



WILLIAM LEONARD FERRAR, 1893–1990

## OBITUARY

### WILLIAM LEONARD FERRAR

William Ferrar, who was elected to the London Mathematical Society in November 1922 and served for ten years in various capacities on its Council, was born in Bristol on 21 October 1893 and died in Oxford on 22 January 1990, aged 96. Although most of his working life was spent in the service of Hertford College and the University of Oxford, through his textbooks and examining he contributed significantly to mathematics courses and syllabuses nationwide.

His father, George William Parsons Ferrar, who was described as a lamplighter on his marriage certificate, and his mother, whose maiden name was Maria Susannah Dale, were not well off, and sometimes found it hard to find the 1d to pay for his school lunch. Bill (as he was known to most colleagues in later years) was the eldest of three brothers. His schooldays were happy and successful and he proceeded by scholarships from his first school, Ashley Down, to Queen Elizabeth's Hospital at Brandon Hill, and from there to Bristol Grammar School. At the Grammar School he was taught by Jeff Westcott, who inspired in his pupil a great love of pure mathematics but failed to inspire a similar feeling towards applied mathematics. In December 1911 Ferrar competed for scholarships at Oxford, and was successful at Queen's College, but failed to win the Balliol award which he had considered more prestigious. At that time a scholarship not only carried substantial academic standing but also was an important financial resource, paying, in Ferrar's case, £80 of the £150 per annum that he needed. (It is perhaps a surprise to many of us that even in those pre-war days the Local Authority made up the difference in the form of a grant of £70 per year from the Bristol Municipal Charities.) Once at Oxford, where he matriculated in 1912, Ferrar set his sights not just on First Class Honours in his first University Examinations (known as Honour Moderations or Mods) but on a better First than that of the Balliol scholar. Although the two of them were far ahead of the other candidates, the Balliol scholar again won. In those days, the next ambition for a clever undergraduate mathematician after success in Mods was the Junior Mathematical Scholarship at Oxford. Ferrar's tutor at Queen's (C. H. Thompson) was sufficiently impressed by his abilities and prospects to send him to Cambridge the following vacation. There he worked for six weeks with G. N. Watson, who introduced him to the study of infinite series and convergence which would later dominate his research work. Back in Oxford, he carried off the Junior Mathematical Scholarship in January 1914, beating the Balliol scholar into second place.

When the Great War broke out in the summer of 1914, Ferrar volunteered for army service. He served as a telephonist in an artillery unit which was sent to France and, with its old-fashioned horse-drawn guns, did a long tour of duty in and around the battlefields of the Somme. During that period of two years or so, he made a point of learning to speak French, and succeeded so well that he was transferred (not, in the first instance, very willingly) to an intelligence unit working a little further west, towards the Pas de Calais. There his job was amongst civilians, and he became fluent

also in the dialect of Picardy. He was allocated a motor-cycle to travel around his region but he was never happy with it, and at one point, roused to passion by finding himself the target of a German airborne gunner, vowed never to have anything more to do with the internal combustion engine for the rest of his life—a vow which he later described as ‘one of the most stupid oaths of my career’, but which he nevertheless kept. When his father died shortly before the war ended, he spent a few days, including Armistice Day, on leave in Bristol. Demobilisation came a few months later, and he returned to England in the spring of 1919.

Back in Oxford for the summer term, Ferrar seems to have been just as hard-working and competitive an undergraduate as before the start of war, little changed by his experiences, save perhaps by being a heavier pipe smoker (he never lost the habit). He quickly regained his mathematical skills and, in June 1920, was placed in the First Class in the Final Honour Schools (the Oxford final examination). It was a matter of some pride that his marks placed him at the top of the list, ahead of a man called Denning, who later turned to law and became very famous as Master of the Rolls. A week before the examination, Ferrar had obtained an appointment as assistant lecturer at the University College of North Wales at Bangor. After spending six weeks of the summer as a prep school master, he started at the College that September at a salary of £250 per annum. The department had a staff of three, a professor, a lecturer and an assistant lecturer, and so this first appointment carried quite a heavy load of teaching. It was here that he established the style of lecturing for which he is clearly remembered. Firmness and the control of his audience were never in doubt, and his material was meticulously prepared. In such a small department, and without any training, it was less easy to embark on a research career. Nevertheless, inspired by a student’s exercise, he wrote a thesis on determinants whose elements are determinants which won for him the Oxford Senior Mathematical Scholarship in 1922, and which, after much revision, was finally accepted for publication by the London Mathematical Society [1]. By September 1923, now a lecturer earning £400 a year (supplemented by £60 earnings from school certificate marking), his prospects were good enough that he could support a family, and he married Edna O’Hara, whom he had met in Nottingham four years before on a visit to his aunt, the widow of his father’s brother who had been a lecturer at the University College there until the war in which he was killed.

