ARTHUR ERDÉLYI 1908–1977

ARTHUR ERDÉLYI

DAVID COLTON

Professor Arthur Erdélyi, F.R.S.E., F.R.S., died suddenly at his home in Edinburgh on December 12, 1977, at the age of 69. His academic life was spent at both the University of Edinburgh and the California Institute of Technology, although the stronger claim on him is perhaps that of Edinburgh where he arrived as a refugee from Czechoslovakia in 1939 and to which he returned in 1964 after sixteen years at Caltech. His greatest contributions to knowledge were in the area of asymptotic analysis and special functions, although he also made major contributions in many other fields, in particular generalized functions, singular perturbations, fractional integration, and the analytic theory of partial differential equations. His breadth of mathematical knowledge was extraordinary, ranging over the entire spectrum of pure and applied mathematics. This wide range of interest was reflected in his distinction as a teacher, and his students now occupy academic posts throughout North America and Great Britain. He was a man of great wisdom and broad vision, and his influence will live on in his published work as well as in the minds of innumerable friends and colleagues throughout the world.

For assistance in preparing this article I am grateful to R. Askey, W. L. Edge, D. S. Jones, A. G. Mackie, A. McBride, F. W. J. Olver, R. E. O'Malley, I. N. Sneddon, J. Todd and J. Wimp. For their help I would like to express my deepest appreciation.

Arthur Erdélyi was born in Budapest, Hungary, on October 2, 1908, the first child of Ignác and Frieda Diamant. After his father's death he was adopted by his mother's second husband, Paul Erdélyi. He attended primary school in Budapest from 1914 to 1918 and secondary school in the same town from 1918 to 1926. During this time in Budapest student " circles " were organized in the schools to pursue topics outside the normal syllabus. Erdélyi played an active role in several of these groups, in particular those devoted to Hungarian literature and mathematics. At the same time in Hungary there existed a mathematics journal which devoted itself exclusively to the needs of students in secondary schools, mainly through the means of expository articles and problems, and Erdélyi was an avid reader of this journal. His life long devotion to mathematics can be traced back to this time.

The problem of pursuing a University education was complicated by the fact that there existed a numerus clausus which made it difficult for Jews to study at Hungarian institutes of higher education. As a result Erdélyi enrolled at the Deutsche Technische Hochschule in Brno, Czechoslovakia, to study electrical engineering. In order to obtain a degree at the Technische Hochschule it was necessary to pass two " State Examinations ", one in mathematics, physics, and related scientific subjects, and the other in professional subjects. Erdélyi passed the first of these with distinction in 1928, but never completed work for the second. This was not at all uncommon in Central Europe at the time, since the need actually to obtain a degree before leaving University was not a prerequisite for a successful career as it is at present. In fact more important to Erdélyi's subsequent career was the fact that during his first year at the Technische Hochschule he was awarded both the first and the second prizes in a mathematics competition organized by the Professor for Algebra and Geometry.

[BULL. LONDON MATH. SOC., 11 (1979), [191–207]

14692120, 1979, 2, Downloaded from https://londmathsoc.onlinelibrary.wiley.com/doi/10.1112/blms/11.2.191 by NICE, National Institute for Health and Care Excellence, Wiley Online Library on [30/10/2025]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License

Based on this virtuoso display, the Professor of Mathematical Analysis urged Erdélyi to devote himself to mathematics. Erdélyi chose to follow his advice and began active research in 1930, his first paper appearing in 1934. In 1937, he matriculated at the German University of Prague, submitted a collection of papers in lieu of a thesis, and was awarded the degree of Doctor rerum naturalium in 1938. Under normal circumstances a successful career in mathematics in Czechoslovakia would have been assured.

Unfortunately, normal circumstances did not prevail in Czechoslovakia in 1938. The graduation ceremony of 1938 was the last the German University of Prague held before being taken over by the Nazis. As a Jew, Erdélyi was ordered either to leave the country by the end of the year or to risk internment in a concentration camp. In desperation he wrote to E. T. Whittaker at Edinburgh to inquire if any means of support were available. The choice of Whittaker was not accidental. Although not yet thirty years of age, Erdélyi had by that time published over twenty papers, most of them concerned with the confluent hypergeometric function which had been discovered by Whittaker in 1904. However providing a place for Erdélyi was not easy since the British government had a policy of not issuing a visa for refugees unless £400 per annum could be guaranteed for support. Through the dedicated efforts of Whittaker and Professor S. Brodetsky of Leeds, these funds were finally collected, and in December, 1938, Whittaker wrote to Erdélyi that the way was now open to securing a visa. The offer came just in time, as evidenced by Erdélyi's correspondence with Whittaker. " Necessity and danger ", he wrote to Whittaker on January 26, 1939, " compel me to trouble you once more . . . You know, perhaps, what it means today if a Jew is to be put on the German or Hungarian frontier ". In the last days of January, 1939, Erdélyi was finally able to depart from Czechoslovakia, and in February appeared at the Mathematical Institute in Edinburgh. Other members of Erdélyi's family were not so fortunate: two brothers and a sister were later to die in concentration camps.

For the next few years, Erdélyi continued work under a research grant from Edinburgh University and financial support from the Society for the Protection of Science and Learning. His publications continued unabated, and in recognition of his widening international reputation the University awarded him the degree of Doctor of Science in 1940. However life was difficult for him at that time, both financially and because of the insecurity of his position. In a letter of January, 1948, Aitken recalled that " for two or three years, from 1939 on, when he escaped from Hitler's descent on Prague and came to us, he lived like Lazarus on the crumbs from Dives' table while doing all kinds of unobtrusive work in our Department ". Finally in 1941, Whittaker managed to persuade the University to make Erdélyi an Assistant Lecturer. His appointment was celebrated by his colleagues in the form of a welcoming tea party and, despite wartime austerities, a large chocolate cake. In 1942 he was appointed Lecturer and, with his security now assured, married Eva Neuburg, daughter of Frederic and Helene Neuburg and second cousin of Max Perutz, F.R.S.

Erdélyi was now able to enter more fully into the life of the Department and to participate actively in the war effort by serving as a consultant to the British Admiralty. In this last connection frequent visits were made to London to consult with G. E. H. Reuter, D. H. Sadler, and J. Todd, and out of these discussions came a proposal for a National Mathematical Laboratory (which ultimately came to pass as a Division of the National Physical Laboratory). However, the steady progress of Erdélyi's career as a British don was soon to be interrupted by the death in America

14692120, 1979, 2, Downloaded from https://londmathsoc.onlinelibrary.wiley.com/doi/10.1112/blms/11.2.191 by NICE, National Institute for Health and Care Excellence, Wiley Online Library on [30/10/2025]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License

of Harry Bateman, who left behind him notes on special functions amounting to a veritable mountain of paper that demanded editing and publication. Since Whittaker was at that time the world's senior authority on special functions, and had known Bateman as a student at Cambridge, he was contacted by the California Institute of Technology and asked if he could recommend someone to supervise the editing and publication of the Bateman manuscripts. Without hesitation he recommended Erdélyi, and on July 1, 1947, Erdélyi took up a position as Visiting Professor of Mathematics at Caltech, thus beginning his sixteen-year association with what was then, and is today, one of the outstanding centres of scientific learning in the world.

Erdélyi's initial appointment at Caltech was for one year, and his duty was to evaluate the contents of the Bateman manuscripts and to determine the time needed to prepare them for publication. After a careful study he reported that the job might take as much as fifteen years! An alternative possibility was that the same job could probably be done by four highly trained mathematicians working over a period of four years. The last proposal was agreed to by Caltech and followed up by an offer to Erdélyi of a permanent Full Professorship at Caltech, with directing the Bateman Manuscript Project as part of his duties. Edinburgh responded by promoting him to Senior Lecturer, but the attraction of a Professorship at Caltech at double his Edinburgh salary proved to be irresistible, and after returning to Edinburgh for the session 1948–49, he resigned his position at Edinburgh and moved to America. With him were to come F. G. Tricomi from the University of Torino, W. Magnus from the University of Göttingen, and F. Oberhettinger from the University of Mainz to form, along with Erdélyi, the famous team that produced the three volumes of *Higher Transcendental Functions* and the two-volume work *Tables of Integral Transforms*. These books were destined to be among the most widely cited mathematical works of all time and a basic reference source for generations of applied mathematicians and physicists throughout the world. The most important part of this work, *Higher Transcendental Functions*, remains to this date the most scholarly and comprehensive treatment of the special functions of mathematical physics that is yet available, and the mathematical community is indeed fortunate that Erdélyi was willing to spend a number of years out of the most productive period of his life to complete this task.

