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OBITUARY

James (Jim) Gourlay Clunie 1926–2013

(colour online)

1. *Life and career*

by W. K. Hayman

Jim was born on 26 October 1926 in St. Andrews and attended Madras College there. He was awarded the college's prestigious Dux (leader) in Science in 1944. He entered the University of St. Andrews in 1945, having won that university's Bursary Competition; he was ranked number one in the competition for all the faculties. He graduated in 1949 with first class honours in Mathematics.

He then went to Aberdeen University for his PhD, supervised by Professor Archibald James Macintyre. In his PhD thesis, Jim developed what quickly became the 'modern' approach to Wiman–Valiron theory. The results were published in *J. London Math. Soc.* (28 (1953) 58–66 and 30 (1955) 32–42).

I saw these two beautiful papers while I was in Exeter, and, when I came to Imperial College, I was determined to get Jim to join the department as soon as possible.

Jim had been appointed to a lectureship at the University of North Staffordshire, Keele in 1952 and worked there until he came to us at Imperial College in 1956. He was elected a member of the LMS on 20 February, 1958 and promoted to a Professorship at Imperial College in 1964.

Our research school there flourished and we were soon joined by Noel Baker, Thomas Kövari, Klaus Roth and Christian Pommerenke. Jim had a number of PhD students including Milne

Anderson, David Brannan, Qazi Ibadur Rahman, Terry Sheil-Small, Derek Thomas and Brian Twomey.

Jim semi-retired from Imperial College in 1981 for a Research Fellowship at the Open University, which he held till 1986. In that year he became an honorary Research Associate at the University of York. There, he and I were together again with other analysts Richard Hall, Terry Sheil-Small, Maurice Dodson and their research students.

Jim visited the Massachusetts Institute of Technology in 1959–1960. He was awarded the degree of Doctor of Science, honoris causa by the National University of Ireland in 1988.

Jim married Nancy Toff in 1955 and many of us remember their hospitality and kindness in London, Milton Keynes and York. Sadly Nancy started to suffer from Alzheimer's and had to move to the Retreat in York, where she died in 2000. Jim visited Nancy every day in the Retreat. He never really recovered from her death.

Jim and Nancy were a wonderfully devoted couple and loving parents to their daughter Fiona, and loving grandparents to their grandson Zack and granddaughter Alex. Jim would have been delighted to hear that Alex received a PhD in Applied Psychology from Heriot-Watt in 2014. Sadly, Zack died a few months before Jim did (after several troubled years), though Jim was fortunately never aware of this.

Jim had Polio when he was four and this left him suffering in his legs all his life. However, he never complained and in spite of his handicap he led an active and energetic life. He was an outdoor person and liked rowing, walking, cycling, swimming and golf.

On one occasion he, Pommerenke and I found ourselves on a miniature golf course in America. He did very well, while Pommerenke's and my results were miserable.

Life became increasingly difficult for Jim after Nancy died and he retired to the care home Lamel Beeches where he was well looked after. He suffered a stroke in 2009, and after this his muscles weakened and he lost the use of his right leg completely. He died on 5 March, 2013.

Jim was the first person I consulted with any academic or mathematical problem. His advice was always wise and sound. I feel privileged to have known him for over half a century and to have collaborated with him. He was a very private person and kept his feelings to himself.

I would like to conclude this section with extracts from a few of the many tributes to him. I will start with two from the funeral oration by Margaret Jenkins. It was non-religious, since Jim has been described as an 'atheist, but a Presbyterian atheist'.

Jim's daughter Fiona Crawford wrote:

'His work and his sociable nature brought him in contact with a huge number of people, all of whom will remember him in different ways. However, he was a father to only one person in the whole world, namely me.

I don't remember any time when Dad was not working very, very hard. And, I might add, enjoying it enormously. Wherever we lived in London, be it north or south of the river, he would walk for 20 min to the Tube station, travel for an hour and a half on the underground, and walk for another 10 min at the other end to his work place, doing the whole journey in reverse at the end of a long day. In fact, he was often not home before 9 o'clock in the evening, having waited till after the rush hour to be sure of getting a seat on the train, since he could not have endured such a journey standing up.