Although happy enough in Bangor, Ferrar had felt isolated from research activity and had on two occasions applied unsuccessfully for posts elsewhere. Therefore when, some time during the summer term 1924, he received a letter from E. T. Whittaker inviting him to become Senior Lecturer in the University of Edinburgh at an annual salary of £500, he jumped at the chance. It was promotion, financial improvement and the chance to work with a world-famous professor. Under the influence of Professor Whittaker, with E. T. Copson and A. C. Aitken as colleagues, and with only six hours a week of lectures to give (in Bangor it had been at least twelve), his interest in research flourished and he began to publish papers in analysis. It was during that year, some time around Easter 1925, that Ferrar received, wholly unexpectedly, a letter from the Principal of Hertford College, asking if he would allow his name to be considered for election to the fellowship in mathematics that had been vacated on the death of J. E. Campbell, FRS (whose name is remembered in the so-called Campbell–Baker–Hausdorff formula for multiplication of exponentials in Lie algebras). He had expected to stay several years at Edinburgh and to develop his career there, but he accepted the Oxford post and returned south in September 1925.

At Hertford College his teaching hours increased again, and he had to cover a much broader range of subjects than had been required at Bangor or Edinburgh. He gave tutorials on all the pure mathematics in the Oxford undergraduate syllabuses and on all the compulsory applied mathematics; only the applied mathematical special subjects for Finals were farmed out to specialist tutors. In addition to his teaching and lecturing, he had substantial responsibilities for the setting and marking of university examinations. This work was shared by all established members, professors and college fellows, of the sub-faculty of mathematics at Oxford, but as they numbered only about a dozen, the chores came round every year. At that time a college salary was barely adequate for married fellows, and financial necessity led Ferrar to seek appointment with the Oxford and Cambridge Joint Examinations Board marking mathematics scripts for school-leaving examinations (at first School Certificate and Higher Certificate, later General Certificate of Education at Ordinary and Advanced levels). He soon became an awardee, which meant taking part in the setting of questions, marking sometimes as many as a thousand scripts, and taking part in the awarding of passes and grades. For fifty years examining was an integral part of his mathematical life, and for many of those years he and E. A. Maxwell of Cambridge dominated the Oxford and Cambridge higher mathematics examinations for schools.

During the first twelve years or so back in Oxford, although he was heavily committed to teaching and examining, Ferrar's first priority was research. Whittaker had drawn his attention to interpolation theory, and he wrote several papers on the subject (see [7, 9, 12]). The basic problems are quite simple to state. Let  $a_1, a_2, a_3, \dots$  and  $z_1, z_2, z_3, \dots$  be two sequences of complex numbers, and suppose that  $|a_r| \rightarrow \infty$  as  $r \rightarrow \infty$ . First, among the regular functions  $f$  such that  $f(a_r) = z_r$ , find a 'best' one, in the sense of minimising a sort of energy integral. Secondly, under what conditions does interpolation a second time give back the same function; that is, if the process is applied to a sequence  $(b_r)$  to find an interpolation function  $g$  such that  $g(b_r) = f(b_r)$  for all  $r$ , under what circumstances will we find that  $g = f$ ? This work was intermingled with some systematic, but even at that time less highly valued work on summability methods, and a number of papers on convergence and other areas within analysis.

