THOMAS MUIRHEAD FLETT

S. J. TAYLOR

Thomas Muirhead Flett was in the prime of life when a rare form of cancer struck him: he died after about a year of ill health on February 13, 1976, though doctors claimed he must have suffered from the disease for about fifteen years before anyone knew. The University of Sheffield lost a Professor of Pure Mathematics and the British mathematical community lost a distinguished analyst who had contributed much to its life. Every task which Tom undertook was done thoroughly, with meticulous attention to detail, and he was always willing to play a full part in any enterprise with which he was concerned. We will be able to describe just a few of the areas in which he made a notable contribution.

Tom Flett was born on 28 July 1923, soon after his parents moved from Scotland to London, and at the age of 11 he won a scholarship awarded by the County of Middlesex from a state primary school to University College School, Hampstead. Little is known of the details of his progress in U.C.S., but Tom himself ascribed his deep interest in mathematics to the teaching he enjoyed in those years. He remembered a particular teacher, the late Mr. Marsden, who inspired him with a love for mathematics that was to shape his life. His school reports from the last two years at U.C.S. make interesting reading, for they accurately describe the qualities which shaped his life. Even at this stage he was obviously aware of his own ability without becoming objectionable due to arrogance. His family used to tease him about the comment " He should, however, realise that the methods of others, even if they appear not so good as his own, may still have merit worthy of notice ".

During the war, Tom was first employed as a laboratory assistant in the Post Office Research Station at Dollis Hill for 3 years, and then as a research physicist at Simmonds Aerocessories in Brentford for 2 years. While working in the Post Office, he studied part-time at Acton Technical College and obtained first class Honours in the London University B.Sc. General Degree in Mathematics and Physics. He then pursued his interest in Mathematics at Chelsea Polytechnic where he obtained first class Honours in the B.Sc. Special Degree in Mathematics in 1945, obtained a mark of distinction in further optional subjects in 1946 and in the M.Sc. in Mathematics of London University which he was awarded in 1947.

Tom's first teaching post was that of Assistant Lecturer at Chelsea in 1945–47. One of his responsibilities in 1946–47 was to teach Analysis to a class of two—the late Rex Tims, and Joseph Dunnage. The latter writes:

" Tom was a brilliant lecturer, and the hour with him was the highlight of my week. He made ' epsilonology ' seem so easy and natural that he must have been the prime influence in turning Rex and myself into analysts. His course was completely worked out and his blackboard technique impeccable. When I started lecturing myself in 1947 at Battersea Polytechnic, I quite deliberately modelled myself on him, and I do not think that any more formal training could have been better. Rex Tims' skill as a teacher was acknowledged in Professor Scott's obituary [S] and I think that a lot of the credit must go to Tom ".

[BULL. LONDON MATH. SOC., 9 (1977), 330–339]

THOMAS MUIRHEAD FLETT 1923–1976

At last, in 1947, Tom won the opportunity to become a full time university student: he had been awarded the Sherbrooke Studentship of London University which he was allowed to hold at Cambridge, and he obtained a D.S.I.R. research grant for 1948–50. He studied under J. E. Littlewood, whose influence on his research interests will become apparent when we consider in detail some of Tom's contributions to mathematics. During this period he also acted as Littlewood's secretary and he started on the mammoth task of rewriting [L]. Though this new edition of [L] was never finished, much of Tom's research was to grow out of problems which became clear as he worked on the revision. Tom was awarded a Ph.D. in 1950 for a thesis entitled "Some applications of estimates of exponential sums to the theory of functions", and he obtained an Sc.D. in 1961 from the University of Cambridge for his major contributions to Analysis.

Tom Flett held permanent teaching posts in only two universities. His first appointment, in 1950, was as Assistant Lecturer in Pure Mathematics at Liverpool, but he was promoted steadily through the ranks of Lecturer and Senior Lecturer to become a Reader in 1963. Geoffrey Walker, who was head of department during this period writes:

"In my opinion Flett was the most useful assistant any Head of Department could have. He did not push his own ideas but would produce them when invited, he was willing to undertake responsibility, and could always be safely left to do a job efficiently and with complete mastery over details. His unfussy but accurate attention to detail in everything he did was perhaps one of his most outstanding personal characteristics and he eventually took charge of all organisational jobs in the department under my general direction. One particularly onerous responsibility he undertook concerned the new building opened in 1962—when he was responsible during several years of planning for liaison with the architects—to see that plans included what the department wanted, and for preparing the detailed schedule of equipment. He was largely responsible for the design of ornamental panels on mathematical topics that decorate the entrance hall".