At weekends Dad would always be outside tending our lovely garden and, often, decorating our house. He took every opportunity to get lots of exercise, including using a cylinder push mower to cut a big lawn. Many times I heard my mother shout in fright when she saw that he'd shinned up a huge ladder to clear the gutters or paint the outside of the house. He never had an accident, though most weekends he'd arrive at the dinner table with a handkerchief wrapped round a finger or thumb, having sliced it while pruning roses or sawing wood.

One of my clearest memories is of when Dad dispatched a very large plum tree in our back garden at Wembley. It had been a beautiful tree providing many pounds of plums every year,

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but then it got silver leaf and had to go so as not to infect others nearby. Ever the do-it-yourselfer, Dad managed to obtain some dynamite and blew the tree up. Of course, I had to be safely indoors at the time, but I do remember a lot of noise and drama, and I feel the same way about it that I do about travelling on steam trains, that I was glad to be privy to something that couldn't be done today.

I thought about Dad last Saturday when I watched the Oxford and Cambridge Boat Race. Dad and I always watched it together every year, in the days when it was just a smallish event that happened around the corner from us when we lived in Wimbledon, only four stops to Putney on the District Line. Dad had superb upper-body strength and was very good at rowing, something we always did on holiday if possible, when not visiting a putting green. He taught me to swim at the age of eight, when we would go to the local open-air pool for an hour every morning before school and work. I'd practise at the shallow end while he powered up and down the pool for 30 lengths or more, his arms turning like millwheels, never seeming to tire.

I'd like to finish with a very short quotation from Rudyard Kipling's "Story of the Gadsbys".

Down to Gehenna
Or up to the throne
He travels fastest
Who travels alone.'

Professor Milne Anderson wrote:

'We have heard about Jim as a father, but he was also a respected mathematician and Fiona has received many messages from former colleagues.

It was very easy to love and admire Jim Clunie. The whole mathematical community appreciated his mathematics as well as his personal qualities. He bore the curse of his bad leg with uncomplaining stoicism and never let it interfere with his life or impair his sense of humour. It was with Imperial College that he was most closely associated. He was there for 25 years and produced a succession of doctoral students, many of whom went on to professorships in various places. In my own experience, he was a great person to go to with a problem, always making some illuminating and helpful comments.

He first sprang to prominence with his work on the Wiman–Valiron theory, but soon spread himself over all complex analysis. Clunie's Lemma plays a vital role in the theory of value distribution, while there is also Clunie's constant, whose precise value is not yet known. There is also the Clunie–Jack Lemma in univalent function theory. These ideas have stimulated much research.

All this was combined with a friendly and concerned attitude. He was a good administrator, being for many years on the editorial board of the London Mathematical Society and being vice-president in 1967. He was an honorary doctor of the National University of Ireland. But he wore all his distinctions lightly and was always approachable. He will be greatly missed and our sympathy goes to his daughter Fiona and granddaughter Alex.'

Professor Linda Sons wrote:

'Jim, as you note, was always rather a private person. The first time I heard him lecture at Imperial was as part of a course in Functional Analysis he was giving in the fall of my first visit there. He was extremely organized and I got a nice set of notes from his talks, the course being centred on topics such as multipliers, etc. But the first class I went to was a test of my ears.... I don't think I understood much of what he said, because his accent was so Scottish... still I got a good set of notes, because he wrote everything on the board in a legible fashion, and by the second class I got used to his accent and "eepsilon".

When I was giving a talk on my research one time and Jim was in the audience, a mathematician of my vintage interrupted me a couple of times with his "superior" attitude of knowledge until Jim spoke up and made a direct remark to the guy with an "of course..." statement which totally shut up the guy. The guy was being a jerk, and Jim had enough of it.

On a couple of rare occasions Jim saw that I was invited to join him and a couple of other mathematicians for a "pint" at the end of a day just to chat. There was some talk of mathematics but mostly it was just informal time on a personal level. It was one of the few times I remember his saying anything about himself. He clearly took pride in Fiona and his granddaughter.

