P. J. DANIELL

C. A. STEWART.

Percy John Daniell, who died in Sheffield on the 25th of May, 1946, was a member of the Council between 1927 and 1932 and was a Vice-President from 1929 to 1931. He was born in Valparaiso, Chile, on the 9th of January, 1889, and came to England with his parents in 1895. They settled in Birmingham and were actively associated at that period with the church of which Daniell's brother is at present the minister. After a year or two at a private school at Fladbury in Worcestershire, he went to King Edward VI High School, Birmingham, in January, 1900. He became an outstanding pupil of the school, not only as a scholar but also as a prefect and a member of the School Rugby XV. He won a major scholarship at Trinity College, Cambridge, and in Part I of the Mathematical Tripos in 1909 he was Senior Wrangler—the last of the long line that achieved this distinction. Other names occupying the leading places in the list for that year were E. H. Neville, L. J. Mordell, W. E. H. Berwick, C. G. Darwin, and G. H. Livens. In the following year he was placed in the First Class of Part II of the Natural Sciences Tripos and in 1912 he was awarded a Rayleigh Prize in the Smith's Prizes Examination. After leaving Cambridge he studied at Göttingen for two years under Born and Hilbert and he also held an appointment for one year as an assistant lecturer at Liverpool University. In 1914 he left England to be Assistant Professor at the Rice Institute, Houston, Texas, and in the same year he married Nancy Hartshorne of Birmingham. She survives him with their family of two sons and two daughters. Shortly after entering the Rice Institute he was appointed Professor of Mathematics there, a post which he held until 1923 when he returned to England as Professor of Mathematics at the University of Sheffield. In the meantime his mathematical work had been recognised by the award of the Sc.D. degree at Cambridge. Shortly before the recent war the state of his health caused some anxiety, but he recovered, though not, it would seem, completely. During the war he worked very strenuously not only for the mathematical department of the University but also on research problems sent to him by the Ministry of Supply. The strain of the war years became evident during the summer of 1945 when he was attacked by serious heart trouble. He recovered to some extent and decided to undertake the work of the session 1945-1946, but there seems little doubt that his life would have been prolonged if he had made a different decision. He continued with his many activities in a spirit of

great fortitude and determination, but early in May, 1946, he collapsed at his home and died a few weeks later without fully recovering consciousness.

Daniell was highly accomplished as a pure mathematician as well as an applied mathematician. His interests extended from the deep fundamental problems of rigorous analysis to the practical problems of statistics and applied science. The change in his studies in earlier days from pure mathematics to natural sciences was not so much, in his case, a change in direction as a broadening of interest that continued throughout the course of his life. He was a prodigious reader of scientific journals and was conversant with the latest developments in Physics, Chemistry and Biology as well as those in most branches of pure mathematics. As a consequence of this dispersion of interest, however, he seldom gave his undivided attention to the systematic development of particular lines of research and therefore the actual number of his publications does not provide a true measure of his intellectual powers and activity. Much of his time and energy was expended in advising and assisting research workers in many fields and it was only on rare occasions that he troubled to make a permanent record of his own contributions to the problems involved. His diverse investigations were related to a variety of topics ranging from generalised integrals, probability and mathematical logic to practical problems arising from electrical oscillations, flame motion and mine ventilation.

The most important of his earlier contributions to mathematical knowledge were those contained in a series of papers published partly in America and partly in England between 1918 and 1928, in which he developed the theory of generalised integrals and derivatives. He introduced a general form of integral (**6**) which was based on the existence of a set T_0 of bounded functions, closed with respect to the operations of multiplication by a constant, of the addition of two functions and of the taking of the modulus. Functional operations $I(f)$, $S(f)$ of integral form were postulated over T_0 satisfying certain conditions. The integral $S(f)$ was a generalised Stieltjes integral and $I(f)$ was analogous to a Lebesgue integral. The S integral was expressible as the difference of two I integrals. This work was in fact an extension of the Radon-Young integral from the field of space of a finite number of dimensions and included, for example, the case in which there were points in an enumerable number of dimensions. He gave, as illustrations of his general integral (**11**), a generalisation of the Lebesgue integral and also an infinitely multiple Stieltjes integral of positive type. The importance of the Daniell integral was recognised by other writers in this field who showed its applicability to a large number of cases and to functionals in particular. The application to functionals was also considered by Daniell who showed (**10**) how Volterra's definition of a

functional derivative can be extended to the case when it is expressible as a Stieltjes integral. Later (**15**) he extended the Volterra integral products in a similar way. A further application of the Daniell integral was made (**12**) to functions of limited variation in an infinite number of dimensions; and later (**18**) he made other generalisations to integrals with respect to functions not of limited variation and also to integrals of set-functions over a set. The more difficult problem of defining appropriately the derivatives that correspond to the generalised integrals was also considered. The problem was to define in any number of dimensions the derivative of an additive function of sets with respect to a second additive function of sets in such a way that the fundamental relationship between integration and differentiation should be preserved. The difficulty of arriving at a completely satisfactory definition produced a certain element of arbitrariness in his original net-derivative (**14**) based on Young's method for one dimension; but various improvements were made by means of a quasi-derivative (**19**), applicable to functionals, and later a core-derivative (**21**) and a central derivative (**26**). None was completely satisfactory to him owing to the restrictions that had to be imposed.

In spite of the very theoretical character of all this work, there was always behind it the background of physical ideas. Thus Wiener*, in an important monograph on Differential-space, obtained the average of a functional as a Daniell integral, citing as an application the Brownian movement of particles in a liquid. Again Daniell's integral product was applied by him to problems that occur in statistical biology and statistical economics (**17**). Also, in his generalisation of Green's theorem (**8**, **9**), the concept of a boundary as the boundary of a set, measurable Borel, and the use of functions of limited variation representing mass or electrical charge provided for him a true description of the physical reality.