"When Flett came to Liverpool in 1950 he was already an established teacher with well considered views on syllabuses and teaching methods in analysis at all levels. He played an important part in the change to modern syllabuses in analysis: his book [3] was the outcome of his work at that time and reflects the polish of his first and second year courses to prospective honours students. The best known of Flett's research students is U. Kuran who came from Turkey to work with Flett for a Ph.D. He later joined the staff and is now a Reader in the department, with a solid reputation in Analysis".

In 1964 Tom spent a term as Visiting Professor at the Middle East Technical University in Ankara, Turkey and in 1966–67 he was Visiting Professor at the University of Washington in Seattle, U.S.A. He took up the post of Professor of Pure Mathematics in the University of Sheffield in 1967 and held it until his death. On going to Sheffield he made an immediate impact on the work of his department and on its relationships with other departments in the mathematical sciences. He was largely responsible for a new degree structure which has worked well. His precise mind combined with a willingness to consider administrative problems of great complexity made him a valuable member of any committee, and he made notable contributions to the work of the Faculty of Pure Sciences as well as the Senate of Sheffield University. Among his diverse responsibilities was the Committee for

Halls of Residence and the Body advising the local Colleges of Education on both the content and standards of mathematical training for teachers.

Tom Flett was a member of both the London Mathematical Society and the Mathematical Association and was one of the Founder Fellows of the Institute of Mathematics and its Applications. In all these organisations he was willing to undertake hard work and responsibility. He served on the I.M.A. Council for 3 years, and was Chairman of the local branch of the M.A., and he put in many hours of detailed and careful work as a member of the Editorial Board for the L.M.S.: at the time of his death he was Editor of the *Proceedings*. When I was Editor of the *Journal* and later the *Proceedings* of the London Mathematical Society, Tom was one of the referees on whose judgement we could always rely. On more than one occasion authors whose papers were "accepted after major revision" wrote to me saying that the anonymous referee (Tom) had made fundamental suggestions which transformed their paper from the mediocre to one with clear impact. So he helped many authors towards the standards of clear exposition which he set himself.

In 1948 Tom married Joan F. Ayers while he was a research student in Cambridge. Joan and Tom had been at primary school together from the age of 7 to 11 and their childhood friendship was renewed and deepened later partly through their common interest in Scottish dancing. It is interesting to see how Tom's early experiences were to play a notable part in moulding his life. His Scottish parents sent him to dancing lessons when he was seven and from that time he never stopped dancing until he fell ill in 1975. He not only enjoyed it as a recreation but, in the small amount of time he could spare from his first love, mathematics, took a serious academic interest in it. He and Joan wrote many papers on dancing and in 1964 they published the unique book [**2**]. This book was the result of thorough and careful research in which facts had to be traced back to their primary source. Much of the information on which this research was based was collected in a systematic series of journeys covering the Hebrides, Orkneys and Shetlands as well as mainland Scotland. For example, during the International Congress of Mathematicians in Edinburgh in 1958, Tom took off one day to collect several dances and relevant background material from villages within easy access of Edinburgh. In 1960 he turned his attention to the traditional step-dancing of the Lake District and made a major contribution to the knowledge of traditional dancing in England. He was well known in the folk dance world for his talks and demonstrations of both Scottish and English dancing. Tom had several other interests outside mathematics which he took seriously. He knew a great deal about eighteenth century glass and had collected some interesting examples; and he also loved Alpine flowers and grew as many as he could in his own garden.

It is interesting to note that Flett's research career might well have been stillborn if he had accepted in 1948 the well paid permanent Lectureship at Chelsea Polytechnic offered to him just at the time he took on the responsibilities of marriage. He turned it down because of his burning interest in mathematics—for he wanted to take advantage of the opportunity to become a research mathematician. In this decision he was supported by his wife, in fact Joan has entered into all his interests and responsibilities and they have had a very happy and purposeful life together. There are two daughters Lindsay Frances, now Mrs. G. H. Smith, and Jane Margaret.

Contributions to Mathematics

The areas of analysis to which Tom Flett contributed are diverse but interrelated.

It is these relationships between different topics that enhance the richness of mathematics and provide the interest to his work. At the same time the fact that he often developed a result in one area and immediately applied it to a different problem makes it difficult to give a coherent account. Many of his results can only be expressed in very technical language, so we will only attempt to describe a small subset of them. Although several papers are relevant to more than one topic, we try to arrange them in main subject areas.

1. *Fourier series and power series* **[6, 7, 12, 22, 23, 24, 25, 31, 40, 41, 46]**.