In 1930 his research entered a new phase when he began a substantial and productive collaboration with A. L. Dixon, then Waynflete Professor of Pure Mathematics. Their work was motivated by a famous lattice-point enumeration problem of number theory. Let  $r(n)$  be the number of representations of the positive integer  $n$  as a sum of two squares. The sum  $R(x) := 1 + \sum_{1 \leq n \leq x} r(n)$  enumerates the lattice points inside and on the circle of radius  $x^{1/2}$ , and therefore, to a first approximation it will be the area  $\pi x$  of the circular disc. Define  $P(x) := R(x) - \pi x$ . The fact that  $P(x) = O(x^{1/2})$  is easy to see. In 1906 Sierpiński had proved that  $P(x) = O(x^{1/3})$ , and in 1915 G. H. Hardy had proved the existence of a positive number  $K$  such that  $|P(x)| > Kx^{1/4}$  for infinitely many values of  $x$ . By 1928 Nieland had proved that  $P(x) = O(x^{27/82})$ , and the conjecture that  $P(x) = O(x^{1/4+\epsilon})$  for any positive  $\epsilon$  was well established. A very similar problem has to do with the average order of  $d(n)$ , the number of divisors of  $n$ . It is known as Dirichlet's divisor problem, and asks for good estimates for the function  $\Delta(x)$  defined by  $\Delta(x) := D(x) - x \log x$ , where  $D(x) := \sum_{n \leq x} d(n)$ . It comes down to estimating the error term for the number of lattice points below the rectangular hyperbola (in the  $u$ - $v$ -plane) with equation  $uv = x$ . The similarity between these two problems goes much deeper than merely the fact that they can be interpreted as questions about lattice points in certain familiar

regions of the plane. For,  $d(n)$  obviously, and  $r(n)$  less obviously, are determined by the prime factorisation of  $n$ . Because of this,  $P(x)$  and  $\Delta(x)$  are closely related to error terms in estimates for integrals of the Riemann zeta-function. As a tool for studying these problems, Voronoi had, in 1904, proved summation formulae of the following kind: for certain (arithmetical) functions  $\tau$  there exist analytic functions  $\delta, \alpha$  such that

$$\frac{1}{2} \sum_{a < n \leq b} \tau(n) f(n) + \frac{1}{2} \sum_{a \leq n < b} \tau(n) f(n) = \int_a^b f(x) \delta(x) dx + \sum_1^\infty \tau(n) \int_a^b f(x) \alpha(nx) dx.$$

The existence of  $\delta$  is not hard to show, and the principal difficulty in proving summation formulae of this type comes in establishing the existence of  $\alpha$ . The papers by Dixon and Ferrar, and some written by Ferrar alone, treat various questions about convergence of certain series and evaluation of certain singular integrals in the theory of Bessel functions that arise in connection with the application of Voronoi summation formulae. They do not deal directly with lattice-point problems, although Ferrar was conscious that that was the ultimate goal. One example must suffice. In his paper [30] in 1935, Ferrar pointed out that in the principal cases for which Voronoi's formula had been proved, namely those in which  $\tau(n) = 1$ ,  $\tau(n) = r(n)$  and  $\tau(n) = d(n)$ , the function  $\alpha$  turns out to be what he recognised as 'a kernel of Fourier type', and he investigated how far this might be true in general. It should be said that although such famous contemporaries of Dixon and Ferrar as Sierpiński, Hardy and Landau were attracted to these lattice-point problems, the conjectures that  $P(x) = O(x^{1/4+\varepsilon})$  and that  $\Delta(x) = O(x^{1/4+\varepsilon})$  are still open—the best result presently known, to the effect that  $P(x)$  and  $\Delta(x)$  are  $O(x^{7/22+\varepsilon})$  for any positive  $\varepsilon$ , is due to H. Iwaniec and C. J. Mozzochi ('On the divisor and circle problems', *J. Number Theory* 29 (1988)).

During this period when Ferrar was dedicated to research, he was also kept very busy contributing his fair share of scholarly routine. His proof-reading reached, for one two-year period, a rate of about 1600 pages a year. He was editor (with Poole and Chaundy) of the new *Quarterly Journal of Mathematics* which, under Hardy's influence, had just been founded at Oxford to replace the *Quarterly of Pure and Applied Mathematics* after the death of Glaisher. Ferrar served from 1930 to 1933, receiving manuscripts, getting them refereed, preparing accepted papers for press and reading the proofs; he remained on the editorial board from 1934 until 1947. He also served for ten years, 1933–42, on the Council of the London Mathematical Society. In 1934 he succeeded G. N. Watson as Secretary, working for five years in tandem with F. P. White of Cambridge. In those days, the job of the secretaries was not merely to run the society, but also to run its publications. Although decisions about acceptance and rejection of manuscripts were taken by Council in full session (a practice that survived until 1968), the two secretaries were effectively the Editors of the *Proceedings* and the *Journal*. After he finished his period as secretary, Ferrar served a further four years on Council, two of them as Vice-President. In a letter to one of us (PMN) dated 15 February 1991, Michael Ferrar (the only child born to Edna and William, in 1927) writes 'The L.M.S. was very much a part of my father's life for quite a number of years. I may have been being too clever by half but I still recall my indignation at not being allowed a mark when as a prepschool boy I put the London Mathematical Society as my answer to the question "what do the initials L.M.S. stand for?"'. (In those days the expected answer would have been London, Midland and Scottish Railway.)

