OBITUARY

PATRICK DU VAL

Patrick Du Val was born in Cheadle Hulme, Cheshire on 26 March 1903 and died on 22 January 1987. He was the son of Bartram Du Val, a cabinet maker, and his wife Margaret; their marriage was unfortunately short-lived and the young Patrick was brought up by his mother. He was a sickly child, suffering from an asthmatic condition that was to plague him throughout his life, and was not judged to be fit enough to attend school; instead, he received most of his education at the hands of his mother who was, by all accounts, a most cultured and remarkable woman. Even she, however, was unable to keep pace with her son's progress in mathematics and science during his later teenage years. He continued his studies through a correspondence course and, with the aid of some private tuition from a certain Miss B. H. Poole, obtained First Class Honours as an external student for the London B.Sc. General degree in 1926.

He evidently had no very clear idea, at that time, of the career he might pursue. He was a talented linguist and, for example, taught himself Norwegian so that he might read *Peer Gynt* in the original; he also had a strong interest in history; but his love of mathematics won the day and he became set upon a career in this subject. As his first publications reveal, his earliest leanings were towards applied mathematics. Meanwhile, however, he and his mother had moved house to a village near Cambridge and, by singular good fortune, had become acquainted with H. F. Baker, then the Lowndean Professor of Astronomy and Geometry in the University. Through Baker's influence, Du Val's interests turned towards algebraic geometry and he entered Trinity College as a research student in this subject in 1927. Baker was, by then, in the last decade of his distinguished academic career, but his research school was still extremely strong and active; the list of its members around that time—Coxeter, Edge, Hodge, Room, Semple, Todd (to mention but a few)— reads like a roll of honour of mid-twentieth century geometers; and Du Val was another of their number. He was an active participant in Baker's famous Saturday afternoon tea-parties (at which the research students would forgather over tea to regale one another with their latest geometrical discoveries) and established there many lifelong friendships, especially with Coxeter and Semple. He obtained his Ph.D. in 1930 and was in the same year elected a Research Fellow of his College. The then Master of Trinity, J. J. Thomson, at the Admission Dinner, drew particular attention to his remarkable feat in gaining a Trinity Fellowship after only three years of serious mathematical study. He held this Fellowship for four years, during which time he spent a period in Rome, learning at the feet of the great Italian geometers of that time; he particularly admired Enriques, to whom he would often later refer as his master. In 1934, by contrast, he visited Princeton as a Rockefeller Foundation Fellow, attending lectures by such famous mathematicians as Alexander, Eisenhart, Lefschetz, Veblen, Wedderburn and Weyl.