The direction of the Bateman Manuscript Project was by no means easy. To begin with, the notes left by Bateman were in a chaotic state and primarily orientated towards the numerical evaluation of special functions. Furthermore, some of the analytic results on which these numerical procedures were based were simply incorrect. Therefore the team of Erdélyi, Tricomi, Magnus and Oberhettinger decided to proceed on their own, collecting, summarising, and often creating new results when the literature proved inadequate. Further problems arose in preserving the delicate working balance between established scholars, each of whom had his own ideas and prejudices on how best to proceed. In this last task Erdélyi was masterful, and by tack, courtesy and compromise he judiciously assigned areas of concentration to each of the collaborators and managed to avoid any crippling disagreements. By 1951 the task was essentially completed and Erdélyi was once again able to devote his full time to the pursuit of his own research.

The Bateman Manuscript Project marked a turning point in Erdélyi's development as a mathematician. Until this time most of his work was in special functions, and although the results were often striking and elegant, the investigations were mainly undertaken for their own sake and not, in general, to illuminate other areas

F

14692120, 1979, 2, Downloaded from https://londmathsoc.onlinelibrary.wiley.com/doi/10.1112/blms/11.2.191 by NICE, National Institute for Health and Care Excellence, Wiley Online Library on [30/10/2025]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License

of mathematics. However as the Bateman Manuscript Project neared completion Erdélyi quickly became involved with investigations into a variety of different areas, in particular the analytic theory of singular partial differential equations ([90], [99]), diffraction theory ([104], [109]), and the asymptotic expansion of solutions to differential equations ([107], [110]). Most important of these investigations was his work on asymptotics, and under a contract with the Office of Naval Research, a long series of papers on asymptotic expansions of integrals and solutions to differential equations began appearing around 1950 authored by Erdélyi and his co-workers. Much of this work was summarised in *Asymptotic Expansions* which Erdélyi published in 1956, and this short paperback soon became the standard work in the area. These investigations on asymptotic analysis were influenced by the work then being undertaken in the Guggenheim Aeronautical Laboratory by Lagerstrom, Kaplun, and their co-workers on the theory of singular perturbations, and Erdélyi's life-long interest in singular perturbation theory can be traced back to this time. All in all, it was an exciting time to be at Caltech, and Erdélyi, at the peak of his powers, soon became one of the outstanding members of what was already an illustrious faculty.

As the years progressed and Erdélyi's international stature grew, he continued broadening and deepening himself as a mathematician, laying the foundations for the rest of his life's work. In the early 1960's his book *Operational Calculus and Generalized Functions* appeared, as well as his early work in singular perturbation theory ([131], [134], [137], [143], [144]), and fundamental papers on the asymptotic evaluation of integrals ([130], [149]). It was at this time (1960–1964) that I first met him as an undergraduate student, and still recall the awe and respect with which he was held by the student body, mainly due to his reputation as a teacher since we were too mathematically immature to appreciate his stature as a mathematician. I took a course on distribution theory from him, grading the papers as well, and was so totally captivated by his lectures that I decided to abandon my infatuation with topology and venture into distribution theory as a graduate student! He had a similar effect on other students, seemingly regardless of what course he taught. At that time it was difficult to imagine him in any other environment than Caltech. However events had occurred at Edinburgh which were to cause Erdélyi, at the age of 55, to make yet another major change in his life.

During the years that Erdélyi was at Caltech, Whittaker had retired and had been succeeded by Aitken. However, by 1963, Aitken was in very poor health and the Department was in desperate need of strong and effective leadership. As a result, a second Chair of Mathematics was created and although Erdélyi did not officially apply he was formally invited to allow his name to be put forward. It was a difficult decision for him to make. Although Erdélyi had a deep attraction to the city of Edinburgh and a strong sense of indebtedness to the University for offering him a place of refuge in 1939, it was not easy to leave the exciting atmosphere of Caltech and the close friends he had made there. There were further problems of the prospect of being burdened with administration and the lack of adequate secretarial help. Equally serious was the question of superannuation, since he had cashed in his F.S.S.U. benefits in 1949. However, in the end, the attractions of Edinburgh were too strong, and on the day before his 55th birthday he wrote to accept the invitation to become a candidate. His nomination was quickly accepted and in July, 1964, Erdélyi arrived in Edinburgh to take up his position as Professor of Mathematics.

His first few years in Edinburgh were a busy time for him. The Department was in a depressed situation, as evidenced by a sadly out-dated mathematical syllabus and

14692120, 1979, 2, Downloaded from https://londmathsoc.onlinelibrary.wiley.com/doi/10.1112/blms/11.2.191 by NICE, National Institute for Health and Care Excellence, Wiley Online Library on [30/10/2025]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License

the lack of an effective postgraduate programme. I recall that when I arrived in 1965 to begin working on my Ph.D under Erdélyi, I was the only graduate student in analysis in the Department, and I couldn't help but compare the situation in Edinburgh with the active mathematical life I had known at Wisconsin (where I had just obtained my Master's degree). Erdélyi must have had similiar thoughts when he recalled his recent years at Caltech. However, in a short period of time, F. F. Bonsall arrived to take up his Professorship at Edinburgh, bringing with him a sizable group of students from Newcastle, and shortly thereafter Jet Wimp arrived from the States to become Erdélyi's second student at Edinburgh. An active postgraduate seminar was initiated and under Erdélyi's guiding hand, with strong support from his staff, the Department began to regain the distinction it held in previous years. With the arrival of A. G. Mackie in 1968 to take up the Chair of Applied Mathematics, the Mathematics Department at Edinburgh became a true multi-professorial department and the administrative burden became somewhat less onerous. Erdélyi now managed to find more time to participate in the cultural life of Edinburgh, take long walks in the Scottish countryside, and involve himself in musical evenings with friends, where he played the violin with virtuoso ability.

Gradually, Erdélyi became the revered elder statesman of Scottish mathematics and, attracted by his name, visitors arrived in Edinburgh from all over the world. Regardless of what area they spoke on, Erdélyi almost always made well-informed questions or comments at the end of the talk which clarified things and often led on to productive discussion. In spite of his duties in the Department and University, his research continued unabated, with a regular stream of papers appearing on singular perturbation theory, singular partial differential equations, fractional integrals of generalized functions, and the asymptotic evaluation of integrals. In 1973 he had a serious illness but, making a remarkable recovery, continued doing research, giving lectures, and actively participating in Scottish mathematical life until he died. His death came suddenly on December 12, 1977, but his work and influence will remain in the numerous contributions he made to mathematics for over forty years.

These contributions had been duly honoured. In 1940 he was awarded the degree of Doctor of Science from the University of Edinburgh and in 1945 he was elected a Fellow of the Royal Society of Edinburgh. In 1953 he became a Foreign Member of the Academy of Sciences of Torino, and in 1975 a Fellow of the Royal Society of London. The Gunning Victoria Jubilee Prize of the Royal Society of Edinburgh was awarded to him in 1977, the only mathematician to receive it in recent years other than Sir William Hodge. During the years he served the mathematical community in a variety of administrative roles, including the Presidency of the Edinburgh Mathematical Society, Council Member of the American Mathematical Society, membership on a variety of advisory bodies appointed by the National Academy of Sciences, and joint or associate editorships of numerous periodicals. He held Visiting Professorships at the Hebrew University, Jerusalem, in 1956–57, and at the University of Melbourne in 1970. In addition to his visiting appointments, he lectured widely throughout the world, one of these being an invited lecture at the 1954 International Congress of Mathematicians in Amsterdam. The work he initiated and encouraged has been carried on by his students, who tc the best of my knowledge are listed below:—

1952 R. H. Owens

1952 P. G. Rooney

14692120, 1979, 2, Downloaded from https://londmathsoc.onlinelibrary.wiley.com/doi/10.1112/blms/11.2.191 by NICE, National Institute for Health and Care Excellence, Wiley Online Library on [30/10/2025]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License

1957	C. A. Swanson
1959	J. Rice
1960	T. Boehme
1963	D. W. Willett
1964	J. W. Macki
1967	D. L. Colton
1968	J. Wimp
1969	J. Searl
1971	A. McBride

For an indication of the range of Erdélyi's interests, and the impact his research has made in the mathematical world, the reader should consult the special issues of *Applicable Analysis* which have been dedicated to his memory, as well as the Proceedings of the 1978 Dundee Conference on Differential Equations which was also dedicated to him. In these special volumes many of Erdélyi's former students and colleagues have come together to provide a lasting memorial to his work and the influence it continues to exert on current mathematical research.

