CLIFFORD HUGH DOWKER 1912–1982

facing p. 535

OBITUARY

CLIFFORD HUGH DOWKER

Clifford Hugh Dowker was born in 1912 in Western Ontario, and grew up in a rural community, where his family owned a small farm. His ancestors on his father's side were of Yorkshire origin, while his mother was a McGregor of Scottish descent.

This rural background might appear unexpected for an important mathematician. Indeed, Hugh was the first Dowker to go to High School. Neither of his brothers had academic careers. His elder brother, Gordon, left school at thirteen and worked in the Canadian forests. His younger brother, Arthur, followed the family tradition of working as a farmer.

Hugh Dowker's first school was a one-room country school to which he had to walk a couple of miles. His next school was the High School in Parkhill, where the mathematics teacher appears to have had little understanding of the subject. Hugh was paid to stay in after school in order to teach mathematics to his teacher!

There was one teacher in Parkhill—not a mathematician—who seems to have had an important influence on Dowker. This was a teacher with a deep knowledge of wildlife, botany and geology, who took him and other pupils to the Muskoka Lakes and the Bruce Peninsula. This experience probably had a lasting effect on Dowker, who, throughout his life, displayed a keen interest in the countryside around him.

When Dowker was seventeen, he went to the University of Western Ontario, having been awarded a scholarship on the basis of his excellent examination results. He intended to be a school teacher. However, unexpectedly for himself and his family, his talents were to lead him into mathematics. This was a period of some penury for Dowker; the room that he shared with another student was heated by a chicken-coop heater, and he lived largely on tinned salmon and carrots—the cheapest foods available.

He studied a variety of subjects, including physics and economics, and received his BA degree in 1933. Then, because of his evident brilliance in mathematics, he was persuaded to continue his studies at the University of Toronto. After obtaining an MA there in the following year, he was advised to go to Princeton University to study under Lefschetz.

It was at Princeton that Dowker became fully aware of the power and beauty of mathematics, and that he became an active topologist, running one of Lefschetz' seminars. He obtained his Ph.D. there in 1938. Apart from Lefschetz, the mathematicians who were to have an important influence on Dowker included Alexandrov, Fox, Hurewicz and Steenrod.

He subsequently held a position as instructor at the University of Western Ontario, and was then an assistant to Von Neumann at the Princeton Institute for Advanced Study. He then went to Johns Hopkins University as an instructor. It was here that he met Yael Naim, whom he was to marry in 1944. Yael, at the time, was a young graduate student who had come to Johns Hopkins from Israel. She is herself a highly gifted mathematician, who was to become well-known for her work in ergodic

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theory. Dowker once remarked that all measure theorists marry topologists—this law being exemplified by the Dowkers, Rudins and Stones!

In 1943 Dowker was seconded to the United States Air Force as a civilian adviser, and carried out work on gunnery and the trajectories of projectiles, which took him to Libya and Egypt. Then, from 1943 to 1946, he and Yael both worked at the M.I.T. Radiation Laboratory. After the war he was an associate professor at Tufts, and then a visiting lecturer at Princeton and at Harvard.

This was the period of McCarthyism, when the atmosphere in North American Universities was very difficult. Several of Dowker's friends in the mathematical community were severely harassed, and one had been arrested. Hugh and Yael decided to leave North America. They came to England in 1950, where Yael had a post at the University of Manchester, and where Hugh was soon appointed to a readership in Applied Mathematics at Birkbeck College.

Although Dowker is famous for his work in the purest and most abstract branches of mathematics, it is a mark of his versatility that he deserved to hold a post as an applied mathematician. Indeed, he has made a real contribution to applied mathematics, through his work on projectiles and on servo-mechanisms [5].

In 1962 Dowker was appointed to a personal chair at Birkbeck College. He remained at Birkbeck until his retirement in 1979. He died in London in 1982, after a long and difficult illness against which he had struggled for seven years. He had been a member of the Society since 1951.

In manner, Dowker was reserved and gentle, with an innate dignity and a penetrating wit. He possessed a high degree of integrity and moral strength which enabled him to endure seven years of illness uncomplainingly. Although supremely tolerant towards others, he had only the highest standards of behaviour for himself. He was totally without ostentation or pretention and totally disinterested in wealth, honours or managerial power.

