, no doubt, on her own already established record of research done abroad. If we accept the view of Hobson's career as a pure mathematician drawn by Hardy§, that his interest in the modern theory of functions was largely due to his intercourse with W. H. Young, and that the present position of real function theory at Cambridge is very largely due to Hobson, then it all began with the Youngs' move from Cambridge to Göttingen. No doubt the ideas of real function theory would have found a place in the Cambridge Tripos sooner or later, but perhaps never such an important place if Mrs. Young had not pushed her way into Göttingen.

Not that either of the Youngs began immediately to work on real function theory when they arrived at Göttingen. Mrs. Young at that time seemed to be more interested in algebraic, geometrical, and even astronomical topics; and W. H. Young began to write on vectors in n dimensions; but about 1901 both W. H. Young and Hobson turned to the theory of functions of a real variable, and from that time it has been the main interest of the mathematical members of the Young family. I do not know who or what directed their attention that way. None of the people connected with Göttingen at that period were particularly interested in the subject, so far as I know, except perhaps Osgood, though the French mathematicians were already writing a great deal about it.

After a period of comparative mathematical inactivity in early married life, when the children were quite young, Mrs. Young's name began to appear, first in 1906 as joint author with her husband of the book *The theory of sets of points*, then as joint author with him of several important papers from 1909 onwards.

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* E. T. Bell, *Men of mathematics* (London, 1937), 474–475.

† *Journal London Math. Soc.*, 17 (1942), 218–237.

‡ *Loc. cit.*, 221.

§ *Journal London Math. Soc.*, 9 (1934), 225–237 (227 and 236).

About 1914 she once more began to write mathematics independently; and the next few years saw the appearance of her most important work—work which has given her a permanent place among those mathematicians who were developing the modern theory of real functions*. Her special topic was the theory of differerentiation and of derivates. The first of this series of papers was one in *Acta Mathematica*, containing the theorem

Except at an enumerable set of points, the lower derivate of any function on either side is not greater than the upper derivate on the other side.

Mrs. Young's paper was written independently of a slightly earlier Habilitationsschrift of Rosenthal† which includes the special case of this theorem in which the function is continuous. This special case lends itself to a treatment which is more geometrical than Mrs. Young's, and it is an analytical discussion substantially on her lines which would normally be given to-day.

In 1915 Girton College awarded Mrs. Young the Gamble Prize for an essay "On infinite derivates" which was published in a modified form in the *Quarterly Journal* in 1916. The introduction to this essay contains a general survey of the theory; the style is in parts exuberant, and effervesces into fantasy on the subject of the ultra-microscope and the atom—"Away with your ordinary curves, the wild atom will none of them" The main theorem proved in the essay is that

The points at which the upper right-hand derivate of a continuous function is $+\infty$ *and the lower left-hand derivate is different from* $-\infty$ *form a set of measure zero.*

The second part of the essay contains a detailed investigation of Weierstrass's and Cellérier's non-differentiable functions.

The complete statement of the relations between the derivates of an arbitrary function was given in Mrs. Young's paper in Volume 15 of the *Proceedings of the London Mathematical Society*, namely,

Except at a set of measure zero, there are three possible dispositions of the derivates of a measurable function $f(x)$, *either*

(1) *they are all equal, and there is a finite differential coefficient,*

* I am indebted to Dr. Burkill for much help in the discussion of Mrs. Young's mathematical work.

† A. Rosenthal, "Uber die Singularitäten der reellen ebenen Kurven", *Habilitationsschrift* (München, 1912).

or (2) *the upper derivates on each side are* $+\infty$, *and the lower derivates on each side are* $-\infty$,

or (3) *the upper derivate on one side is* $+\infty$, *the lower derivate on other side is* $-\infty$, *and the two remaining extreme derivates are finite and equal.*

This statement—for a continuous $f(x)$—was contained in the first part of Denjoy's exhaustive study of differentiation and integration, which appeared at about the time of the award of the Gamble Prize, and so the first published account is his. Mrs. Young's work is, however, the more general in that she assumes only that $f(x)$ is measurable, and these striking results are fittingly associated with the names of both of them.

Many years later Mrs. Young wrote another substantial paper on the foundations of the differential calculus (*Fundamenta Mathematicae*, 14), but it is marred by a mistake (in Theorem 5).

Her energy and enthusiasm must have been quite extraordinary. She was a good tennis player in her younger days, and her interests, recorded in *Who's Who*, include music, domestic occupations, vine-culture, literature and languages, history (especially the sixteenth century), philosophy, chess, and formerly tennis, croquet and billiards. The domestic occupations involved in bringing up six children (two of whom, L. C. Young and R. C. Young, are mathematicians) would have been sufficient for most women, not to mention her collaboration with her husband in so much of his work, and her independent work on derivates. But she studied medicine at Göttingen and Geneva; she wrote articles in *Nature*, and poems; and her paper "On the solution of a pair of simultaneous Diophantine equations connected with the nuptial number of Plato" shows a more than superficial knowledge of the history of Greek mathematics.