Below I attempt to describe Erdélyi's major contributions to mathematics under five major headings. Due to the volume and breadth of his work I have only attempted to outline the highlights, and this evaluation is, of course, biased by my own limitations and prejudices. I have already acknowledged, on the first page of this article, the help of several of Erdélyi's former friends and colleagues in the preparation of this survey. I, of course, accept responsibility for any errors, omissions, or incorrect judgments. The grouping is in some aspects rather arbitrary. In particular it would not have been unreasonable to include a separate heading under " generalized functions " since his book *Generalized Functions and Operational Calculus* was one of the first to appear on this topic, and both interpreted and extended the original work of Mikusinski. However since his most important work in this area was so intimately connected with the theory of fractional integration, I have, with some hesitation, decided to include his work on generalized functions under this latter heading.

Special Functions

Erdélyi began his mathematical career with the confluent hypergeometric function and before arriving in Edinburgh in 1939 had already established himself as a leading expert in the area of special functions. In Edinburgh he continued to pursue his investigations, broadening his interests into generalized hypergeometric functions, classical orthogonal polynomials, and in particular Lamé functions where he published a series of fundamental papers ([**64**], [**70**], [**74**], [**75**], [**87**]). This interest in solutions of Lamé's equation was extended to include other equations of Heun type ([**72**], [**73**], [**76**], [**79**]). By the time he arrived at Caltech he probably had more knowledge of the special functions of mathematical physics than any other person alive at that time, with the possible exception of Whittaker. As such he was the obvious choice to lead the Bateman Manuscript Project, and the completion of this Gargantuan task firmly established his world wide reputation in this area. Due to the breadth and volume of his work a proper evaluation of his contributions to the theory

14692120, 1979, 2, Downloaded from https://londmathsoc.onlinelibrary.wiley.com/doi/10.1112/blms/11.2.191 by NICE, National Institute for Health and Care Excellence, Wiley Online Library on [30/10/2025]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License

of special functions is difficult, but it is perhaps fair to say that until his arrival at Caltech much of his work was in special functions for their own sake, in particular the derivation of often remarkable identities, integral representations, and expansion formulae. In this sense his work before 1950 can probably best be viewed as a preparation for his work on the Bateman Manuscript Project. (There are obviously many exceptions to this statement, for example, his early work on fractional integration ([**59**], [**60**]) which laid the groundwork for much of his later investigations in this area, and his paper of 1947 ([**84**]) where he first introduced the concept of an asymptotic scale which was to strongly influence his later work on the asymptotic evaluation of integrals. These developments will be discussed under the appropriate headings). For the mathematical community the years invested in preparing himself for the Bateman Manuscript Project were well spent. The publication of *Higher Transcendental Functions* provided the bridge that made it possible for many people to go beyond what was in Whittaker and Watson's *Modern Analysis* and educated a new generation who had been brought up thinking that abstract methods would solve all problems. The three volumes of *Higher Transcendental Functions* provided in the 1950's the one source that gave enough facts in a useful way so that potential users of this material could see what was known and what was still needing to be discovered. For a more complete appraisal of the impact that *Higher Transcendental Functions* made on the scientific community the reader is urged to consult the remarks by Richard Askey in the special issue of *Applicable Analysis* dedicated to Erdélyi.

It should perhaps be mentioned that in his later years Erdélyi tended to look less favourably on his early work in special functions and instead emphasised his subsequent investigations into asymptotic analysis, fractional integration, singular perturbations, and generalized functions. From a broad mathematical perspective this was probably correct. However without the work he carried out in the 1930's and 1940's he could not have brought the Bateman Manuscript Project to its masterful conclusion, and for this task the mathematical community must forever be in his debt.

After the completion of *Higher Transcendental Functions* Erdélyi remained interested in special functions, but turned more in the direction of asymptotic analysis. This aspect of his work will be discussed under the appropriate heading.

Singular Perturbation Theory

In the early 1950's the Guggenheim Aeronautics Laboratory at Caltech was heavily involved in the development of an improved boundary layer theory for viscous fluid flow past obstacles. This effort, led by Kaplun and Lagerstrom, produced an amazingly general philosophy for attacking singular perturbation problems for ordinary differential equations which came to be known as the method of matched asymptotic expansions. At the same time, in the mathematics department, a research team under the direction of Erdélyi was re-examining the theory of linear differential equations with large parameters, and the proximity of these closely related projects inevitably caused them to be profitably intermixed.

Erdélyi's first publication in the area of singular perturbations appeared in 1961 ([**134**]) where he presented a simplified mathematical treatment of the Kaplun–Lagerstrom matching principle. His paper of 1961 was followed ([**137**], [**143**]) by an investigation of two point boundary value problems for nonlinear scalar equations of the form

$$\varepsilon y'' = f(t, y, y'; \varepsilon).$$

14692120, 1979, 2, Downloaded from https://londmathsoc.onlinelibrary.wiley.com/doi/10.1112/blms/11.2.191 by NICE, National Institute for Health and Care Excellence, Wiley Online Library on [30/10/2025]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License

Based on integral equation techniques Erdélyi was able to show rigorously that the solution had the composite form of an outer solution, boundary layer, and uniform error term. Similiar results had been given earlier by Coddington, Levinson and Wasow, but under more restrictive conditions. Extensions of these results were subsequently given by Erdélyi's students Willett and Macki, and a rigorous discussion of the necessary topics in singular integral equations was provided by Erdélyi in **[152]**. In a related direction Erdélyi showed how his theorems could be applied to construct approximate solutions to singular nonlinear boundary value problems for ordinary differential equations (**[159]**).

A modification of the method of matched asymptotic expansions is to attempt to construct the expansion directly without developing outer and inner expansions and matching. This process is known as the two variable expansion, and initial steps in the development of a rigorous mathematical theory for this approach were given by Erdélyi in **[158]**. In this analysis Erdélyi invoked his theory of general asymptotic expansions and asymptotic scales, a subject which I shall return to in my discussion of Erdélyi's work in asymptotic analysis.

As in all the other areas he worked in, Erdélyi's influence extended far beyond that generated by his published papers. Of equal importance to his publications must be ranked his frequent reassessments of current progress and problems (c.f. **[144]**, **[171]**, **[174]**), his careful efforts at editorial work and refereeing chores, and his encouragement of younger workers. This aspect of his work, combined with his publications, made Erdélyi one of the leading world figures in the theory of singular perturbations.