I saw Jim's approach to mathematics as being different from that of many other mathematicians. If you were to ask him whether he thought a certain theorem or fact was true, most mathematicians I know would begin to outline a proof, or a series of statements as to how one might approach a proof for that theorem. Not Jim.... He would often muse on what it would take to produce a counterexample. And, as we know, he produced some great examples. Jim was certainly always open to discussing mathematics which interested you and seldom talked about what was his major interest at the time.... And his ways were always kind and unassuming.'

Professor Q. I. Rahman wrote:

'My main recollection of my relationship with him as a student at the Imperial College is that, as advisor, he was always available and very generous with his help.

After my departure from London in 1961 I had relatively little contact with him until the summer of 1966, when we met each other in La Jolla, California for several weeks for the Symposium on Entire Functions and Related Parts of Analysis. Since that time our relationship became increasingly close. He visited Montreal on several occasions.

He was always eager to know about the problems I was interested in and often had some very useful observations to make after thinking about them for a few days. Some of his remarks and suggestions helped me make considerable progress with the problems I was trying to resolve. I did publish one paper with him in 1998 and another with him and W. H. Walker (Auckland) in 2000, but that does not tell the full extent of the relationship I had with him. In a way he was my lifelong advisor.

Since about 1983, I saw Professor Clunie as a family friend and someone whom I could ask for advice about personal matters too. His words of wisdom were always to the point and very reassuring. Until a few years ago, I used to talk to him by telephone on a fairly regular basis and always looked forward to the next conversation, but then one day in 2006 I felt that it might be stressful for him even to pick up the phone. After that I only wrote to him and often it was Fiona who replied on his behalf.'

Sadly, soon after writing the above, Professor Rahman died.

Professor Finbarr Holland wrote to Fiona:

'Your father will be well remembered by many of the Irish mathematical community, especially by those who were staff members of Colleges of the National University of Ireland when he acted over an extended period of time as Extern Examiner in Mathematics for this university. His twice-yearly scrutiny of draft papers composed by members of these colleges for the Summer and Autumn NUI examinations were greatly appreciated, and his advice was carefully followed. In addition, his follow-up trips to Ireland to assess marking standards of these examinations were warmly anticipated. He played a full part in assisting the local examiners to dispense justice to the various degree candidates fairly and evenly between the Colleges. In deciding difficult cases, his professional judgement was always relied upon to resolve matters. These were also great social occasions as well and following long and arduous hours dealing with the examination results of students following different degree programmes, he would join the local examiners for a meal and entertain them with gossip and humour. He enjoyed such occasions hugely. Many of us benefited from his insights and knowledge which he imparted freely and graciously at all times.

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In recognition of his research work and duties as Extern Examiner the university awarded him an honorary doctorate in 1988.

Many Irish postgraduate students who went to London/Milton Keynes to study for their PhD, and others who went there on sabbatical leave, were helped by him in many different ways and will not forget him for that.

I myself recall with great fondness the many Sunday evenings I sat down with you and your parents to a lovely meal prepared by your mother in Wimbledon about 40 years ago, when I was on leave of absence from University College Cork and attached to Chelsea College at the time and away from my wife and children. Those visits were a lifeline for me at the time, and I'm eternally grateful for the warmth and hospitality shown to me when I was treated as one of the family, something which I also experienced subsequently in Milton Keynes and York. I treasure these memories.'

Professor David Brannan wrote:

'I first met Jim when I became a graduate student at Imperial College in 1964. We met at the Monday morning seminar, and then once a week for an hour or so to discuss what I had been doing in the previous week and what I might work on in the coming week. For the first six months, I was really catching up on a lot of complex analysis (and other mathematics) that was standard stuff but new to me. At the same time, there was a good active student group:

Derek Thomas (1964–1967), who had done a degree at Chelsea College and so was allowed by the rules to finish in 2 years; he then went in 1966 to a lectureship at Swansea under J. D. Weston; after some years he did a PhD in ornithology, and that became his main academic interest though he stayed in the Mathematics Department.