Mrs. Young's educational ideas are worth some consideration. For the fact that cheaper education for the six children, and education more in harmony with her ideas, could be obtained abroad than in England was probably an important factor in determining the Youngs to settle in Geneva permanently. She wrote three educational books for children, *The first book of geometry* with her husband, and *Bimbo* and *Bimbo and the frogs* by herself. The two latter are really lessons on the elementary biology of plants and animals, including cell structure seen under a microscope, with a story about a family with marked resemblances to the Young family as jam to make the pill go down. They apparently had some success, and were probably a product of the days when she was studying medicine. The geometry was not successful in England, but went better in translation in Germany. All the books were in some ways in advance

of their time, but unequal and only suited for children taught individually, with the inclination and sufficient intelligence to study things scientifically at an earlier age than most. For instance, I think that it is generally recognized now that the teaching of geometry should begin with very elementary solid geometry and the use of models as in *The first book of geometry*, but an average child in a class could not follow the book to the later proofs, which are also based on models and paper folding, until he was of an age to learn the grown-up proofs; and the English examination system was quite sufficient to kill the book.

The end of her life was saddened by the tragic separation from her husband by the collapse of France. She had intended to leave him for a few days only, but she was never able to rejoin him. He died in July, 1942, in Switzerland, and she on 29 March, 1944, at Croydon. The Fellows of Girton College had just recommended her for election to an honorary fellowship, and it is a matter of great regret to them that she died before the Governors were able to elect her.

A list of Grace Chisholm Young's mathematical papers and books.

Papers.

1. "Algebraisch-gruppentheoretische Untersuchungen zur sphärischen Trigonometrie", *Dissertation* (Gottingen, 1895) (as Grace Chisholm).
2. "On the curve $y=\left\{\frac{1}{x^2+\sin^2\psi}\right\}^{\frac{3}{2}}$, and its connection with an astronomical problem", *Monthly Notices, R.A.S.*, 57 (1897), 379–387 [as Mrs. W. H. Young (Miss Grace Chisholm)].
3. "Sulla varietà razionali normale M_3^4 di S_6 rappresentante della trigonometria sferica", *Atti d. Reale Accad. d. Torino*, 34 (1898), 587–596.
4. "On the form of a certain Jordan curve", *Quarterly Journal*, 37 (1904), 87–91.
5. "A note on derivates and differential coefficients", *Acta Math.*, 37 (1914), 141–154.
6. "Sur les courbes sans tangente", *Enseignement math.*, 17 (1915), 348.
7. "On infinite derivates", *Quarterly Journal*, 47 (1916), 127–175.
8. "Sur les nombres dérivés d'une fonction", *Comptes Rendus*, 162 (1916), 380–382.
9. "On the derivates of a function", *Proc. London Math. Soc.* (2), 15 (1916), 360–384; see also (2) 19 (1921).
10. "A note on a theorem of Riemann's", *Messenger of Math.*, 49 (1919–20).
11. "Démonstration du lemme de Lebesgue sans l'emploi des nombres de Cantor", *Bulletin des Sciences Math.* (2), 43 (1919), 245–247.
12. "On the partial derivates of a function of many variables", *Proc. London Math. Soc.* (2), 20 (1922), 182–188.
13. "On the solution of a pair of simultaneous Diophantine equations connected with the nuptial number of Plato", *Proc. London Math. Soc.* (2), 23 (1925), 27–44.
14. "Pythagore, comment a-t-il trouvé son théorème?", *Enseignement math.*, 25 (1926), 248–255.
15. "On functions possessing differentials", *Fundamenta Math.*, 14 (1929), 61–94.

With W. H. Young.

16. " Note on Bertini's transformation of a curve into one possessing only nodes ", *Atti d. Reale Accad. d. Torino*, 42 (1906), 82–86.
17. " On derivates and the theorem of the mean ", *Quarterly Journal*, 40 (1909), 1–26.
18. "An additional note on derivates and the theorem of the mean ", *Quarterly Journal*, 40 (1909), 144–145.
19. " Discontinuous functions continuous with respect to every straight line ", *Quarterly Journal*, 41 (1910), 87–93.
20. " On the determination of a semi-continuous function from a countable set of values " *Proc. London Math. Soc.* (2), 8 (1910), 330–339.
21. " On the existence of a differential coefficient ", *Proc. London Math. Soc.* (2), 9 (1911), 325–335.
22. " On the theorem of Riesz-Fischer ", *Quarterly Journal*, 44 (1912), 49–88.
23. " The reduction of sets of intervals ", *Proc. London Math. Soc.* (2), 14 (1915), 111–130.
24. " Sur la frontière normale d'une région ou d'un ensemble ", *Comptes Rendus*, 163 (1916), 509–511.
25. " On the internal structure of a set of points in space of any number of dimensions ", *Proc. London Math. Soc.* (2), 16 (1918), 337–351.
26. " On the inherently crystalline structure of a function of any number of variables ", *Proc. London Math. Soc.* (2), 17 (1918), 1–16.
27. " On the discontinuities of monotone functions of several variables ", *Proc. London Math. Soc.* (2), 22 (1924), 124–142.

Books.

1. *The first book of geometry* (with W. H. Young; London, Dent, 1905).
2. *The theory of sets of points* (with W. H. Young; Cambridge University Press, 1906).
3. *Der kleine Geometer* (Deutsche Ausgabe, besorgt von S. and F. Bernstein, Leipzig, 1908).