Asymptotic Analysis

Erdélyi's investigations in the 1950's on the asymptotic theory of differential equations with transition points or singularities was of course closely related to his later work on singular perturbation problems. This research was the outgrowth of the efforts made by a research team (led by Erdélyi) in the mathematics department under a grant from the Office of Naval Research, and a lucid survey of this work was provided by Erdélyi in **[125]**. A further excellent account of the general theory of asymptotic solutions of ordinary differential equations was given in **[133]** and it seems a pity to me that these notes were not given a wider circulation. The basic problem considered by Erdélyi and his co-workers was the uniform asymptotic approximation to solutions of the differential equation

$$y'' + [\lambda^2 p(x) + r(x, \lambda)]y = 0$$

where λ is a large parameter, $r(x, \lambda)$ is " small " in comparison with $\lambda^2 p(x)$, x is a real variable ranging over a finite or infinite open interval (a, b), and $p(x)$ is real and changes sign at c, $a < c < b$. At c itself $p(x)$ has either a simple zero or simple pole. Investigations were also made in the case where x is a complex variable. This study was of course related to the fundamental work carried out in this field by Langer, Cherry, and Olver. Erdélyi's contribution was to present for a reasonably general class of problems a systematic treatment of the singular Volterra integral equations which arose in showing that a formal solution was in fact an asymptotic approximation. Such a contribution was of course precisely what the practitioner desired, since such results could be applied in a number of cases of practical importance, while at the same time they reduced the ad hoc investigations of specific integral

14692120, 1979, 2, Downloaded from https://londmathsoc.onlinelibrary.wiley.com/doi/10.1112/blms/11.2.191 by NICE, National Institute for Health and Care Excellence, Wiley Online Library on [30/10/2025]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License

representations or other special formulae to a minimum. These results were subsequently applied by Erdélyi to obtain asymptotic representations of Bessel functions ([**125**]), parabolic cylinder functions ([**110**]), Whittaker's confluent hypergeometric function ([**122**]), and Laguerre polynomials ([**135**]). A systematic account of the treatment of the singular integral equations arising in the asymptotic theory of ordinary differential equations can be found in [**128**], [**142**] and [**152**].

Erdélyi's most important contributions to asymptotic analysis were perhaps made in the area of asymptotic evaluation of integrals. Fundamental to much of his investigations was the idea of an asymptotic scale and generalized asymptotic expansion, an idea which dates back at least to H. Schmidt but which Erdélyi was the first to exploit on a systematic basis. The basic concept is simple: $\{\phi_n\}$ is called an asymptotic scale if $\phi_{k+1} = o(\phi_k)$ as $z \to z_0$ in a sector S and the formal series $\sum_{n=0}^{\infty} F_n$ of functions $F_n = F_n(z)$ is said to be an asymptotic expansion of the function $F = F(z)$ with respect to the scale $\{\phi_n\}$ if for each fixed $N \geqslant 0$

$$F - \sum_{n=0}^{N} F_n = o\,(\phi_N).$$

If, in particular $F_n = c_n\phi_n$, where c_n is independent of z, then the asymptotic expansion is said to be of Poincaré type, for which a general theory had been available for a considerable time. The idea of a generalized asymptotic expansion was first used by Erdélyi in 1947 ([**84**]) and was gradually developed by him over the years in his investigations on the asymptotic expansion of integrals, as well as in his study of singular perturbation problems. (For an excellent survey of this area of Erdélyi's work the reader is referred to the survey paper "Uniform Scale Functions and the Asymptotic Expansion of Integrals" by Jet Wimp which will appear in the Proceedings of the 1978 Dundee Conference on Differential Equations dedicated to Erdélyi). In the course of his investigations into the asymptotic expansion of integrals Erdélyi obtained a series of striking and useful results. Of particular note were his extensions of the method of Laplace and the method of stationary phase to integrands with logarithmic and algebraic singularities ([**114**], [**119**], [**130**], [**169**]) and his paper with Tricomi generalizing Watson's lemma to loop integrals ([**97**]). In the former connection, his papers on the method of stationary phase helped remove the last vestiges of mystery that had attended this method since its introduction by Stokes and Kelvin in the nineteenth century. However of all his work in this area, his papers [**130**] and [**149**] are probably the most impressive. In these papers Erdélyi presented his final version of the concept of a general asymptotic expansion and applied this theory to the uniform asymptotic expansion of Laplace integrals as well as to more general integrals involving several independent parameters. An application of these results yielded new theorems on the asymptotic expansion of Laplace integrals involving logarithms and exponential functions, as well as an elegant and unified treatment of Watson's lemma, Darboux's method, and the asymptotic behaviour of functions in transition regions. In particular it is shown that the Poincaré type definition of an asymptotic expansion is much too narrow for a satisfactory discussion of the asymptotic behaviour of functions depending on more than one parameter.

Any discussion of Erdélyi's work on asymptotic analysis would be incomplete without mentioning his book *Asymptotic Expansions*, which at the time of its appearance was the only modern work on this topic available in English. In addition to presenting many new results, it gave a masterful survey of the major themes of

14692120, 1979, 2, Downloaded from https://londmathsoc.onlinelibrary.wiley.com/doi/10.1112/blms/11.2.191 by NICE, National Institute for Health and Care Excellence, Wiley Online Library on [30/10/2025]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License

asymptotic analysis, and is by now regarded as one of the classic monographs on the subject. A Russian and Polish translation of this work appeared in 1962 and 1967 respectively.

Singular Partial Differential Equations

Erdélyi's interest in singular partial differential equations arose out of his earlier work on hypergeometric functions, and in particular his study of Appell series. These series satisfy a singular system of partial differential equations and although ten solutions of this system were known as early as 1893, they were insufficient to provide fundamental systems in a neighbourhood of all the singular points. For this another fifteen solutions are necessary, and these were obtained by Erdélyi in a paper appearing in Acta Mathematica in 1950 ([**90**]). The basic problem which needed to be solved was how to obtain a fundamental system of solutions in a neighbourhood of singular points where three singular curves intersect, and Erdélyi's paper of 1950 was the first to show how this could be done. A survey of this and related work was presented in a talk to the American Mathematical Society in 1951 and published in [**99**].

Erdélyi did not return to the area of singular partial differential equations for five years. His interest was finally revived in this subject by the publications of Alexander Weinstein on the generalized axially symmetric potential equation (GASPE)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{k}{y}\frac{\partial u}{\partial y} = 0$$

where k is a real parameter. In addition to its appearance in a variety of areas of applied mathematics, this equation is of interest since it is probably the simplest example of a partial differential equation with a regular singular line. Erdélyi's first paper on this equation appeared in 1956 ([**120**]) and related the location of singularities of regular solutions of GASPE to the location of singularities of $u(x, 0)$ in the complex x-plane. Shortly before this time, following a suggestion put to him by Erdélyi, Gabor Szegö had published his paper "On the Singularities of Zonal Harmonic Expansions", and these two papers together laid the foundation for a large part of the later developments in the analytic theory of partial differential equations. In particular the work of Szegö and Erdélyi led to subsequent publications by Nehari, Henrici, and Gilbert, and ultimately to R. P. Gilbert's book *Function Theoretic Methods in Partial Differential Equations*.

The basic tool used in Erdélyi's classic paper of 1956 ([**120**]) was fractional integration operators, and he continued making use of these operators in all his future work on singular equations of the GASPE type. In 1958 ([**123**]) Erdélyi and Copson used fractional integration operators and the Mellin transform to study a singular hyperbolic equation with two intersecting singular lines, and in 1965 Erdélyi returned to his study of fractional integration and the generalized axially symmetric potential equation ([**154**], [**155**]). His last paper on this subject appeared in 1970 ([**163**]) in which he utilized fractional integration operators to study the Euler–Poisson–Darboux equation.

Erdélyi's interest in operators of fractional integration of course dated back to well before his application of these operators to the study of axially symmetric potential theory, and he continued his investigation of these operators until his death. This aspect of Erdélyi's work will be discussed in the following section.