Dick London (1964–1967), with Thomas Kövari; he went in 1967 to a lectureship at Swansea.

Brian Twomey (1965–1967); he had done an MSc in Cork with Paddy Barry, then a year (1964–1965) at Royal Holloway with Frank Keogh; when Frank went to Kentucky he left Brian with Jim; when he finished Brian did 3 years in USA at Syracuse and South Florida, before returning to Cork.

Terry Sheil-Small (1962–1965); he went to a lectureship at York under Paddy Kennedy, retired to Cyprus with his French wife and returned to York in due course.

Dick Hornblower (1965–1968) with Hayman; he had posts at London School of Economics and Albany before returning to UK; I then lost track of him.

I once went to tea at Jim's house in Wembley. On another occasion he took Brian and me to the LMS meeting in Burlington House in 1965 to celebrate the LMS centenary; and in 1969 to a dinner in Beit Hall after his inauguration. I remember that he took as his inaugural theme that in doing mathematics he was "just playing a game according to its rules"; I argued with him about this at the time, but maybe he was right!

As a supervisor Jim suggested to me the topic of univalent polynomials (started by Jean Dieudonné in 1931) that had made rather little progress by then; I made some but was somewhat overtaken by Ted Suffridge immediately after I had done my bit. Jim did suggest quite a few ideas and proofs but was content to largely let you get along at your own pace on things that struck your fancy, until you needed help.

In 1979, Jim and I organized a NATO Instructional Conference in Durham on Complex Analysis and Potential Theory whose proceedings were published by Academic Press. This was one of the last such LMS conferences, but had a stellar cast of participants including Ahlfors and Doob. Jim's role was mainly on the academic programme while I concentrated on the practical side. It was a great success and the papers in the proceedings are regularly referenced by people even now. In 1983, we ran a Durham Symposium, but I don't remember much about it.

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At one of these conferences in Durham, we ran a conference outing to Beamish Open-Air Museum and the Vaux Brewery in Sunderland (both thoroughly enjoyed!!!) At another conference, we ran a trip to Hadrian's Wall and Vindolanda.

In the 1970s, I visited Jim's house in Wimbledon a few times, twice with my wife and children. On one (summer) visit, we went to a fete at the local Catholic school, where I won a set of china in a raffle (it lasted us for years at a time when we were rather short of cash in inflationary 1970s England). On another (winter) visit, one of our children fell into a pond on Wimbledon Common. So while Nancy made everyone's tea and we ate it, my 2-year old son's trousers were put up to dry in front of a roaring fire in their sitting room.

Brit Kirwan visited IC in 1971–1972. He and I were working on V_k, the class of functions

$$f(z) = z + a_2 z^2 + \cdots$$

analytic and univalent in $|z| < 1$ of bounded boundary rotation. This had been introduced by V. Paatero in 1931 in his PhD thesis and the coefficient bounds had only been obtained (by l971) for a_2, a_3 and a_4. Following Jim's suggestion of using the notion of "extreme points" brought into univalent function work by Wilken, Brickman and MacGregor, we were able to solve the problem for coefficients up to a_{13}; and in 1973 I finished the job for all coefficients.

In 1979, I came to the Open University. Then in 1981, Jim, who had never been especially fond of undergraduate teaching noticed that under USS rules he could retire from Imperial College to a lower-paid academic job for up to 13 years without his pension being affected (it would still be based on his IC professorial salary). In spite of the terrible financial position at the time I was able to persuade the OU to appoint Jim to a 6-year Research Fellowship in 1981. He and Nancy sold their house in Wimbledon and bought a house in Milton Keynes (MK) as well as another house there for their recently married daughter Fiona. Jim thoroughly enjoyed the change to the country atmosphere.

At IC, he had a fixed routine. He came into IC early, and had a swim before going to work in the old Huxley Building. He left the office late so that he could get a seat on the Tube (direct line to Wimbledon); and on his way back home he would then drop into a local hostelry for "a quick pint". In MK he and Nancy bought bikes for cycling round the MK "red-ways for cycles"; he simply walked across one field between his house and the campus; he joined in the life of the Plough pub in Simpson (his MK village), getting to know all the regulars and going on regular coach trips to things in London from the pub.