Hugh Dowker was unfailingly kind and generous and was always ready to spend time in aiding others. Over the course of some thirteen years, he and Yael did a great deal of work with children who were sent to them by the National Association for Gifted Children. The Dowkers were very committed to this work and were highly successful in it. They wrote an interesting joint paper [32], in which they describe how they had helped more than thirty gifted children—many of whom had difficulties in school—to experience the delight of mathematical discovery. It should not be thought that it was only with gifted children that Dowker was concerned. He had an affection for all young people and was known among his students for his helpfulness and patience. Even children in his neighbourhood would come to him for help with their homework.

Hugh and Yael were well known for their kindness and hospitality, which earned them many deep and lasting friendships in the mathematical community.

Dowker was widely travelled. In his early twenties he had twice "hoboed" across the United States and Canada, jumping on and off freight trains at suitable points. Later, as a mathematician, he held posts as a visiting professor in Russia, Israel, India and Canada. He also spent some time working on a kibbutz in Israel. He was able to speak Russian and knew some Georgian and some Hebrew. He loved the countryside, and often went walking or mountain-climbing in the national parks and in Switzerland.

That Dowker was impressively knowledgeable in mathematics was widely known; his letters and papers contain a wealth of answers to other mathematicians' questions. What might be less widely known, is that he had a deep knowledge of many subjects, including Georgian culture, the early history of Christian religions, the geography of many lands and localities and the history and anthropology of arithmetic in different societies. He always had a thorough knowledge of the history and culture of whatever area he happened to live in.

He had a deep love of mathematics, and continued to work even when he was weak and in pain. I visited him during the last days of his life, and was very moved by his determination to discuss mathematics even though he scarcely had the strength to speak.

He was a man who was respected and loved by his students, his many friends and the world-wide mathematical community.

Mathematical work

Dowker's mathematical work lay mainly in the field of topology. Although the number of his published papers is not large, they have been remarkably influential. His name is very widely quoted among topologists—indeed, it has been said that his best known paper [**9**] is one of the most frequently quoted in the whole of mathematics.

Dowker's papers are always striking, not only for their fundamental importance, but also for the elegance and clarity with which they are written. They contain a wealth of ingenious examples, often answering difficult problems posed by other mathematicians. He was constantly concerned to find the "right" basic definitions and axioms, and this led to his proving very general results under very few assumptions.

The results of his Ph.D. thesis were announced in [**1**] and presented, with additional material, in [**3**]. The theme of these papers was the extension of basic theorems in homotopy theory from compact metric to normal or paracompact spaces, a key role being played by the concept of uniform homotopy. [**3**] contains a pioneering exposition of Čech cohomology from a geometric point of view, and includes a proof of the surprising fact that the first Betti number of the real line is c. It also provides a proof of the important fact that the same covering dimension of a normal space is obtained whether finite, star-finite or locally finite covers are used. The techniques employed, involving the use of canonical maps to nerves of covers, led naturally to [**6**], in which it was shown that a canonical map of a space into the nerve of an arbitrary open covering exists if and only if the space is paracompact and normal. Although there are two different ways of topologising the nerve of a covering (called "geometric" and "natural"), they are shown to be equivalent in this context.

This observation led to a systematic study of metrisable topologies on infinite complexes in [**10**], in which the important fact is established that all the various reasonable topologies have the same homotopy type. In this paper, a question of J. H. C. Whitehead is answered by an example of two CW complexes whose product is not CW. The study of infinite complexes was continued in subsequent papers ([**18**, **21**]). An interesting corollary of theorems proved in [**21**] is that isomorphic Euclidean complexes are homeomorphic.

Dowker's interest in locally finite covers led to [**4**], in which it was shown that paracompact metric spaces are precisely the spaces that can be embedded in Hilbert spaces. (It was not yet known that all metric spaces are paracompact.) This paper now provides a key step in the famous Bing–Nagata–Smirnov metrisation theorem.