Back in England in 1936, he took up a temporary Assistant Lectureship at

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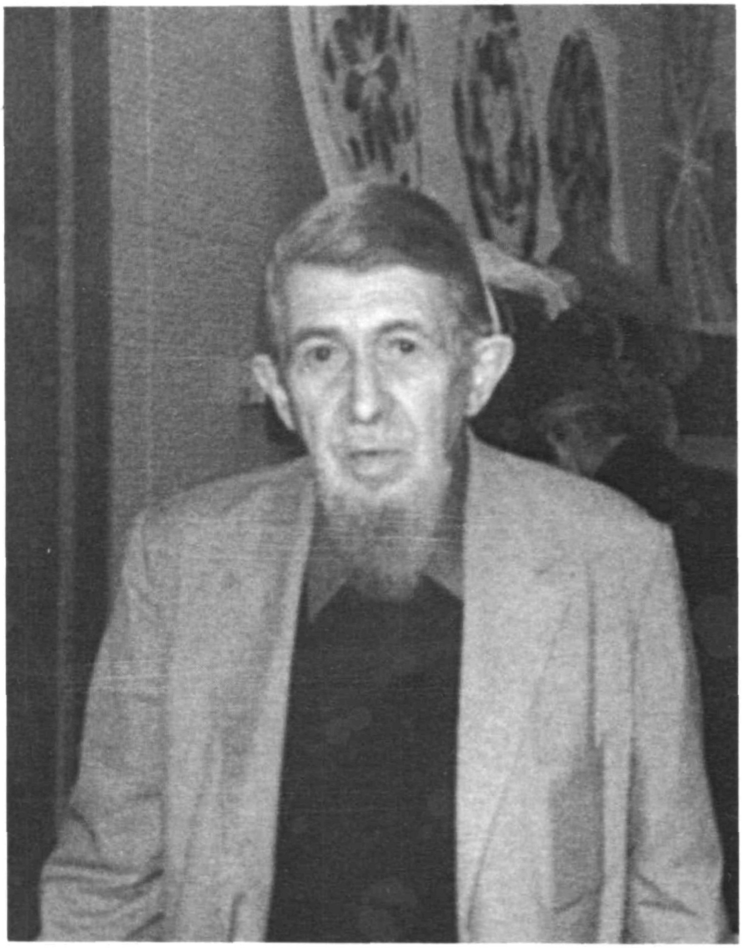
Manchester University, staying there eventually for five years before being invited, through a British Council war-time scheme, to go to Istanbul as Professor of Pure Mathematics. Despite the uncertain world conditions of the time, he very much enjoyed this period of his life. He developed a strong interest in Byzantine culture and quickly mastered the Turkish language, in which some of his work is written, including an elementary text-book on coordinate geometry of which he was rather proud. (I have unfortunately been unable to inspect a copy of this rather rare book, nor can I find its title and publication details, so it is not included in the list of publications at the end of this notice; but I have been assured of its existence by a number of reliable sources!) In 1945, during a brief visit back to Cambridge, he was married to Isobel Shimwell and they began their family in Istanbul during the post-war years. A move to the USA in 1949, where Du Val spent three years as Professor at the Georgia State University at Athens, proved to be rather uncongenial and he returned with his family to England. Here, he was Senior Lecturer at Bristol University until 1954 and then Reader in Mathematics at University College London until he retired in 1970. During this last period, he co-operated with his old friend Semple in leading the London Geometry Seminar and also undertook the Librarianship of the London Mathematical Society from 1957 to 1965. After his retirement from London, he returned to an 'Old Chair' at Istanbul for a few years before fully retiring in 1974 and living in Cambridge for the rest of his life. He was made an Honorary Doctor of Science of the University of Istanbul in 1979.

In his academic work, Du Val was little attracted to administrative matters, preferring always to spend his time on teaching and research. As a lecturer, his raucous asthmatic delivery gave him a somewhat forbidding manner, yet he was most kind and sympathetic to his students and always willing to spend time coaching the weaker ones among them. He was a keen maker of models and other visual aids and used these to great effect in illustrating his lectures; many of his models may still be seen in the Mathematics Department at University College London. At other people's lectures, he was noted for the quickness with which he grasped new ideas; and his courteous interventions were often the makings of what would otherwise have been rather dull seminars. As a person, he displayed an enigmatic combination of the modest and the flamboyant and his doings were the subject of a number of amusing stories. He is still remembered at Manchester, for example, for his work during the early war years as a fire warden, when his cloaked figure, striding along the parapets, was apparently a familiar sight. Later on, he loved to startle the travelling public in London by carrying around his large and garishly coloured models of stellated icosahedra in string bags—as boys do their footballs. My own favourite Du Val story concerns a fine pocket-watch that he carried at one time. He swore that it never lost a second and, when once I asked him the time, he consulted it and announced that it was twenty past six. Noticing that the hands actually pointed to five o'clock, I asked him why he did not make it keep the correct time. He replied that it was *his* watch and he saw no reason why everybody else should be able to tell the time by it! Away from mathematics, his principal recreation was reading, particularly about history. His early fascination with this subject (reinforced no doubt by the influence of his personal tutor at Trinity, who was the historian D. A. Winstanley) stayed with him throughout his life; and he often confessed to enjoying a few pages from Gibbon as his favourite bed-time reading. He is reputed, once, to have been passing an open lecture room door when he noticed the class waiting for a lecturer who had forgotten to turn up. He asked one of the students what the subject was and, being told that it was history, immediately obliged by giving them an extempore hour on the History



PATRICK DU VAL 1903–1987

Patrick Du Val, with his wife Isobel, on the occasion of his Honorary Doctorate from the University of Istanbul, October 1979.

of Mathematics. Many other (sometimes apocryphal) anecdotes are told of him, but these few must suffice.

Du Val was a remarkable man to know and a most amusing companion. His many friends were saddened to hear that his last years were troubled by illness, both of himself and of his wife, who survived him by little more than a year. They leave a son Nicholas and two daughters, Paula and Belinda.

