EVAN TOM DAVIES

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Evan Tom Davies was born on 24 September, 1904, at the small village of Pencader, Carmarthenshire, the younger son of a farmer, Thomas Davies, and his wife Elizabeth. His parents were Welsh-speaking and throughout his life Ianto, as he was affectionately known to his many friends, retained a passionate regard for the Welsh culture.

He received his secondary schooling at Llandyssul County School. From here he went on to University College, Aberystwyth and graduated with honours in applied mathematics in 1924. He then moved to University College, Swansea where he had the distinction of being the first student to graduate with honours in pure mathematics. He stayed on to write a thesis on n-dimensional geometry, and for this he was awarded an M.Sc. in 1926.

Acting on the advice of Paul Dienes, who was lecturing in Swansea at this time, Ianto then turned his attention to the absolute differential calculus. This was being developed as one of the chief mathematical tools in the theory of relativity. So in August 1926 he travelled to Rome to study under Tullio Levi-Civita.

This was the beginning of many eventful visits to Europe and in later years he had many tales to tell of his experiences. He used to recall that he was arrested three times, although he was lucky enough to escape gaol. On the first occasion in Rome, a party of students were making merry in the Colosseum and since many nationalities were represented among them, they decided that each should sing a song of his native land. As Wales begins with a W, Ianto had the honour of singing last. The strains of "Land of my Fathers" were ringing strongly through the Colosseum when the police arrived.

Nearly all the examinations in Rome were oral and open to the public. By now Ianto spoke fluent Italian. He was examined by eleven professors on a day when the temperature was over 100°F. However, he achieved success and was awarded his doctorate. But overwork had brought on a temporary breakdown in health and another year was to pass before he was completely fit and ready to travel again.

In 1928 he left home again to spend a year in Paris at the Sorbonne and the Collège de France. As in Italy, he met many young differential geometers who later became leading mathematicians and were to remain life-long friends.

He was appointed assistant lecturer at King's College, London in 1930 and so commenced his career as a University teacher and creative mathematician. In those days young lecturers had to be prepared to teach the junior classes. Ianto became very accomplished in teaching generally, and especially in dealing with the Engineers who could become very lively at times. He travelled extensively in Europe in the