[9] is probably Dowker's best known paper. This is the paper which introduced the important concept of countable paracompactness. In homotopy theory, the properties of a product space $X \times I$. where I denotes the closed unit interval, are fundamentally important. In [9], Hugh showed that $X \times I$ is normal precisely in the case in which X is countably paracompact. He gave several striking and useful characterisations of countable paracompactness, and asked whether there were any normal spaces which failed to be countably paracompact. This question turned out to be one of the most challenging in general topology. It was finally answered twenty years later by Mary Ellen Rudin, who constructed an ingenious and intricate example of a normal space which was not countably paracompact. Spaces of this kind—which play a significant role in the study of non-metric spaces—are now known as "Dowker" spaces.

Dowker was constantly concerned with the fundamental definitions of homology and cohomology groups in general spaces. In [7] he showed how the Čech construction of direct and inverse limits can be used to deal with singular homology and cohomology. This work showed the identity of the Alexander and Čech cohomology groups in a wide class of spaces. In [8] he showed that the Eilenberg–Steenrod axioms were satisfied by Čech cohomology theory based on infinite open coverings. (It was known that the homotopy axiom failed to hold if finite covers were used.) The identity of the Čech and Vietoris homology groups and the Čech and Alexander cohomology groups was established in [11] for *arbitrary* spaces. A corollary of this is the fact that the Eilenberg–Steenrod axioms are satisfied by Alexander cohomology theory.

In [20, 22, 23], Dowker developed results in excision theory. In [20] he gave very general conditions for the strong excision property to hold for Čech cohomology. He also showed that some conditions, going beyond normality, are needed, thus proving a conjecture due to A. D. Wallace.

An interest in the extension of maps was implicit in [2, 3] and developed further in [12, 17, 18]. Dowker's main contribution was given in [17], where he gave very general conditions sufficient for extensibility, and gave criteria for spaces to be ANR and NES (neighbourhood extension) spaces. In an earlier paper [12], he had improved and generalised Hanner's characterisation of the separable metric spaces which are ANRs for normal spaces, and had extended it to non-separable spaces.

Dimension theory was a recurring theme in Dowker's work. [14] contains a thorough study of large inductive dimension (Ind) in completely normal spaces, and introduces the slightly more restrictive class of totally normal spaces, to which many of the principal theorems of classical dimension theory are extended. This study was continued in [15], where the subset theorem is extended to totally normal spaces. In this paper, Dowker shows that local dimension and dimension coincide in a wide class of spaces, and gives an example of a normal space in which they differ. In [19] it is shown that the dimension of a metric space can be defined in terms of a sequence of open covers satisfying the condition that the closures of the sets in the $(i+1)$-st cover form a refinement of the i-th cover. This paper also contains a new proof of the Katetov–Morita theorem which states that dim and Ind coincide for metric spaces.

[24] contains a generalisation of the concepts of proximity and uniformity, in which symmetry is not required. By discarding the property of symmetry, it becomes possible to define quotient, open and closed maps. Product structures are also defined and investigated. A question of Smirnov is answered by an example of a (symmetric) proximity space which has no finest consistent (symmetric) uniformity.

In joint papers with myself, published from 1966 onwards, Dowker did pioneering work in the study of frames. A frame is a complete lattice which satisfies the infinite distributivity condition: $x \wedge \bigvee_{\alpha} x_a = \bigvee_{a} (x \wedge x_a)$. Lattices of this kind are significant in topology, as the lattice formed by the open subsets of a topological space is an example of a frame. They are also significant in other fields; for example, they furnish models for intuitionist logic. In recent years, they have aroused interest because of their applications in sheaf theory and in the new field of topos theory. Dowker was responsible for establishing some of the basic features of the category of frames, including the properties of quotients and co-products. (It should be noted that some of these ideas were developed independently by other mathematicians, including J. R. Isbell.)

[34] gives an indication of the versatility of Dowker's work. In this paper, written with M. Thistlethwaite, a complete classification of all 12,765 knots with at most thirteen crossings is given. The authors pioneered the use of the computer for the purpose of tabulating knots, and developed some strikingly ingenious new techniques.