In 1986, after five years at the OU he decided finally to retire to York, where Hayman and his former student Sheil-Small were then based. I was very sorry about him going, but the USS rules meant that he could not have stayed on much longer anyway as a research fellow at the OU. Another attraction of York was that it was where Nancy's brother lived. I believe that unfortunately he died not long after Jim had gone to York. However, Fiona moved to York to be near them, so that seemed to compensate.

While at the OU he helped to attract Phil Rippon from Cork to OU. We had an irregular series of complex analysis visitors and seminars (including Brian Twomey, Abdullah Lyzzaik (Lebanon) and Tom MacGregor (Albany)). He also, jointly with me, ran several of the UK's One Day Function Theory Meetings, that had started in 1981, getting some OU funding to help prime the pump in the early years.

I saw little of him once he had gone to York. There was a One Day meeting at York in honour of his 70th birthday. But by then Nancy needed his care and attention, and Jim had to miss the evening pub dinner in his honour. That was very sad indeed.

He took care of Nancy at home for some years, then visited her daily in a nursing home in York till she died.

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In recent years, he told me that he was becoming a bit of a recluse, though he still liked to hear about the various mathematicians in the OU whom he had known and gossip about any other complex analysts across the globe that he had known. More recently, he started not to answer the phone, and to make excuses to ring off when you did get him. I think he rather lost heart after Nancy's death.'

David Brannan also e-mailed about 45 people about Jim's death. Among twenty replies were the following.

From Peter Barnes, a former neighbour and OU colleague:
'I didn't know Jim as a Mathematician and scholar, but I remember him as a neighbour in Simpson in the 1980s. My abiding memory of him was in the garden raking the moss out of his lawn in a determined and enthusiastic fashion. He was probably thinking maths as he set to.'

And from Professor Mike Grannell:
'I remember him as one of our lecturers at IC. One particularly amusing incident was occasioned by the vertical sliding blackboards. He obviously had great strength in his arms and used to pull these up and down with one hand. They had two handles about four feet apart. One day he pulled so hard, that the board came free from its runner and fell to the floor with a loud crash, narrowly missing him.'

The above tributes show how much Jim Clunie was loved and respected as a great Mathematician and a great friend.

2. *Mathematical contribution*

by P. J. Rippon

Jim Clunie was a hugely accomplished complex analyst who made highly influential contributions to the areas of entire functions, meromorphic functions, univalent functions, polynomials, harmonic functions and univalent harmonic functions. Many of his results continue to be used by complex analysts and his papers are regularly cited as the definitive sources for their ideas and techniques.

At Aberdeen, Jim's PhD supervisor Archibald James Macintyre encouraged him to work on entire functions, then known as integral functions, and in particular the problem of describing the behaviour of a transcendental entire function f near a point $z = re^{i\theta}$ where f is relatively large compared to its maximum modulus $M(r) = \max_{|z|=r} |f(z)|$. Results on this behaviour had been obtained earlier by distinguished authors such as Wiman, Valiron, Saxer and Macintyre himself. Jim's thesis was entitled 'On certain topics concerning the extremal behaviour of functions' and led to two papers, [**2**, **7**], in the Journal of the LMS, where he was to publish many papers subsequently. In these papers, he worked with the 'maximum term'

$$\mu(r) = \max\{|a_n|r^n : n \geqslant 0\}, \quad r > 0,$$

of the Taylor series of f to prove the strongest results so far on this problem and established what became the 'modern' approach to the subject, now known as Wiman–Valiron theory, a fine achievement for a PhD thesis. His approach differed from previous work on the problem and involved lengthy complicated calculations, which he modestly described as follows.

'It seems desirable to describe the general outlines of the arguments developed below, in order to indicate these differences and also because their underlying simplicity is obscured in the formal account by calculations, which from the logical point of view are relatively trivial.'

This combination of new ideas, intricate calculations, thoughtful explanations and modesty characterized Jim's work throughout his distinguished career.