It is easy to see that the series

$$\sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{t}{n} = Q(t)$$

converges for each fixed t, and the sum $Q(t)$ has the same order as

$$Q_1(t) = \sum_{n \leqslant t} \frac{1}{n} \sin \frac{t}{n} \quad \text{as } t \to \infty.$$

In his very first paper **[6]**, Flett shows his understanding of the intricate arguments typifying hard analysis by improving on existing Hardy–Littlewood estimates for $Q_1(t)$ to obtain $Q_1(t) = O[(\log t)^{3/4} (\log\log t)^{1/2+\varepsilon}]$ for each $\varepsilon > 0$. He goes on to apply this result to a study of the zeta function near the critical line $\sigma = 1$. In [7] he discusses estimates for the size of coefficients for $\phi(z) = \sum a_n z^n$ when ϕ is regular in $|z| < 1$ but does not take the values $\pm 2\pi k i$, and for the corresponding conjugate function $\psi(z)$ with coefficients $\{b_n\}$. For example, he proves that

$$|b_n| < A(a_0) (\log N \log\log N)^{3/4},$$

where $A(a_0)$ denotes a constant depending only on the first term a_0 of the power series for ϕ.

Suppose $E \subset [0, 1]$ has positive measure, and p is a positive integer. In collaboration with Carefoot **[40]**, Flett shows that there is an integer $M = M(E, p)$ such that if $\sum_{n=0}^{\infty} a_n r_n(t) = f(t)$ is a Rademacher series converging on E, then

$$\sup_{m \geqslant M} \sum_{n=m}^{m+p} |a_n| \leqslant \sup_{t \in E} |f(t)|.$$

This result has the interesting corollary that any Rademacher series which converges to zero on a set of positive measure must be a polynomial. The paper **[41]** is a unified account of the properties of mean values on circles of radius $p < 1$ for power series convergent in the unit circle. If $\phi(z) = \sum a_n z^n$, then

$$M_p(\phi, \rho) = \left\{\frac{1}{2\pi} \int_{-\pi}^{\pi} |\phi| \rho e^{i\theta} |^p d\theta\right\}^{1/p}$$

is increasing in ρ, and the Hardy class H^p consists of those functions ϕ for which $\mathcal{M}_p(\phi) = \lim_{p \to 1-} M_p(\phi, \rho)$ is finite. Flett obtains a collection of useful inequalities involving the fractional derivative $\theta^\beta \phi(z) = \sum n^\beta a_n z^n$ ($\beta \geqslant 0$), and when $\phi(0) = a_0 = 0$, the fractional integral $\theta_\alpha \phi(z) = \sum_{n=1}^{\infty} n^{-\alpha} a_n z^n$.

2. *Summability* [**12, 15, 16, 17, 22, 23, 24, 29, 30, 31**].

When α is not a negative integer, put

$$E_n^\alpha = \binom{\alpha+n}{n}, \quad n > 0, \quad E_0^\alpha = 1; \quad \sigma_n^0 = A_n^0 = a_0+a_1+\ldots+a_n,$$

$$A_n^\alpha = \sum_{\nu=0}^{n} E_{n-\nu}^{\alpha-1} A_\nu^0, \quad \sigma_n^\alpha = A_n^\alpha/E_n^\alpha.$$

Then the limit of σ_n^α as $n \to \infty$ is called the (C, α) sum of the series. In Flett's first paper involving summability he considers the Fourier series of a function $f \in L(-\pi, \pi)$. It is known that, if f belongs to the class lip $\alpha(0 < \alpha < 1)$, then the Fourier series is Cesaro summable, that is $\sigma_n(x) = \sigma_n^1(x) \to f(x)$ for all x and in fact the error

$$\sigma_n(x) - f(x) = O(n^{-\alpha}).$$

Flett shows that, if $\phi_x(t) = f(x+t)+f(x-t)-2f(x)$ and $\int_0^t |d\phi_x(u)| \leqslant At^\alpha$ for $0 \leqslant t \leqslant \delta$, then $\sigma_n^\alpha(x) - f(x) = O(n^{-\alpha})$, and nothing essentially weaker will imply this inequality.