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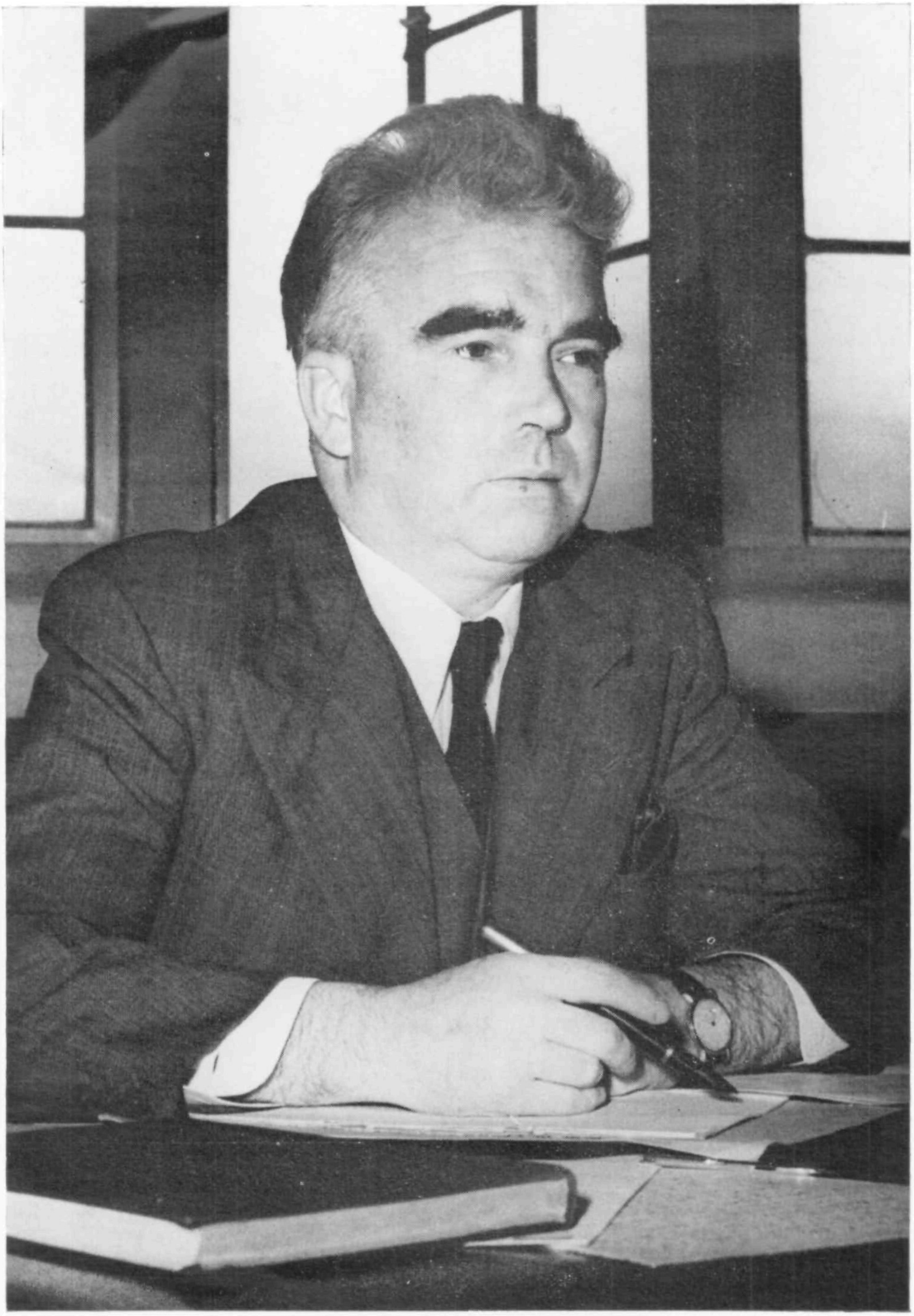

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EVAN TOM DAVIES (1904–1973)

vacations and became fluent in five languages. Every Christmas he would join a party of friends for winter sports and he soon acquired some ability as a skier. In those pre-war days in Switzerland the atmosphere in the evening was elegant and relaxed. Ianto was an excellent raconteur and these were the hours in which he shone most brilliantly. He also became a keen Wimbledon tennis fan and for many years he would take an early place in the queue for the Centre Court on the First Thursday and the Second Wednesday of the tournament. All this time his mathematical research was developing. He was promoted to lecturer in 1935 and became a University Reader in 1946.

A few months later Ianto accepted the Chair of Mathematics at University College, Southampton. At that time he was the only professor of mathematics in the country who had not studied at either Oxford or Cambridge. With his natural gaiety, his flair for dancing and for college functions generally, he quickly became a highly popular and respected figure. In those early days his department was small and its annual meeting would take place somewhere in the country after a strawberry and cream tea provided by the professor. In 1952 University College became Southampton University, and the department grew from a staff of six to a multi-professorial one of over sixty. He played an increasing role in the higher administration of the University and acted as Deputy Vice-Chancellor (1954–1957) and Dean of Science (1965–1967).

During the early years of the War, King's College had been evacuated to Bristol. Ianto made friends as readily here as elsewhere and in 1941 he married Margaret Helen Picton. It was a devastating blow to him when she died in London in 1944. Happily, however, after moving to Southampton he met Hilda Gladys Boyens. They married in 1955 and had one son, Geraint. Their home at once became a centre for Ianto's hospitality and the department still enjoyed its strawberry and cream teas, though its official meetings were held elsewhere. He enjoyed many associations outside the University and perhaps the most important to him was the Rotary movement, of which he was a very loyal supporter. In his travels he met Rotarians and addressed Rotary Clubs all over the world.