Mathematical work

Du Val's first published papers were written before he graduated; three of these [**1**, **3**, **4**] are notes on the theory of relativity, including a geometrical description of de Sitter's world, while another [**2**] develops the notion of the Grassmann product of two linear tensors and gives examples of its physical application. Two more papers were written while he was a research student; one is a brief note [**5**] clarifying the literature to do with the real shape of space quartic curves (of the kind which lie on two quadric surfaces); the other [**6**] describes a simple way of setting up the Hesse–Cayley algorithm, by which the twenty-eight bitangents of a non-singular plane quartic curve are indexed by the unordered pairs taken from a set of eight symbols.

Next came his Ph.D. thesis [**7**], subsequently published as [**11**], in which he generalised a result of Schoute ⟨3⟩, namely that the twenty-seven lines on a cubic surface F can be paired off against the vertices of a certain polytope P in six-dimensional space, in such a manner that the incidences between the lines on F are mirrored by metrical properties of P (so that the famous 'group of the twenty-seven lines' is simply isomorphic with the symmetry group of P). Taking F to be the projective model of a system Σ of cubic curves in the plane, with six base points $K_1, \ldots, K_6$, Du Val views Schoute's result as a correspondence between the vertices of P and the *directrices* of Σ, a directrix being a curve whose multiplicities at the points K_i are such as to reduce both its genus and its freedom to zero. With a like definition of directrix for an arbitrary finite set of points, he develops similar correspondences between the directrices of sets of n points in the plane and the vertices of certain polytopes in n-dimensional space. He goes on to discuss many examples of this correspondence and shows how Cremona transformations based on the points in the plane give rise to symmetries of the corresponding polytopes. These ideas were substantially extended and elaborated in [**26**] and, many years later, he returned to the subject in two further papers [**40**, **60**], where he particularly emphasised their crystallographic aspects.

After his sojourn in Rome in the early 1930s, he developed a strong interest in the theory of algebraic surfaces, a subject that was to inspire much of his later published work. He wrote on problems of classification of such surfaces, whether by means of their genera [**8**, **9**, **12**, **34**, **36**], by properties of their prime sections [**10**, **15**, **16**], or by means of their canonical systems [**27**, **35**]. Also, in two other papers [**14**, **37**], he tackled similar problems for varieties of higher dimension. He was particularly interested in the structure of multiple planes and enumerated [**13**, **17**, **22**, **23**] every type of curve, up to order fourteen, that can occur as the branch-curve of a triple plane. The behaviour of multiple planes is central, also, to his (relatively little known) work on the resolution of singularities of an algebraic surface. This was first set out as the opening section of an essay that he submitted, in 1936, for the Adams Prize at Cambridge (unsuccessfully, as it turned out; the prize that year went to Hodge for his seminal work on harmonic integrals). The existence of a non-singular birational transform of any given algebraic surface was, at about that time, not proven. Many attempts at proof had been published, but none had survived critical analysis (see **[31]** and, for a more general survey, Zariski ⟨**6**⟩). Du Val's idea was to fuse together two results which he regarded as unexceptionable; one, due to Albanese ⟨**1**⟩, states that any surface can be transformed into one that has no point of multiplicity greater than two; the other, due to Severi ⟨**4**⟩, states that any surface with only *improper* singularities can be transformed into a non-singular surface. Du Val claimed to fill in the missing step by showing that a surface having no points of multiplicity greater than two can be birationally transformed into one without proper singularities. To the best of the present writer's knowledge, however, this work has never been subjected to the detailed scrutiny that such an important claim demands (lying, as it does, at the very root of the invariantive theory of surfaces). It was twelve years, indeed, before it was properly published **[32]** and then only in a highly condensed form (and, it has to be said, in a rather obscure place!). Du Val was naturally disappointed that his work was overshadowed by the appearance of the proof by Walker ⟨**5**⟩ which came to be generally accepted. He remained convinced, however, that his geometrical approach was much to be preferred to the more abstract work that came later. He lectured enthusiastically in support of this thesis, particularly in London in 1954 (with, unfortunately, only privately circulated notes) and again in Messina a few years later (partially published in **[39]**).