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Wiman–Valiron theory itself has become a subject of great importance with many applications, for example, to complex differential equations, and detailed accounts of the many aspects of the subject have been given by Fuchs ⟨**7**⟩ and Hayman ⟨**9**⟩, with more recent treatments in ⟨**3**, **5**⟩, for example.

Jim wrote many further papers on entire functions and often used the maximum term of their Taylor series to great effect, as in [**30**] where he showed that if ϕ is any convex increasing function such that $\phi(t)/t \to \infty$ as $t \to \infty$, then there exists a transcendental entire function f such that

$$\log M(e^t) \sim \phi(t) \quad \text{as } t \longrightarrow \infty,$$

a result later refined in the paper [**43**] with Kövari. This result provides a convenient means of obtaining examples of entire functions for which $M(r)$ has particular properties; see ⟨**16**⟩, for example. He also wrote two papers with Hayman, [**29**, **32**], which carefully compared the size of the maximum term $\mu(r)$ of f with its maximum modulus $M(r)$ and with various means of f.

Jim's early papers showed his great fascination for entire functions; for example, in 1955 he published eight papers, six of which were on various aspects of the behaviour of entire functions. However, after moving to Imperial College in 1956 he was inevitably drawn to work on univalent functions (also known as schlicht functions), which at the time was a highly active area of study dominated by the celebrated Bieberbach conjecture from 1916. This states that if f is in the class S, that is, $f(z) = z + a_2z^2 + a_3z^3 + \cdots$ is analytic and one-to-one in the open unit disc $\{z : |z| < 1\}$, then $|a_n| \leqslant n$, for $n \geqslant 2$.

In 1959, it was known only that $|a_n| \leqslant Cn$ for $n \geqslant 2$, where $C > 1$, that the inequality $|a_n| \leqslant n$ holds for $n = 2, 3$ and 4, and that it is true asymptotically. The conjecture was also known to be true for subclasses of S such as 'starlike' univalent functions, ones for which every point of the image of f can be joined to the origin by a line segment in the image.

Later Jim proved coefficient results for many subclasses of S, in joint work with others, but at first he considered the analogous coefficient problems for univalent meromorphic functions, those of the form $g(z) = z^{-1} + b_0 + b_1z + b_2z^2 + \cdots$ that are defined and one-to-one in the open unit disc. At the time, these problems appeared to be even more difficult than those for the class S but by an ingenious transformation he was able in [**22**] to obtain the sharp estimate $|b_n| \leqslant 2/(n + 1)$ for $n \geqslant 0$ in the starlike case, a result he liked so much that he gave the proof in his inaugural lecture at Imperial College in 1966!

At about the same time Jim also published the paper [**23**] in *Annals of Mathematics*, which made progress on the coefficient problem for such univalent meromorphic functions in general by constructing an example to show, quite unexpectedly, that *no* estimate of the form $b_n = O(n^{-1})$ as $n \to \infty$ is possible for this larger class. On the other hand, in [**37**] he and Pommerenke showed that the estimate

$$b_n = O(n^{-1/2-1/320}) \quad \text{as } n \longrightarrow \infty$$

does hold. Without the small constant $-\frac{1}{320}$, this estimate is 'obvious' from simple area considerations. These results led to speculation about what the best possible estimate for the coefficients b_n of the form $O(n^{-c})$ is. The corresponding value of c, the so-called 'Clunie constant', is still unknown.

Eventually, the Bieberbach conjecture itself was proved to be true by Louis De Branges in 1984, with a highly complicated argument later simplified somewhat by Milin, and by Fitzgerald and Pommerenke. This solution came as a great surprise to complex analysts, including Jim; see ⟨**15**, **6**, **10**⟩ for accounts of the work on univalent functions at various stages of the subject's development.