Dowker was also interested in the general theory of categories. In **[26]** he showed how the connecting morphism of homology theory arises from a functor definable in any abelian category. In his final paper **[35]**, written during the last weeks of his life, he proved the interesting fact that two categories must be isomorphic if there are functors between them which are injective on objects and have composites naturally equivalent to the identity functors.

Finally Dowker's influential lectures on sheaf theory **[16]** should be mentioned among his publications. They provide a very clear and careful exposition of the subject, with a wealth of illustrative examples, and were for some years the only source for several important results about sheaves and sheaf cohomology. Mathematicians, who are now experts in sheaf theory, have told me that it was these lecture notes which first awakened their interest in the subject.

This description of Dowker's mathematical work is not exhaustive. His command of mathematics was so wide that he has contributed to fields of which I am unaware. For example, I have recently learned of a theorem in pure geometry which bears his name.

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Bibliography

1. 'Hopf's Theorem for non-compact spaces', *Proc. Nat. Acad. Sci. U.S.A.*, 23 (1937), 293–294.

2. 'On minimum circumscribed polygons', *Bull. Amer. Math. Soc.*, 50 (1944), 120–122 [*Math. Rev.* 5, #153 (John)].

3. 'Mapping theorems for non-compact spaces', *Amer. J. Math.*, 69 (1947), 200–242 [*Math. Rev.* 8, #594 (Freudenthal)].
4. 'An imbedding theorem for paracompact metric spaces', *Duke Math. J.*, 14 (1947), 639–645 [*Math. Rev.* 9, #196 (Dieudonné)].
5. (with R. Phillips) Chapter VIII, *Theory of servo-mechanisms* (McGraw Hill, 1947), 340–360. (Publication of the M.I.T. Radiation Laboratory.)
6. 'An extension of Alexandroff's mapping theorem', *Bull. Amer. Math. Soc.*, 54 (1948), 386–391 [*Math. Rev.* 9, #523 (Dieudonné)].
7. (with W. Hurewicz and J. Dugundji) 'Connectivity groups in terms of limit groups', *Ann. of Math.*, 49 (1948), 391–406 [*Math. Rev.* 9, #606 (Freudenthal)].
8. Čech cohomology theory and the axioms', *Ann. of Math.* (2), 51 (1950), 278–292 [*Math. Rev.* 11, #450 (Freudenthal)].
9. 'On countably paracompact spaces', *Canad. J. Math.*, 3 (1951), 219–224 [*Math. Rev.* 13, #264 (A. H. Stone)].
10. 'Topology of metric complexes', *Amer. J. Math.*, 74 (1952), 555–577 [*Math. Rev.* 13, #965 (Spanier)].
11. 'Homology groups of relations', *Ann. of Math.* (2), 56 (1952), 84–95 [*Math. Rev.* 13, #967 (Spanier)].
12. 'On a theorem of Hanner', *Ark. Mat.*, 2 (1952), 307–313 [*Math. Rev.* 14, #296 (Michael)].
13. 'A problem in set theory', *J. London Math. Soc.*, 27 (1952), 371–374 [*Math. Rev.* 13, #924 (Gustin)].
14. 'Inductive dimension of completely normal spaces', *Quart. J. Math.*, *Oxford Ser.* (2), 4 (1953), 267–281 [*Math. Rev.* 16, #157 (Katětov)].
15. 'Local dimension of normal spaces', *Quart. J. Math. Oxford Ser.* (2), 6 (1955), 101–120 [*Math. Rev.* 19, #157 (M. Henriksen)].
16. *Lectures on sheaf theory.* Notes by S. V. Adavi and N. Ramabbadran (Tata Institute of Fundamental Research, Bombay, 1956, v+212+iv+iii pp. mimeographed) [*Math. Rev.* 19, #301 (M. F. Atiyah)].
17. 'Homotopy extension theorems', *Proc. London Math. Soc.* (3), (1956), 100–116 [*Math. Rev.* 17, #518 (J. Dugundji)].