It was natural that Daniell with his special gifts should be attracted to the study of symbolic logic and the foundations of mathematics, and he thought that this study should be an essential element in university training. He himself was skilled in the technique of the Principia Mathematica and he made contributions to discussions in philosophical journals.

The various investigations conducted by Daniell into problems of a practical and numerical character were in strong contrast to his earlier research into fundamental theory. In much of this work, carried on principally from 1930 and right through the war years, he acted often as an adviser to other workers rather than as an independent investigator, and it

* Wiener, "Differential-space", *Journal of Math. and Phys., Mass.*, 2 (1923), 131–174.

is natural that the permanent records of all his contributions should be few in number. He assisted, for example, in heat-conduction problems arising in the manufacture of steel, in questions relating to the safety of mines, in the clarification of difficulties encountered in the interpretation of statistics and, during the war, in oscillation problems connected with the automatic control of instruments. His paper to the Royal Society in 1930 on the Theory of Flame Motion (**28**) was directly related to experimental work carried out by the Safety in Mines Research Board in Sheffield. In this he obtained an improved formula for the theoretical velocity of the horizontal propagation of flame through a still mixture under such circumstances that the pressure produced by the combustion was vented freely into the atmosphere. Apart from such papers he did not publish much after 1930, but in 1940 he wrote an interesting paper on the region of convergence of double series (**29**), in which he extended the work of Lemaire.

The part that Daniell played during and after the war in research organised under the Ministry of Supply was significant and effective; and it is remarkable that, in spite of the deterioration of his health, he should have been so actively engaged in this work even up to the time of his death. He investigated problems that arose from the control of instruments which were designed to pick up and follow targets such as aeroplanes and ships and which required a high degree of accuracy, rapidity and sensitiveness. The problem of automatically tracking a target by radar methods is complicated by the fact that the reflected signals are disturbed by unwanted fluctuations. The problem of determining and overcoming these fluctuations was approached by the study of the frequency spectra of the radar information and a study of the various types of 'band pass filters' with their effect on the operation of the automatic control. In this work Daniell contributed appreciably not only to the mathematical techniques required but also to the more practical aspects of the problem. He very quickly familiarised himself with some very elaborate automatic control equipments incorporating metadynes and suggested how improved performance might be achieved. In addition, he was one of the foremost exponents in this country of the frequency response approach to the synthesis of automatic control equipment and he gave two papers to the Servo Panel on this subject. Daniell also tackled the problem of back-lash, an essentially non-linear problem that had scarcely been touched before he started work on it. His contribution could not have come at a more appropriate time and as a result of his work and his collaboration with Professor Tustin of Birmingham University a new approach to problems involving the dynamics of closed loop systems has been initiated. Daniell assisted materially the Mathe-

matics Branch of the Radar Research and Development Establishment on problems relating to the determination of frequency spectra of random information (e.g. radar misalignment signals). He was also consulted by the Admiralty on ultra-high frequency detection methods. One of the principal mathematical tools used by Daniell was the Laplace Transform and at the time of his death he was writing a comprehensive account of the application of frequency response methods to the design of servo systems. It is hoped that this work will be published by the Stationery Office in the not too distant future. The contributions that Daniell made in all this research were not only of the highest value in the prosecution of the war; they will also help us to understand many of the instrumentation problems which will arise in peace-time industry.

In educational affairs generally, Daniell took an important and active part. To the university itself he gave loyal and devoted service in the Faculties, the Senate and the Council, and he took a significant share in the fashioning of academic policy. His interests extended beyond the university, however, in many directions—to the training of teachers, for example, to the Mathematical Association, and to the School Certificate Examination. Of these activities, the most outstanding were those associated with the Joint Matriculation Board of the Northern Universities, on which he served as the representative of Sheffield for more than twenty years. He served on the awarding committee for thirteen years, a proof of the high regard the Board had for his fine judgment not only in mathematical matters but also in discussions involving wide educational issues. He was appointed Vice-Chairman in 1944 but with reluctance had to limit his activities after the onset of his first serious illness.

Daniell impressed all who came into contact with him by his great integrity of character and his sincerity of purpose. His decisions were made without prejudice and his judgments were unclouded by considerations of personal interest. In the many committees on which he served he was always much esteemed, not only for his personal qualities but also for the valuable contributions he made to the business under discussion. He disliked publicity and his tastes were simple. He delighted in good music, in books, in friendly discussion, in country walks and in the quiet pleasures of a happy family life. He made a valiant effort to see the fulfilment of his hopes, not merely to complete the mathematical work on which he was engaged but also to take an important part in the great development of the University now in progress. But the end came before these hopes were realised, depriving the University of a distinguished servant and his many associates of a counsellor and friend.

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Publications.

[The following abbreviations are used: *PLMS* for *Proc. London Math. Soc.* (2); *JLMS* for *Journal London Math. Soc.*; *PM* for *Phil. Mag.*; *BAMS* for *Bulletin American Math. Soc.*; *AM* for *Annals of Math.* (2); *AJM* for *American Journal of Math.*; *TAMS* for *Trans. American Math. Soc.*; *AMM* for *American Math. Monthly*; *PRS* for *Proc. Royal Soc.* (A); *QJ* for *Quarterly Journal* (Oxford); *MG* for *Math. Gazette.*

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30. " Remainders in interpolation and quadrature formulae ", *MG*, 24 (1940), 238–244.