Jim also proved fundamental theorems about general meromorphic functions, many of which can be found in the book ⟨**8**⟩, such as the following result, known as the Tumura–Clunie

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theorem: if f and g are entire functions such that

$$a_n f^n + a_{n-1} f^{n-1} + \cdots + a_0 = b e^{ng},$$

where $a_n, a_{n-1}, \ldots, a_0$ and b are entire functions of smaller growth than that of f, and $a_n \not\equiv 0$, then

$$a_n f^n + a_{n-1} f^{n-1} + \cdots + a_0 = a_n \left(f + \frac{a_{n-1}}{n a_n} \right)^n .$$

The above theorem had been stated by Tumura in a paper in 1937 but his proof was based on an unjustified assumption, and Jim [**27**] gave a correct proof of a much more general result. Jim's proof was based on Nevanlinna theory, in particular on a new result in Nevanlinna theory, now known as 'Clunie's lemma', which has had a huge range of applications in the theory of complex differential equations. For further developments of this area, see ⟨**14**, **12**⟩.

The paper [**35**] with Hayman is also concerned with entire and meromorphic functions, giving estimates for their spherical derivatives. These results have recently become important in the theory of normal families, in particular in relation to applications of the widely used 'Zalcman lemma'.

Another notable paper is [**40**]. Here Jim used a completely ad hoc and ingenious argument to prove that if f is a transcendental entire function, then ff' takes every finite non-zero value infinitely often. This finished off a conjecture of Hayman for entire functions, which was later extended in full to meromorphic functions by Bergweiler and Eremenko in ⟨**4**⟩.

Another result that carries Jim's name is the 'Clunie–Jack lemma'. This states that if f is analytic in a neighbourhood of $\{z : |z| \leqslant r\}$, with a zero of order m at 0, and $|f|$ takes its maximum value on $\{z : |z| = r\}$ at ζ, then

$$\frac{\zeta f'(\zeta)}{f(\zeta)} \in [m, \infty).$$

This result was given by Jack in the paper ⟨**11**⟩ as part of a technique learned from Jim, and various versions of it are used extensively, for example, in the theory of differential subordinations; see ⟨**13**⟩.

The paper [**47**] has also seen many applications. In this paper, Jim compared the maximum modulus and the Nevanlinna characteristic of the composition of a transcendental meromorphic function f and a transcendental entire function g with those of f and g. The estimates he obtained remain the definitive ones of this type.

Jim's approachable manner and skill at solving 'hard' problems meant that he was an ideal co-author, and he wrote papers with at least 35 other mathematicians, including one with Paul Erdős and ten with Milne Anderson, who had been Jim's PhD students. One of Jim's most influential papers [**50**] was written jointly with Anderson and Pommerenke, while the latter was visiting Imperial College. This paper concerned Bloch functions and normal functions. These had been considered quite extensively previously, but in this paper the authors established a firm basis for many aspects of their future study.

A function f that is analytic in $D = \{z : |z| < 1\}$ is called a *Bloch function* if

$$\sup_{z \in D} (1 - |z|^2)|f'(z)| < \infty,$$

or equivalently if the radius of the largest schlicht disc (a disc that is a one-to-one image under f) in the Riemann surface of f with centre $f(z)$ is uniformly bounded for $z \in D$. Bloch functions have a close relationship with univalent functions, and they form a linear space and indeed a Banach space under the norm

$$\|f\|_{\mathcal{B}} = |f(0)| + \sup_{z \in D} (1 - |z|^2)|f'(z)|.$$

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In the paper [50], there is first an account of the functional analytic aspects of the space $\mathcal{B}$, of its subspace $\mathcal{B}_0$, where $(1-|z|^2)|f'(z)| \to 0$ as $|z| \to 1$, known as 'little Bloch', and of the conjugate space of $\mathcal{B}$. The authors then prove new results about the distribution of the zeros of Bloch functions, including the fact that the zeros in any disc in D that is tangent to ∂D at just one point satisfy the Blaschke condition, and new results about the behaviour of the Taylor coefficients a_n of f, for example, showing that a lacunary series is in $\mathcal{B}$ if and only if (a_n) is bounded. Finally, they discuss results about the boundary behaviour of Bloch functions and more generally of normal functions. A function f that is meromorphic in D is called a *normal function* if

$$\sup_{z\in D}(1-|z|^2)\frac{|f'(z)|}{1+|f(z)|^2} < \infty.$$

For example, they show that if

$$(1-|z|^2)\frac{|f'(z)|}{1+|f(z)|^2} \longrightarrow 0 \quad \text{as } |z| \longrightarrow 1,$$

and Γ is an arc in D ending at $e^{i\theta}$, then the angular cluster set of f at $e^{i\theta}$ is contained in the cluster set of f at $e^{i\theta}$ along Γ, thus generalizing Lindelöf's classic theorem about the boundary behaviour of univalent functions.