18. 'Imbedding of metric complexes', *Algebraic geometry and topology*, A symposium in honour of S. Lefschetz (Princeton University Press, Princeton, N.J., 1957), pp. 239–242 [*Math. Rev.* 18, #920 (J. Dugundji)].
19. (with W. Hurewicz) 'Dimension of metric spaces', *Fund. Math.*, 43 (1956), 83–88 [*Math. Rev.* 18, #56 (Haskell Cohen)].
20. 'The excision theorem' (Russian), *Dokl. Akad. Nauk. SSSR*, 125 (1959), 1190–1192 [*Math. Rev.* 21, #3840 (Isbell)].
21. 'Affine and Euclidean complexes' (Russian), *Dokl. Akad. Nauk SSSR*, 128 (1959), 655–656 [*Math. Rev.* 22, #8483 (Kahn)].
22. 'The Kolmogorov–Aleksandrov duality theorem' (Russian), *Mat. Sb.* (*N.S.*), 50 (92) (1960), 247–255 [*Math. Rev.* 22, #12518 (Kahn)].
23. 'The map excision theorem' (Russian), *Soobshch. Akad. Nauk. Gruzin. SSR*, 24 (1960), 649–654 [*Math. Rev.* 24, #A1124 (Isbell)].
24. 'Mapping of proximity structures', *General topology and its relations to modern analysis and algebra*, Proc. Sympos., Prague, 1961 (Academic Press, New York), pp. 139–141; Publ. House Czech. Acad. Sci., Prague; 1962 [*Math. Rev.* 26, #4312 (Krishnan)].
25. (with Dona Papert Strauss) 'Quotient frames and subspaces', *Proc. London Math. Soc.* (3), 16 (1966), 275–296 [*Math. Rev.* 34, #2510 (D. F. Brown)].
26. 'Composite morphisms in abelian categories', *Quart. J. Math. Oxford Ser.* (2), 37 (1966), 98–105 [*Math. Rev.* 34, #2652 (A. Heller)].
27. (with D. P. Strauss) 'On Urysohn's Lemma', *General topology and its relations to modern analysis and algebra, II*, Proc. Second Prague Topological Sympos., 1966 (Academia, Prague, 1967), pp. 111–114 [*Math. Rev.* 39, #108 (O. Frink)].
28. (with D. P. Strauss) 'Separation axioms for frames', *Topics in topology*, Proc. Colloq., Keszthely, 1972, *Colloq. Math. Soc. Janos Bolyai*, 8 (North Holland, Amsterdam, 1974), pp. 223–240 [*Math. Rev.* 52, #15360, 54D10 (06A23) (E. P. Rozycki)].
29. (with D. P. Strauss) 'Paracompact frames and closed maps', *Symposia Mathematica, Vol. XVI*, Convegno Sulla Topologia Isiemistica e Generale, Indam, Rome, 1973 (Academic Press, London, 1975), pp. 93–116 [*Math. Rev.* 53, #14411 54D20 (54F05, 06A23) (Stephen Willard)].
30. (with D. Strauss) 'Products and sums in the category of frames', *Categorical topology*, Proc. Conf. Mannheim, 1975, Lecture Notes in Mathematics 540 (Springer, Berlin, 1976), pp. 208–219 [*Math. Rev.* 55, #11216, 54F05 (E. J. Braude)].
31. (with D. Strauss) 'Sums in the category of frames', *Houston J. Math.*, 3 (1977), no. 1, 17–32. [*Math. Rev.* 56, #1275, 54F05 (06A23) (Stephen Willard). See also Problems (in Topology) *Math. Rev.* 50, #8399].
32. (with Yael Dowker) 'Helping gifted children with mathematics', *Journal of the gifted child*, No. 1, I (1979), 52–66.
33. (with M. Thistlethwaite) 'On the classification of knots', *Comptes Rendus Mathématiques de l'Académie de Science* (*Royal Soc. of Canada*), vol. VI 2, (1982), 129–131.

34. (with M. Thistlethwaite) 'Classification of knot projections', *Topology and its applications*, 16 (1983), 19–31.
35. 'Isomorphism of categories', *J. Pure Appl. Algebra*, 27 (1983), 205–206.
36. (with D. Strauss) 'T_1- and T_2-axioms for frames', *Aspects of topology* (Cambridge University Press), to appear.

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