He took a keen and friendly interest in all his students and research flourished under his guidance. In 1954 and 1962 he made extensive tours in South East Asia visiting many countries, meeting many mathematicians, most of them already personal friends, and addressing many Universities. What he saw and heard about the economic difficulties which face the developing countries concerned him deeply, and he was often to speak about this on his return. These tours had their lighter side as well. On one occasion he found himself billed to appear on the Japanese television to discuss geometrical problems relating to the design of the kimono with his old friend and colleague Kentaro Yano.

On his retirement in 1969 at the age of 65 the University recognised its indebtedness to him by conferring on him the title of Emeritus Professor, and establishing the E. T. Davies Prize for the best honours graduate in mathematics. But the retirement was in name only, for he then proceeded to Canada and became a professor in the University of Calgary. He quickly established himself in the Department of Mathematics, and enjoyed both the countryside and the way of life in that hospitable department. Finally in 1971 he accepted a similar position in the University of Waterloo, Ontario. Once more he soon made many friends and was prominent in their Friday afternoon ritual of saying goodbye to the week over a mug of beer in the Faculty Club. It was on this campus that he was taken ill on 8 October 1973, and within hours he died. He was then in his sixty-ninth year. Thus he completed his life, as he would have wished, in harness.

Several of Ianto's friends have kindly helped me prepare this notice. For help with the following appreciation of his mathematical work I am particularly grateful to Dr. F. Brickell.

Suppose that J is a geometric object field on a differentiable manifold M. A local transformation $\phi : U \to M$ of M induces on U a local geometric object field $\phi_* J$ of the same type as J itself. ϕ is said to be a local automorphism of J if $J = \phi_* J$ on U. A vector field X on M is called an infinitesimal transformation of M. Each point m of M admits a neighbourhood on which X induces a local one-parameter group of local transformations ϕ_t. If all such transformations are local automorphisms of J then X is said to be an infinitesimal automorphism of J.

If X is any infinitesimal transformation of M the Lie derivative $L_X J$ of a tensor field J is defined by

$$(L_X J)_m = \lim_{t \to 0} \frac{1}{t} (J_m - (\phi_{t*} J)_m)$$

and it is a tensor field of the same type as J itself.
The same definition is used when J is a linear connection and then $L_X J$ is a tensor field of type (1, 2).

The Lie derivative had just been defined when Davies commenced his research, and he was to exploit it in a series of early papers [1], [2], [3], [4], [5], [6], [7]. He used it, for example, to study the effect of an infinitesimal transformation on a submanifold of a Riemannian manifold. In this way he obtained generalisations of the second and third fundamental forms of a surface in euclidean space and of the Gauss and Codazzi equations. He was also led to some analogues of the Frenet equations for curves.

If g is a Riemannian metric tensor on a manifold M then X is an infinitesimal isometry if it satisfies Killing's equations

$$L_X g = 0.$$

Such a vector field must necessarily satisfy the equations

$$L_X \Gamma = 0$$

where Γ is the Riemannian connection on M. Using the associated convariant derivation ∇ and the curvature tensor R, W. Slebodzinski showed that the integrability conditions of this mixed system of partial differential equations are

$$L_X g = 0, \quad L_X R = 0, \quad L_X(\nabla R) = 0, \quad L_X(\nabla^2 R) = 0, \ldots$$

Several generalisations of Riemannian geometry had been proposed, and during this later period at King's College Davies played a leading part in their development. In a Finsler manifold M we have on each tangent space $T_m M$ a scalar product which depends on the choice of a line element at m. In a Cartan manifold it depends on a hypersurface element at m. Davies extended much of his previous work to these spaces **[8]**, **[11]**, **[12]**, **[15]**. In particular he introduced Lie derivation and obtained analogues of Slebodzinski's results.

Finsler and Cartan geometries have their origins in the calculus of variations, and they have many formal similarities. They were regarded from a single point of view by J. A. Schouten and J. Haantjes, who considered a manifold M with a scalar product on $T_m M$ depending on the choice of a covariant or a contravariant relative vector at m. Davies also wrote on this topic **[14]** and it led him to a unified presentation of his earlier work.