14692120, 1979, 2, Downloaded from https://londmathsoc.onlinelibrary.wiley.com/doi/10.1112/blms/11.2.191 by NICE, National Institute for Health and Care Excellence, Wiley Online Library on [30/10/2025]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License

Fractional Integration

Although Erdélyi used fractional integration in a variety of papers on special functions published in 1939 and 1940 ([**50**], [**57**], [**58**]), his first major contribution appeared in [**59**] and [**60**], partly in collaboration with H. Kober. In these papers Erdélyi and Kober introduced "homogeneous" modifications of the Riemann–Liouville and Weyl fractional integrals and discussed their connection with the Hankel transform. These generalized fractional integration operators are now normally called Erdélyi–Kober operators, and are defined by the formulae

$$I_{\eta,\alpha}f(x) = \frac{x^{-\eta-\alpha}}{\Gamma(\alpha)} \int_0^x (x-y)^{\alpha-1} y^{\eta} f(y)dy$$

$$K_{\eta,\alpha}f(x) = \frac{x^{\eta}}{\Gamma(\alpha)} \int_x^\infty (y-x)^{\alpha-1} y^{-\eta-\alpha} f(y)dy.$$

If the modified operator $S_{\eta,\alpha}$ of Hankel transforms is defined by

$$S_{\eta,\alpha}f(x) = x^{-\alpha/2} \int_0^\infty y^{-\alpha/2} J_{2\eta+\alpha}(2\sqrt{xy}) f(y)dy$$

then Erdélyi and Kober derived a series of relationships between the operators $S_{\eta,\alpha}$, $I_{\eta,\alpha}$, and $K_{\eta,\alpha}$, typical of which are

$$I_{\eta+\alpha,\beta}S_{\eta,\alpha} = S_{\eta,\alpha+\beta}$$
$$K_{\eta,\alpha}S_{\eta+\alpha,\beta} = S_{\eta,\alpha+\beta}$$
$$S_{\eta+\alpha,\beta}S_{\eta,\alpha} = I_{\eta,\alpha+\beta}$$
$$S_{\eta,\alpha}S_{\eta+\alpha,\beta} = K_{\eta,\alpha+\beta}.$$

For over twenty years the results of these papers lay dormant, until in 1961 Erdélyi and I. N. Sneddon came together as research lecturers at the Canadian Mathematical Congress in Montreal. At that time, Sneddon was giving a series of lectures on mixed boundary value problems, and mentioned that a unified treatment of the dual integral equations that arose in such problems depended on developing certain relationships between Hankel transforms and operators of fractional integration. This of course was precisely the topic treated by Erdélyi and Kober in 1940, and hence the paper [**139**] was born. This paper presented for the first time a systematic and unified treatment of a rather general class of dual integral equations appearing in various areas of application. Further applications of operators of fractional integration to problems involving integral equations were made by Erdélyi in [**151**] and [**160**]. For an excellent survey of Erdélyi's (and others) work in this area, I refer the reader to I. N. Sneddon's paper "The Use in Mathematical Physics of Erdélyi–Kober Operators and Some of Their Generalizations" in Springer–Verlag Lecture Notes in Mathematics, Volume 457.

All of the above work was carried through within the context of classical functions. However in his later years, influenced by Zemanian's book *Generalized Integral Transformations*, Erdélyi began to extend fractional calculus to generalized functions

14692120, 1979, 2, Downloaded from https://londmathsoc.onlinelibrary.wiley.com/doi/10.1112/blms/11.2.191 by NICE, National Institute for Health and Care Excellence, Wiley Online Library on [30/10/2025]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License

([**166**], [**167**], [**170**]). The approach adopted by Erdélyi was based on the observation that the Riemann–Liouville and Weyl operators are adjoint to one another and hence constructing a space of testing functions on which one of these operators is continuous enables one to define the other for a corresponding class of generalized functions. This work on fractional integrals of generalized functions was subsequently extended to include the Stieltjes transform ([**175**]), and at the time of his death Erdélyi was actively developing and refining his investigations in this area. His work is now being carried on by his student, Adam McBride, and a research monograph by Dr. McBride will appear shortly with Pitman Publishing.

One cannot leave the subject of Erdélyi's research without emphasising several points. One of these is the sheer breadth and quantity, as well as quality, of his efforts. In the above discussion I have only been able to comment briefly on the main directions of his research, and have not explicitly referred to many of his papers, for example, his early work on special functions and his papers on functional transformations ([**93**]), variational principles in diffraction theory ([**104**]), etc. He was an excellent expositor, and with is broad interests had something to say in many areas of mathematics, ranging from non-standard analysis ([**147**]) to distribution theory ([**129**]). His reputation was based on much more than his published papers, although this alone would have sufficed to make him one of the leading analyists of his day. It was rather a combination of his mathematical scholarship, his interest and enthusiasm for mathematics, his concern for younger workers, and his willingness to devote his time in aid of the mathematical community that won Erdélyi the admiration and respect of an entire generation of mathematicians. He will be deeply missed and the role he played in the mathematical world will not easily be replaced.

Publications of Arthur Erdélyi

BOOKS

1. (With W. Magnus, F. Oberhettinger, F. G. Tricomi) *Higher transcendental functions*, 3 volumes, (McGraw–Hill, 1953–55). Russian translation 1965–66.
2. (With W. Magnus, F. Oberhettinger, F. G. Tricomi) *Tables of Integral Transforms*, 2 volumes, (McGraw–Hill, 1954).
3. *Asymptotic expansions*. (Cal. Inst. Tech. 1955, Dover 1956). Russian translation, 1962. Polish translation, 1967.
4. *Operational calculus and generalized functions*. (Holt, Rinehart and Winston, 1962). French translation, 1971.

PAPERS, ARTICLES, ETC.

1. " Ueber die freien Schwingungen in Kondensatorkreisen mit periodisch veranderlicher Kapazität ", *Ann. Physik*, (5) 19 (1934), 585–622.
2. " Ueber die kleinen Schwingungen eines Pendels mit oszillierendem Aufhängepunkt ", *Z Angew. Math. Mech.*, 14 (1934), 235–247.
3. " Ueber Schwingungskreise mit langsam pulsierender Dampfung (Zur Theorie des Pendelruckkopplungsempfängers) ", *Ann. Physik.*, (5) 23 (1935), 21–43.
4. " Ueber die rechnerische Ermittlung der Schwingungsvorgäange in Kreisen mit periodisch schwankenden Parametern ", *Arch. Elektrotechnik*, 29 (1935), 475–489. Cf. also *Elektrotechnische Zeitsch.*, 56 (1935), 1128.
5. " Ueber die freien Schwingungen in Schwingungskreisen mit periodisch veranderlicher Selbstinduktivität ", *Hochfrequenztechnik und Elektroakustik*, 46 (1935), 73–77.
6. " Schwingungskreise mit veränderlichen Paramtern. Bemerkungen zu einer Arbeit von C. L. Kober ", *Hochfrequenztechnik und Elektroakustik*, 46 (1935), 178.
7. " Bemerkungen zur Ableitung des Snelliusschen Brechungsgesetzes ", *Z. Physik*, 95 (1935), 115–132.
8. " Ueber einige bestimmte Integrale in denen die Whittakerschen $M_{k,m}$ Funktionen auftreten ", *Math. Z.*, 40 (1936), 693–702.

14692120, 1979, 2, Downloaded from https://londmathsoc.onlinelibrary.wiley.com/doi/10.1112/blms/11.2.191 by NICE, National Institute for Health and Care Excellence, Wiley Online Library on [30/10/2025]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License