At first Ferrar had avoided the major administrative chores of College and University, although he became Senior Tutor of his college for a few years from 1934. But late in 1937 he became Bursar of Hertford College and, although initially he believed he was only standing in for a colleague on leave, this was a post he held for twenty-two years. The beginning of the Second World War threw him into the activities of the intercollegiate Domestic Bursars' Committee, and this, in its turn, led to his being elected to Hebdomadal Council (the body which is, in effect, the governing body of Oxford University) and thence to its two main executive bodies, the General Board of the Faculties, which is responsible for the academic well-being of the university, and the Curators of the University Chest, which deals with its finances. Nominally, the Vice-Chancellor chairs these bodies; in fact, they were at that time already chaired more often by vice-chairmen who thereby became key administrators in the university and who, in virtue of their positions, had to chair large numbers of committees. Ferrar was at different times Vice-Chairman of the Curators of the Chest and Vice-Chairman of the General Board. He became an expert on University policy and finance, and its frequent spokesman.

There was now no time for research and, indeed, Ferrar believed that he would have found no satisfaction in mathematical niceties at a time when his College and University depended so heavily upon him. In later life, his vigorous advice to younger mathematicians was that administration is the enemy of research—and vice versa. Instead, he wrote textbooks, and he found this activity (unlike mathematical research) to be an excellent complement to administrative responsibilities. The idea of writing a textbook had been in his mind for some time before it eventually took shape as *A textbook of convergence*, which was first published in 1938, and which was reprinted many times. This was followed in 1941 by *Algebra: a textbook of determinants, matrices and algebraic forms*. His books, of which there were ten in all, grew out of the needs of pupils in schools as he saw them in the scripts he marked, and in universities as he discovered when he was called upon to lecture on subjects for which no adequate textbook was available. It was his experience as an examiner, for example, that led him to write his *Higher algebra for schools*, which sold some 32000 copies and which was translated into German. *Finite matrices* was the most advanced of the books, aimed at a postgraduate readership. His success as a textbook writer was the result of his sympathetic understanding of the difficulties encountered by young mathematicians and his constant search for greater clarity of exposition.

At one point he applied for the post of Principal of University College, Dundee (now Dundee University, at that time still a subsidiary of the University of St Andrews), but was not elected. His friend E. T. Copson, by then Regius Professor at St Andrews, had suggested that this lack of success was due to his having no degree higher than the Oxford MA and had urged him to take his doctorate. Ferrar was still undecided about applying for the degree when, as he recalled, he overheard some member of the university jibe 'What does a Bursar know about academic matters?'. In 1947 he submitted thirty-five research papers [1–35] and two books [38, 39] to the faculty, and was awarded the degree of Doctor of Science by the University of Oxford. This had the desired effect of establishing his academic credentials at a time when he was conscious that he was all too easily categorised as an administrator.

Ferrar gave up the Bursarship of Hertford in 1959 and, with only a year or two to go before retirement, he began to look forward to a time of relative calm as a college tutor. But this plan was swept aside when Hertford invited him to become its Principal. After his retirement as Principal at the age of 70, an event marked by his

election to an Honorary Fellowship of Hertford College and recorded in the *Hertford College Magazine* in the words 'He retires as a vigorous septuagenarian who is still to be seen on the dance floors of Oxford', he continued writing textbooks and examining until the style of mathematics he enjoyed was no longer taught in schools or universities. Doing nothing was totally uncharacteristic of Ferrar's life. In his long retirement his time was filled with writing, solving mathematical problems, examining (even after he gave up mathematical examining at the age of 80, he continued to examine in Use of English), swimming, dancing (he and Edna had taken their first lesson in old time dancing on his 65th birthday), the theatre, music and talking. And he was an excellent talker—with spice but without malice.

In his long career Ferrar had achieved plenty to talk about. He had contributed his fair share to each of the three areas of academic work, research and writing, teaching, and administration, and he took legitimate pride in his achievements.

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