His first publications **[16]**, **[17]** after appointment to the Chair of Mathematics at Southampton contained calculations relating the first and second variations of the length integral in a Finsler manifold to the second fundamental form of a submanifold. This work was inspired by the early chapters of the book by Marston Morse on the calculus of variations. Although Davies published little in global differential geometry, he was alive to the possible applications of the calculus of variations in this field. The famous sphere theorem of H. E. Rauch was a particular centre of interest, and he gave an invited lecture on this topic to the British Mathematical Colloquium in 1961. A course of lectures on variational spaces given later in that year at the University of Rome provides further evidence of his appreciation of global problems.

It was in 1950 that Davies began to work on contact transformations, a subject that was to lead to a long collaboration with Yano. The starting point was a study of papers published by T. C. Doyle and by L. P. Eisenhart in the Annals of Mathematics in 1941 and 1948 respectively. The paper by Doyle contained a geometrical theory of contact transformations. Davies realised that the methods used were ones in which he had long been expert, and he was able to generalize Doyle's theory **[19]**, **[20]**. Yano had also worked in this field and they gave a unified presentation of this topic in a joint paper in 1954 **[21]**. The last chapter contained new viewpoints on Eisenhart's use of contact transformations to deduce Finsler geometry from Riemannian.

Davies' collaboration with Yano continued until his death. It was mainly concerned with the geometry of the tangent bundle TM of a manifold M and in particular, with the relation of this geometry to a given Finsler structure on M. The situation is as follows. Let n denote the dimension of M and let $\pi : M' \to M$ be the natural projection of the open submanifold M' of non-zero vectors in TM onto M. The induced vector bundle $\pi^{-1}(TM)$ is the set of pairs (u, v) where $u \in M'$, $v \in T_{\pi u} M$. It has a canonical cross-section $\eta : u \to (u, u)$. Elie Cartan showed that the Finsler structure on M determines a scalar product and a linear connection in $\pi^{-1}(TM)$. The tangent vectors X to TM' for which the covariant derivative $\nabla_X \eta$ is zero provide an n-dimensional distribution H transversal to the fibres of M'. Let V denote the complementary distribution tangent to the fibres of M'. We can regard H and V as subbundles of TM' and, as they are both naturally isomorphic to $\pi^{-1}(TM)$, they carry the scalar product and linear connection determined by the Finsler structure. Consequently TM', which is the Whitney sum of H and V, carries a Riemannian metric and a linear connection determined by the Finsler structure. This connection is a metric one with torsion. The relations between these structures and an almost complex structure which can be defined using the distribution H were discussed in a series of papers by Yano and Davies [22], [28], [29], and these ideas motivated several other papers [24], [30], [31], [32], [33], [37].

Concurrently with this work Davies continued his study of a general areal structure on M [25], [27], [34], [35], [36]. To describe this structure some further notation must be introduced. A set of p linearly independent tangent vectors is called a p-frame of M and the set M^p of all such frames is a fibre bundle over M. It is acted on by the group $L_p{}^+$ of $p \times p$ matrices $\psi = [\psi_{\beta\alpha}]$ with positive determinants, the action being

$$e = [e_1, \ldots, e_p] \to e\psi = [f_1, \ldots, f_p]$$

where $f_\alpha = \sum_\beta e_\beta \psi_{\beta\alpha}$. Suppose that $p < n$. An area measure on M is a positive function L on M^p with the homogeneity property $L(e\psi) = (\det \psi)\, L(e)$. An areal space is a manifold together with an area measure satisfying a regularity condition (which I will not make explicit). The spaces of Finsler and Cartan arise when $p = 1$ and $p = n-1$ respectively. In both of these cases Cartan established a satisfactory local theory. Although it is probably true to say that no really satisfactory local theory has yet been found in the general case, substantial progress has been made by several authors including A. Kawaguchi, H. Rund and Davies himself. The subject was a main topic of Davies' lectures in Japan during his visit of 1962.

I hope that the above paragraphs give some impression of the width of E. T. Davies' interests in differential geometry. He was a leading member of the group of classical differential geometers whose far ranging applications of the tensor calculus provided one springboard for the outstanding success of modern global differential geometry.

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