9. " Ueber die kleinen Schwingungen eines Pendels mit oszillierendem Aufhängepunkt. Zweite Mitteilung ". *Z. Angew. Math. Mech.*, 16 (1936), 171–182.
10. " Sulla generalizzazione di una formula di Tricomi ", *Rendiconti dei Lincei*, 24 (1936), 347–350.
11. " Ueber die Integration der Mathieuschen Differentialgleichung durch Laplacesche Integrale ", *Math. Z.* 41 (1936), 653–664.
12. " Ueber eine Methode zur Gewinnung von Funktionalbeziehungen zwischen konfluenten hypergeometrischen Funktionen ", *Monatshe. Math. Physik.*, 45 (1936), 31–52.
13. " Funktionalrelationen mit konfluenten hypergeometrischen Funktionene. Erste Mitteilung. Additions und Multiplikationstheoreme ", *Math. Z.*, 42 (1936), 125–143.
14. " Entwicklung einer analytischen Funktion nach Whittakerschen Funktionen ", *Proc. Akad. Amsterdam*, 39 (1936), 1092–1098.
15. " Ueber eine Integraldarstellung der $W_{k,m}$ Funktionen und ihre Darstellung durch die Funktionen des parabolischen Zylinders ", *Math. Ann.*, 113 (1936), 347–356.
16. " Ueber eine Integraldarstellung der $M_{k,m}$ Funktionen und ihre asymptotische Darstellung für grosse Werte von $R(k)$ ", *Math. Ann.*, 113 (1936), 357–362.
17. " Sulla transformazione di Hankel pluridimensionale ", *Atti R. Accademia di Torino*, 72 (1936/37), 96–108.
18. " Funktionalrelationen mit konfluenten hypergeometrischen Funktionen. Zweite Mitteilung. Reihenenwicklungen ", *Math. Z.*, 42 (1937), 641–670.
19. " Ueber gewisse Funktionalbeziehungen ", *Monatsh. Math. Physik.*, 45 (1937), 251–279.
20. " Ueber die Integration der Whittakerschen Differentialgleichung in geschlossener Form ", *Monatsh. Math. Physik.*, 46 (1936), 1–9.
21. " Gewisse Reihentransformationen die mit der linearen Transformations-formel der Thetafunktion zusammenhange ", *Compositio Math.*, 4 (1937), 406–423.
22. " Untersuchungen über Produkte von Whittakerschen Funktionen ", *Monatsh. Math. Physik.*, 46 (1937), 132–156.
23. " Der Zusammenhang swischen verschiedenen Integraldarstellungen hypergeometrischer Funktionen ", *Quart. J. Math. (Oxford)*, 8 (1937), 200–213.
24. " Integraldarstellungen hypergeometrischer Funktionen ", *Quart. J. Math. (Oxford)*, 8 (1937), 267–277.
25. " Beitrag zur Theorie der konfluenten hypergeometrischen Funktionen von mehreren Veränderlichen ", *Wiener Sitzungsberichte*, 146 (1937), 431–467.
26. " Zur Theorie der Kugelwellen ", *Physica (Haag)*, 4 (1937), 107–120.
27. " Ueber die erzeugende Funktion der Jacobischen Polynome ", *J. London Math. Soc.*, 12 (1937), 56–57.
28. " Inhomogene Saiten mit parabolischer Dichteverteilung ", *Wiener Sitzungsberichte*, 146 (1937), 589–604.
29. " Sulla connessione fra due problemi di calcolo delle probabilitá ", *Giorn. Ist. It. Attuari*, 8 (1937), 328–337.
30. " Eigenfrequenzen inhomogener Saiten ", *Z. Angew. Math. Mech.*, 18 (1938), 177–185.
31. " A Sturm-Liouville féle határértékfeladat sajátértekeiröl ". *Magy. Tud. Akademia Mat. es Természettudományi Értesitöje.*, 57 (1938), 1–6.
32. " Bemerkungen zur Integration der Mathieuschen Differentialgleichung durch Laplacesche Integrale ", *Compositio Math.*, 5 (1938), 435–441.
33. " Eine Verallgemeinerung der Neumannschen Polynome ", *Monatsh. Math. Physik.*, 47 (1938), 87–103.
34. " Asymptotische Darstellung der Whittakerschen Funktionen für grosse reele Werte des Argumentes und der Parameter ", *Caspois Mat. a Fys. (Praha)*, 67 (1938), 240–248.
35. " Bilineare Reihen der verallgemeinerten Laguerreschen Polynome ", *Weiner Sitzungsberichte*, 147 (1938), 513–520.
36. " Ueber eine erzeugende Funktion von Produkten Hermitescher Polynome ", *Math. Z.*, 44 (1938), 201–211.
37. " Die Funksche Integralgleichung der Kugelflächenfunktionen und ihre Uebertragung auf die Ueberkugel ", *Math. Ann.*, 115 (1938), 456–465.
38. " The Hankel transform of a product of Whittaker functions ", *J. London Maht. Soc.*, 13 (1938), 146–154.
39. " On some expansion in Laguerre polynomials ", *J. London Math. Soc.*, 13 (1938), 154–156.
40. " Einige Integralformeln für Whittakersche Funktionen ", *Proc. Akad. Amsterdam*, 41 (1938), 481–486.
41. " On certain Hankel transforms ". *Quart. J. Math. (Oxford)*, 9 (1938), 196–198.
42. " The Hankel transform of Whittaker's function $W_{k,m}(z)$ ", *Proc. Cambridge Phil. Soc.*, 34 (1938), 28–29.
43. " Integral representations for products of Whittaker functions ", *Phil. Mag.*, (7) 26 (1938), 871–877.

14692120, 1979, 2, Downloaded from https://londmathsoc.onlinelibrary.wiley.com/doi/10.1112/blms/11.2.191 by NICE, National Institute for Health and Care Excellence, Wiley Online Library on [30/10/2025]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License

44. " Einige nach Produkten von Laguerreschen Polynomen fortschreitende Reihen ", *Wiener Sitzungsberichte*, 148 (1939), 33–39.
45. " An integral representation for a product of two Whittaker functions ", *J. London Math. Soc.*, 14 (1939), 23–30.
46. " Infinite integrals involving Whittaker functions ", *J. Indian Math. Soc. New Series*, 3 (1939), 169–181.
47. " Integraldarstellungen für Produkte Whittakerscher Funktionen ", *Nieuw Arch. Wisk.*, 20 (1939), 1–34.
48. " Integral representations for Whittaker functions ", *Proc. Benares Math. Soc. New Series*, 1 (1939), 39–53.
49. " Note on the transformation of Eulerian hypergeometric integrals ", *Quart. J. Math. (Oxford)*, 10 (1939), 129–134.
50. " Transformation of hypergeometric integrals by means of fractional integration by parts ", *Quart. J. Math. (Oxford)*, 10 (1939), 176–189.
51. " Transformation einer gewissen nach Produkten konfluenter hypergeometrischer Funktionen fortschreitenden Reihe ", *Compositio Math.*, 6 (1939), 336–347.
52. " Transformation of a certain series of products of confluent hypergeometric functions. Applications to Laguerre and Charlier polynomials ", *Compositio Math.*, 7 (1939), 340–352.
53. " On a paper of Copson and Ferrar ", *Proc. Edinburgh Math. Soc.*, (2) 6 (1939), 11.
54. " Two infinite integrals ", *Proc. Edinburgh Math. Soc.*, (2) 6 (1939), 94–104.
55. " Integration of a certain system of linear partial differential equations of the hypergeometric type ", *Proc. Royal Soc. Edinburgh*, 59 (1939), 224–241.
56. " On Lambe's infinite integral formula ", *Proc. Edinburgh Math. Soc.*, (2) 6 (1940), 147–148.
57. " On some biorthogonal sets of functions ", *Quart. J. Math. (Oxford)*, 11 (1940), 111–123.
58. " Some integral representations of the associated Legendre functions ", *Phil. Mag.*, (7) 30 (1940), 168–171.
59. (With H. Kober) " Some remarks on Hankel transforms ", *Quart. J. Math. (Oxford)*, 11 (1940), 212–221.
60. " On fractional integration and its application to the theory of Hankel transforms," *Quart. J. Math. (Oxford)*, 11 (1940), 293–303.
61. " A class of hypergeometric transforms ", *J. London Math. Soc.*, 15 (1940), 209–212.
62. " Some confluent hypergeometric functions of two variables ", *Proc. Royal Soc. Edinburgh*, 60 (1940), 344–361.
63. " On some generalisations of Laguerre polynomials ", *Proc. Edinburgh Math. Soc.* (2), 6 (1941), 193–221.
64. " On Lamé functions ", *Phil. Mag.*, (7) 31 (1941), 123–130.
65. " On the connection between Hankel transforms of different order ", *J. London Math. Soc.*, 16 (1941), 113–117.
66. " Generating functions of certain continuous orthogonal systems ", *Proc. Royal Soc. Edinburgh A*, 61 (1941), 61–70.
67. " Integration of the differential equation of Appell's function F_4 ", *Quart. J. Math. (Oxford)*, 12 (1941), 68–77.
68. (With I. M. H. Etherington) " Some problems of non-associative combinations II ". *Edinburgh Math. Notes*, 32 (1941), 7–12.
69. " Note on Heine's integral representation of associated Legendre functions ", *Phil. Mag.*, (7) 32 (1941), 351–352.
70. " On algebraic Lamé functions ", *Phil. Mag.*, (7) 32 (1941), 348–350.
71. Tullio Levi-Civita. Obituary Notice. *Year Book of the Royal Soc. of Edinburgh*, 1941–1942.
72. " On certain expansions of the solutions of Mathieu's differential equation ", *Proc. Cambridge Phil. Soc.*, 38 (1942), 28–33.
73. " The Fuchsian equation of second order with four singularities ", *Duke Math. J.*, 9 (1942), 48–58.
74. " Integral equations for Lamé functions ", *Proc. Edinburgh Math. Soc.* (2), 7 (1942), 3–15.
75. " On certain expansions of the solutions of the general Lamé equation ", *Proc. Cambridge Phil. Soc.*, 38 (1942), 364–367.
76. " Integral equations for Heun functions ", *Quart. J. Math. (Oxford)*, 13 (1942), 107–112.
77. " Inversion formulae for the Laplace transformation ", *Phil. Mag.*, (7) 34 (1943), 533–537.
78. " Note on an inversion formula for the Laplace transformation ", *J. London Math. Soc.*, 18 (1943), 72–77.
79. " Certain expansions of the solutions of the Heun equation ", *Quart. J. Math. (Oxford)*, 15 (1944), 62–69.
80. (With W. O. Kermack) " Note on the equation $f(z)K_n'(z)-g(z)K_n(z) = 0$. *Proc. Cambridge Phil. Soc.*, 41 (1945), 74–75.
81. (With J. Cossar). *Dictionary of Laplace Transforms*. Five parts. 1944–46.