This foundational paper on Bloch functions and normal functions has been referred to many times subsequently, partly because the authors ended their paper with a list of tantalizing questions about the functions, but mainly because they demonstrated in the paper that these classes have a rich structure that demands further investigation, with significant relationships to many other classes of interest, and also because these classes of functions are capable of generalization to other contexts such as the unit ball of $\mathbb{C}^n$.

Yet another hugely influential paper was [66] with Terry Sheil-Small, another of Jim's former PhD students. In this paper, the authors develop the theory of harmonic univalent functions, which arise in the study of minimal surfaces and in many branches of applied mathematics. Theirs was the first paper to study such functions in an analogous way to the class S, thus providing a firm foundation for huge amounts of later work.

A complex function $f = u + iv$ defined in a domain D is said to be *harmonic* if u and v are real harmonic functions. Such a function can be written in the form

$$f = \bar{g} + h,$$

where g and h are analytic in D. When D is the unit disc, f can be expanded as

$$f(re^{i\theta}) = \sum_{-\infty}^{\infty} a_n r^{|n|}\, e^{in\theta},$$

where

$$g(z) = \sum_{1}^{\infty} \overline{a}_{-n} z^n \quad \text{and} \quad h(z) = \sum_{0}^{\infty} a_n z^n.$$

Let S_H denote the class of univalent harmonic functions in D such that $a_0 = 0$ and $a_1 = 1$, and S_H^0 be the subclass such that $a_{-1} = 0$, which contains the class S of univalent analytic functions defined above. The paper develops the theory of these functions along the same lines as the theory of functions in S, seeking distortion theorems and coefficient estimates for functions in the class and in various subclasses. For example, in the full class S_H they obtain the estimate that $|a_2| < 12.173$, which leads to distortion theorems as in the case of the class S, and for the subclass of functions in S_H^0 for which the image $f(D)$ is convex they obtain the best possible estimates

$$|a_{-n}| \leqslant \tfrac{1}{2}(n-1) \quad \text{and} \quad |a_n| \leqslant \tfrac{1}{2}(n+1),$$

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for $n = 2, 3 \ldots$. An extensive survey of the work done since the paper [**66**] can be found in ⟨**1**⟩.

Many of Jim's later papers deal with problems about entire functions, his first mathematical love. For example, in [**75**] he gave by far the best result in the direction of a conjecture of Pólya on the final set of an entire function, using his cherished Wiman–Valiron theory in a highly effective and ingenious way to prove that if f is a real entire function of order greater than 2, then either the unit disc contains a zero of the nth derivative for arbitrarily large n, or f grows essentially as fast on the real axis as it does in the plane.

Then in [**82**] Jim joined with Bergweiler and Langley to prove a conjecture due to Baker ⟨**2**⟩ concerning the periodic points of a transcendental entire function f, which stated that for any line L in the plane the second iterate of f has infinitely many fixed points that do not lie on L. In [**82**], the authors show that this is the case for the nth iterate of f for *any* value of $n \geqslant 2$. Not surprisingly a key role is played in the proof of this result by Wiman–Valiron theory.

3. *Research students*

At Imperial College Jim Clunie supervised ten research students, namely:

Qazi Ibadur Rahman (1961)
James Milne Anderson (1963)
Terry Sheil-Small (1965)
Derek Keith Thomas (1966)
David Alexander Brannan (1967)
John Brian Twomey (1967)
Raj Rani Mathur (1972)
Peter Oliver (1975)
Stephen Tudor Davies (1977)
Dinesh Singh (1981)

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