A happier tale may be told about another of his interests—the structure of singularities. In the geometry of a surface F of order n in three-dimensional space, fundamental importance attaches to the systems of *adjoint surfaces*; those of order $n-4$, for example, cut out the canonical system on F. Adjoint surfaces are those which pass with multiplicity $r-1$ through any curve of multiplicity r on F, and with multiplicity $s-2$ through any isolated s-ple point of F (and there may be further 'conditions of adjunction' if the singularities of F are of a complicated kind). Hence an isolated double point of F may or may not affect the conditions of adjunction—depending, roughly speaking, on whether it is of a complicated or uncomplicated type. In **[18]**, Du Val sets out to enumerate all types of double points which are uncomplicated in this sense. By studying the structure of the trees of rational curves into which they may be resolved (on a non-singular transform of F), he finds that they comprise the conic node, the binodes and unodes, and the three so-called exceptional unodes (which he was to study further in **[49]**). Also, in **[19, 20]**, using his earlier ideas about directrices of sets of points in the plane, he elucidates the connection between his classification and the work of Coxeter ⟨**2**⟩ on the enumeration of groups generated by reflections. Although not exactly forgotten, these papers lay somewhat dormant for almost thirty years before they were 'rediscovered' in the 1960s during a resurgence of interest in singularity theory (and the double points listed above have now come to be known as *Du Val singularities*). These developments prompted Du Val to write what is surely his finest work **[44]**, known affectionately in the profession as *Homs Quats and Rots*. In it, he draws together all sorts of apparently diverse ideas from the theories of one-dimensional homographies, of quaternions, of rotations in three and four dimensions, and of regular polyhedra and polytopes; he brings out the complex of relations and correspondences that exist between these different disciplines and links them with more modern work on three-dimensional topological spaces, on singularities of algebraic surfaces and on involutions in the complex projective plane. It is a beautifully produced book, with lots of excellent diagrams, and should have its place on every geometer's bookshelf.

In much the same vein as his work on binodes and unodes, Du Val wrote on the subject of exceptional curves [24] obtaining the conditions for a set of rational curves on a surface F to be transformable into the neighbourhoods of a set of simple points on a birational model of F. Arising out of this, he investigated [29] the famous principles of *scaricamento* (unloading) and *scorrimento* (smoothing) which govern the behaviour of curves on a surface if one attempts to make them have given multiplicities at given points *when some of the points are in the neighbourhoods of others*. He also wrote briefly [30, 33] on curve branches in space whose multiplicity sequences are different from those of their general projections. His facility in handling neighbourhoods and proximity relations was put to good effect in [41], where he shows how to construct a non-singular model of the third order curve-elements $P_0 P_1 P_2 P_3$ in the plane with origin at a fixed point P_0 (each subsequent P_i being proximate to its immediate predecessor). In a masterly follow-up paper [42] he completely generalises this work, showing that the nth order curve-elements in space S_r (as also those with a fixed origin) can be represented by the points of a non-singular variety.

In his later years, he developed a strong interest in the theory of elliptic functions. Apart from two short notes of a technical nature [51, 52], his work here was mainly concerned with the modular relation that exists between the values J, J' of the absolute invariant for two lattices Ω, Ω' of complex numbers, each of which is similar to a sublattice of index n in the other (J being the ratio $g_2^3/27g_3^2$, where g_2 and g_3 are the usual constants associated with the Weierstrass function having Ω as period-lattice). These modular relations represent rational curves (in the affine plane of the coordinates J, J') and their shapes and singularities, for $n \leqslant 4$, are fully worked out in [46, 48, 50]. A summary of this work may also be found in his book [55] which is the best source, in English, for the geometrical applications of elliptic functions; there is, in particular, a chapter on *ternary elliptic functions*, which he had invented [43] in order to provide a natural parametrisation for cubic curves with equations in triangularly-symmetric form; and another on elliptic quartic curves, including spheroconics (see also [56]) as well as the Cartesian and Cassinian ovals. He also wrote two papers on surfaces having elliptic parametrisations; one [53], written jointly with Semple, is a general account of normal elliptic scrolls; the other [54] is a detailed description of the geometry of the normal elliptic quintic scroll in four-dimensional space.

Other publications worthy of note, not included under the general headings considered above, are the joint work [28] with Coxeter, Flather and Petrie on the fifty-nine icosahedra (still in print fifty years after its first publication); another joint work [57] with Saban on the n-dimensional generalisation of the theorem of Archimedes which relates the volume and surface area of a sphere to those of its circumscribing cylinder; and his detailed account [58] of multiple tetrahedroids and Kummer surfaces. Finally, mention should be made of his extensive contributions to *Mathematical Reviews* and the large quantities of lecture notes that he produced over the years. He once contemplated putting these notes together to form (what would have been) an enormous work of reference on classical geometry; sadly, it never came to fruition.

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