14692120, 1979, 2, Downloaded from https://londmathsoc.onlinelibrary.wiley.com/doi/10.1112/blms/11.2.191 by NICE, National Institute for Health and Care Excellence, Wiley Online Library on [30/10/2025]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License

82. (With John Todd). " Advanced instruction in practical mathematics ". *Nature*, 158 (1946), 160–000.
83. Harry Bateman. Obituary Notice. *Journal London Math. Soc.* 21 (1946) 300-310.
84. " Asymptotic representation of Laplace transforms with an application to inverse factorial series " *Proc. Edinburgh Math. Soc.* (2) 8 (1947) 20–24.
85. " On certain discontinuous wave functions " *Proc. Edinburgh Math. Soc.* (2), 7 (1947), 39–42.
86. Harry Bateman, Obituary Notices of Fellows of the Royal Soc. 5 (1947), 591-618.
87. " Expansions of Lamé functions into series of Legendre functions ", *Proc. Royal Soc. Edinburgh A*, 62 (1948), 247–267.
88. " Transformation of hypergeometric functions of two variables ", *Proc. Royal Soc. Edinburgh A*, 62 (1948), 378–385.
89. " Lamé–Wangerin functions ", *J. London Math. Soc.*, 23 (1948), 64–69.
90. " Hypergeometric functions of two variables ", *Acta Math.*, 83 (1950), 131–164.
91. " The inversion of the Laplace transformation ", *Mathematics Magazine*, 24 (1950), 1-6.
92. " The general form of hypergeometric series of two variables ", *Proc. Intern. Cong. of Math.*, 1 (1950), 413–414.
93. " On some functional transformations ", *Rend. Sem. Mat. Universita e Politecnico di Torino*, 10 (1950/51), 217–234.
94. " Note on the paper ' On a definite integral ' by R. H. Ritchie ", *Mathematical Tables and Other Aids to Computation*, 4 (1950), 179.
95. " The general form of hypergeometric series of two variables ", *Proc. Amer. Math. Soc.*, 2 (1951), 374–379.
96. Operational Calculus. By B. van der Pol and H. Bremmer (Book Review). *Bull. Amer. Math. Soc.*, 57 (1951), 319–323.
97. (With F. G. Tricomi). "The asymptotic expansion of a ratio of gamma functions", *Pacific J. Math.*, 1 (1951), 133–142.
98. " Parametric equations and proper interpretation of mathematical symbols ", *Amer. Math. Monthly*, 58 (1951), 629–630.
99. " The analytic theory of systems of partial differential equations ", *Bull. Amer. Math. Soc.*, 57 (1951), 339–353.
100. " Nota ad una lavoro di L. Toscano ", *Rend. Acc. dei Lincei* (8), 11 (1951), 44–45.
101. Die zweidimensionale Laplace-Transformation. By D. Voelker and G. Doetsch (Book review). *Bull. Amer. Math. Soc.*, 58 (1952), 88–94.
102. (With Maria Weber). "On the finite difference analogue of Rodrigues' formula ", *Amer. Math. Monthly*, 59 (1952), 163–186.
103. Randwertprobleme und andere Anwandungsgebiete der Hoeheren Analysis fuer Physiker, Mathematiker und Ingenieure, by F. Schwank. (Book review). *Bull. Amer. Math. Soc.*, 58 (1952), 274–276.
104. " Variational principles in the mathematical theory of diffraction ", *Atti d. acc. delle Scienze di Torino*, 87 (1952/53), 1–13.
105. Bessel functions. Part II. Functions of positive integer order. By W. G. Bickley *et al.* (Book review). *Bull. Amer. Math. Soc.*, 59 (1953), 189–191.
106. " Funzioni epicicliodali ", *Rend. dei Lincei* (8), 14 (1953), 393–394.
107. (With H. F. Bohnenblust *et al.*). " Asymptotic solutions of differential equations with turning points. Review of the literature ". Technical Report 1. Ref. No. NR 043-121. Pasadena, 1953.
108. Lezioni sulle funzioni ipergeometriche confluenti by F. G. Tricomi. Die konfluento hypergeometrische Funktion mit besonderer Berucksichtigung ihrer Anwendunger by Herbert Buchholz. (Book review) *Bull. Amer. Math. Soc.*, 60 (1954), 185–189.
109. (With C. H. Papas). " On diffraction by a strip ", *Proc. Nat. Acad. Sci.*, 40 (1954), 128–132.
110. (With M. Kennedy and J. L. McGregor). "Parabolic cylinder functions of large order", *J. Rat. Mech. and Analysis*, 3 (1954), 459–485.
111. Hypergeometric and Legendre functions with application to integral equations of potential theory by Chester Snow. (Book review). *Bull. Amer. Math. Soc.*, 60, (1954), 580–582.
112. " On a generalization of the Laplace transformation ", *Proc. Edinburgh Math. Soc.* (2), 10 (1954), 53–55.
113. (With M. Kennedy, J. L. McGregor and C. A. Swanson). " Asymptotic forms of Coulomb wave functions. I." Technical Report 4, ref. no. NR 043–121. Pasadena, 1955, 29 p.
114. " Asymptotic representations of Fourier integrals and the method of stationary phase ", *J. Soc. Indust. Appl. Math.*, 3 (1955), 17–27.
115. (With C. A. Swanson). "Asymptotic forms of Coulomb wave functions. II ". Technical Report 5. Ref. no. NR 043-121. Pasadena 1955. 24 p.
116. "Differential equations with transition points. I. The first approximation ". Technical Report 6. Ref. no. NR 043-121, Pasadena, 1955, 22 p.

14692120, 1979, 2, Downloaded from https://londmathsoc.onlinelibrary.wiley.com/doi/10.1112/blms/11.2.191 by NICE, National Institute for Health and Care Excellence, Wiley Online Library on [30/10/2025]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License

117. Review of F. G. Tricomi's book " Funzioni ipergeometriche confluenti ". *Bull. Amer. Math. Soc.*, 61 (1955), 456–460.
118. " Asymptotic factorization of ordinary linear differential operators containing a large parameter ". Technical Report No. 8 Pasadena Calif., February 1956. 25 p.
119. " Asymptotic expansions of Fourier integrals involving logarithmic singularities ", *J. Soc. Indust. Appl. Math.*, 4, (1956), 38–47.
120. " Singularities of generalized axially symmetric potentials ", *Comm. Pure and Appl. Math.*, IX, (1956), 403–414.
121. " Asymptotic solutions of differential equations with transition points ". *Proc. Intern. Cong. of Mathematicians*, 3 (1956), 92–101.
122. (With C. A. Swanson). " Asymptotic forms of Whittaker's confluent hypergeometric functions ", *Mem. Amer. Math. Soc.*, 25 (1957), 1–49.
123. (With E. T. Copson). " On a partial differential equation with two singular lines ". *Arch. Rational Mech. Anal.*, 2 (1958), 76–86.
124. " On the principle of stationary phase ", *Proc. Fourth Can. Math. Cong.* Toronto, 1959, 137–146.
125. " Asymptotic solutions of differential equations with transition points or singularities ", *J. Mathematical Phys.*, 1 (1960), 16–26.
126. " Elliptic function and integral ". McGraw–Hill Encyclopedia of Science and Technology. (1960), 561–563.
127. " Spherical harmonics ". McGraw–Hill Encyclopedia of Science and Technology. (1960). 609–611.
128. " Singular Volterra integral equations and their use in asymptotic expansions ", MRC Technical Summary Report No. 194. Mathematics Research Centre, U.S. Army, University of Wisconsin, Madison, Wisconsin, September 1960. 87 p.
129. " From delta functions to distributions ". In: *Modern Mathematics for the Engineer*, second series. Edited by E. F. Beckenbach. (McGraw–Hill, 1961), 5–50.
130. " General asymptotic expansions of Laplace integrals ", *Arch. Rational Mech. Anal.* 7, (1961), 1–20.
131. " An example in singular perturbations ". *Monatsh. d. Math.*, 66 (1962), 123–128.
132. Review of F. G. Tricomi's book " Vorlesungen ueber Orthogonalreihen ". *Bull. Amer. Math. Soc.*, 67 (1961), 447–449.
133. " Asymptotic Solutions of Ordinary Linear Differential Equations ". (Mimeographed). California Institute of Technology, 1961, 75 p.
134. " An expansion procedure for singular perturbations ", *Atti Acc. Sci. Torino*, 95 (1960–61), 651–672.
135. " Asymptotic forms for Laguerre polynomials ", *J. Indian Math. Soc.*, 24 (1960), 235–250.
136. Review of " Transzendente Funktionen " by A. Kratzer and W. Franz. *Bull. Amer. Math. Soc.*, 68 (1962), 51–55.
137. " Singular perturbations ", *Bull. Amer. Math. Soc.*, 68 (1962), 420–424.
138. " On a problem in singular perturbations ". Report on the second Summer Research Institute of the Australian Mathematical Society, 1962, 11–21.
139. (With I. N. Sneddon). " Fractional integration and dual integral equations ", *Canad. J. Math.*, 14 (1962), 685–693.
140. Review of " Polynomials Orthogonal on a Circle and Interval " by Ya. L. Geronimus. *Scripta Mathematica*, 26 (1963), 264–265.
141. " Note on a paper by Titchmarsh ", *Quart. J. Math.* (2), 14 (1963), 147–152.
142. " A result on non-linear Volterra integral equations ". In: Studies in *Mathematical Analysis and related Topics* (Essays in Honour of George Polya). (Stanford University Press, 1962). 104–109.
143. " On a nonlinear boundary value problem involving a small parameter ", *J. Australian Math. Soc.*, 2 (1962), 425–439.
144. " Singular perturbations of boundary value problems involving ordinary differential equations ", *J. Soc. Industr. Appl. Math.*, 11 (1963), 105–116.
145. " An integral equation involving Legendre's polynomial ", *Amer. Math. Monthly*, 70 (1963), 651–652.
146. " Some applications of fractional integration ". Mathematical Note No. 316, Boeing Scientific Research Laboratories, 1963, 23 p.
147. " An extension of the concept of real number ". *Proc. Fifth Canadian Mathematical Congress*, 1963, 173–183.
148. " Remarks at a Research Policy Seminar ". *Proc. Fifth Canadian Mathematical Congress*, 1963, 82–86.
149. (With M. Wyman). "The asymptotic evaluation of certain integrals", *Arch. Rational Mech. Anal.*, 14 (1963), 217–260.
150. (With R. P. Gillespie). Thomas Murray MacRobert (Obituary Notice). *Proc. Glasgow Math. Ass.*, 6 (1963), 57–64.

14692120, 1979, 2, Downloaded from https://londmathsoc.onlinelibrary.wiley.com/doi/10.1112/blms/11.2.191 by NICE, National Institute for Health and Care Excellence, Wiley Online Library on [30/10/2025]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License

151. " An integral equation involving Legendre functions ", *J. Soc. Industr. Appl. Math.*, 12 (1964), 15–30.
152. " The integral equations of asymptotic theory ". In: *Asymptotic Solutions of Differential Equations and Their Applications*, ed. by Calvin H. Wilcox, (Wiley, 1964), 211–229.
153. Review of " Les transformations integrales a plusieurs variables et leurs applications " by Mlle. Huguette Delavault. *Scripta Mathematica*, 27 (1964), 173.
154. " An application of fractional integrals ". *J. d'Analyse Math.*, 14 (1965), 113–126.
155. " Axially symmetric potentials and fractional integration ", *J. Soc. Ind. Appl. Maths.*, 13 (1965), 216–229.
156. " Some integral equations involving finite parts of divergent integrals ", *Glasgow Math. J.* 8 (1967), 50–54.
157. Review of " Partial Differential Equations of Parabolic Type " by Avner Friedman 1964 (Prentice–Hall). *Math. Gaz.* (1967).
158. " Two-variable expansions for singular perturbations ", *J. Inst. Math. Appl.*, 4 (1968) 113–119.
159. " Approximate solutions of a nonlinear boundary value problem ", *Arch. Rational Mech. Anal.*, **29** (1968), 1–17.
160. " Some dual integral equations ". *SIAM J. Appl. Math.*, 16 (1968), 1338–1340.
161. Review of " Formulaire pour le calcul operationnel " by V. A. Ditkin and A. P. Prudnikov 1967 (Masson & Cie, Paris). *Math. Gaz.* (1969), 53 110–111.
162. " Uniform asymptotic expansion of integrals ". In: *Analytic Methods in Mathematical Physics*, edited by R. P. Gilbert and R. G. Newton (Gordon and Breach, 1970), 149–168.
163. " On the Euler–Poisson–Darboux equation ", *J. Analyse Math.*, 23 (1970), 89–102.
164. Reviews of " Special Functions for Scientists and Engineers " by W. W. Bell, and " Basic Equations and Special Functions of Mathematical Physics " by V. Ya. Arsenin, translated from the Russian by S. Chomet, *Math. Gaz.*, 54 (1970), 97–98.
165. " Lectures on Generalized Functions and Integral Transformations ". In Report on the 10th Summer Research Institute of the Australian Mathematical Society 1970, 20–44.
166. (With A. C. McBride). " Fractional integrals of distributions ", *SIAM J. Math. Anal.*, 1 (1970), 547–557.
167. " Fractional integrals of generalised functions ", *J. Australian Math. Soc.*, 14 (1972), 30–37.
168. Reviews: " The Functions of Mathematical Physics " by Harry Hochstadt and " Solved Problems in Analysis " by O. J. Farrell and Bertram Ross. *Math. Gaz.*, 56 (1972), 354–355.
169. " Asymptotic evaluation of integrals involving a fractional derivative ", *SIAM J. Math. Anal.*, 5 (1974), 159–171.
170. " Fractional integrals of generalised functions ". In: *Fractional Calculus and its Applications*, ed. B. Ross. (Springer–Verlag Lecture Notes in Mathematics no. 457, 1975), 151–170.
171. " A case history in singular perturbations ". In: *International Conference on Differential Equations*, ed. H. A. Antosiewicz. (Academic Press, 1975), 266–286.
172. " Fourier transforms of integrable generalized functions ", *Philips Research Reports*, 30 (1975), 23–30.
173. Review of " Constructive Methods for Elliptic Equations " by R. P. Gilbert, *Bull. Amer. Math. Soc.*, 81 (1976), 1036–1037.
174. " Singular perturbations ". In: *Trends in Applications of Pure Mathematics to Mechanics*, ed. G. Fichera (Pitman Publishing, 1976), 53–62.
175. " Stieltjes transforms of generalized functions ", *Proc. Royal Soc. Edinburgh A*, 76 (1977), 221–249.
176. " An extension of a Hardy–Littlewood–Polya inequality ", *Proc. Edin. Math. Soc.*, 21 (1978), 11–15.
177. " The Stieltjes transformation on weighted L_p spaces ", *Applicable Anal.*, 7 (1978), 213–219.
178. (With E. F. Baxter and S. Holgate). Joseph Langley Burchnall. Obituary Notice. *Bull. London Math. Soc.*, 10 (1978), 111–115.

14692120, 1979, 2, Downloaded from https://londmathsoc.onlinelibrary.wiley.com/doi/10.1112/blms/11.2.191 by NICE, National Institute for Health and Care Excellence, Wiley Online Library on [30/10/2025]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License