Whenever $\sum a_n$ is such that $\phi(z) = \sum a_n z^n$ converges for $|z| < 1$ and $\lim_{x\to 1-} \phi(x) = \Lambda$ exists, then we call Λ the Abel sum of the series $\sum a_n$. It is said to be absolutely Abel summable if, in addition $\int_0^1 |\phi'(x)|\, dx < \infty$. Flett [**15**] introduces the notion of being absolute Abel summable with index $k \geqslant 1$ by requiring that

$$\int_0^1 (1-x)^{k-1} |\phi'(x)|^k\, dx < \infty,$$

and calls such a series summable $|A|_k$. An example of the type of result obtained is the main theorem of [**16**] where it is shown that, if $\sum a_n$ is lacunary and summable $|A|_k$, then $\sum |a_n|^k$ converges and $\sum |a_n|^k \leqslant A(c,k) \int_0^1 (1-x)^{k-1} |\phi'(x)|^k\, dx$ where $A(c, k)$ depends only on k and the lacunary index c. Flett explores some implications of this result in function theory. In [**29, 30**] Flett introduces and explores yet more refinements of the hierarchy of summability methods. For a series summable (C, α), and $k \geqslant 1$, we say it is absolutely summable (C, α) or summable $|C, \alpha|_k$ if, in addition to the convergence of σ_n^α we have $\sum n^{k-1} |\sigma_n^\alpha - \sigma_{n-1}^\alpha|^k$ convergent. The case $k = 1$ reduces to that of summability $|C, \alpha|$ discussed in [**15**]. In addition, he calls a series strongly summable $(C, \alpha+1)$ with index k or summable $\{C, \alpha\}_k$ to the sum s if $\sum_{n=0}^{m} |\sigma_n^\alpha - s|^k = O(m)$; and it is strongly Abel summable, or summable $\{A\}_k$ if

$$\int_0^R (1-x)^{-2} |\phi(x) - s|^k\, dx = O\left(\frac{1}{1-R}\right) \quad \text{as} \quad R \to 1-.$$

It is hard work, involving many intricate arguments, to justify all these definitions and establish their relationships. The concepts are applied to several different problems: in particular, in [**31**], Flett obtains new information about the power series in the Hardy class H^p.

3. *Function theoretic identities and inequalities* **[8, 11, 13, 16, 17, 20, 26, 27, 33, 38, 47, 48, 50, 51, 53, 57]**.

Perhaps this is the area where Flett made the most substantial contributions. In trying to revise the book [L], many interesting questions arose, and most of his work in other areas eventually led back to some identity or inequality for functions of an appropriate class. Flett's contributions are mostly given in classical terms, but many of them turn out to be relevant to the study of dual spaces and in particular, B.M.O., the space of functions of bounded mean oscillation where recent progress has been rapid.

Suppose $\lambda > 0$, $0 < \eta < 1$ and Ω_θ denotes the plane domain bounded by the tangents to $|z| = \eta$ from $e^{i\theta}$ and the larger arc of the circumference of $|z| = \eta$. For a function $\phi(z)$ of Hardy class H^λ, put

$$s(\theta) = \left\{ \int_{\Omega_\theta} |\phi'|^2 \, d\omega \right\}^{1/2}.$$

The study of H^λ uses extensively the inequality

$$\left\{ \int_{-\pi}^{\pi} s^\lambda(\theta) \, d\theta \right\}^{1/\lambda} \leqslant A(\lambda, \eta) \left\{ \int_{-\pi}^{\pi} |\phi(e^{i\theta})|^\lambda \, d\theta \right\}^{1/\lambda},$$

for which Flett obtains a new and simpler proof in **[13]**. The paper **[20]** considers periodic functions U in $L(-\pi, \pi)$ and their conjugates V and shows, for example, that when U is an odd function, $p > 1$, $-(1/p) < \alpha < 2-(1/p)$ then

$$\int_{-\pi}^{\pi} |x|^{p\alpha} |V|^p \, dx \leqslant A(p, \alpha) \int_{-\pi}^{\pi} |x|^{p\alpha} |U|^p \, dx.$$

On the other hand, when U is even and has bounded variation,

$$\left(\int_0^\pi \frac{1}{x} |V|^p \, dx \right)^{1/p} \leqslant A(p) \int_0^\pi |dU|.$$

In **[33]**, Flett obtains properties of

$$L_{k,\lambda}(w, \theta) = \sup_{0 \leqslant \rho < 1} \left\{ \delta^{\lambda-1} \int_{-\pi}^{\pi} \frac{w^k(\rho, \theta+t)}{|1-\rho e^{it}|^\lambda} \, dt \right\}^{1/k}$$

for $w(\rho, \theta)$ non-negative and subharmonic in $\rho < 1$. He uses these to simplify results of Stein concerning the function

$$M_\lambda(\theta) = \sup_{0 \leqslant \rho < 1} \left\{ (1-\rho)^{\lambda-1} \int_{1-\rho \leqslant |t| \leqslant \pi} t^{-\lambda} |u(\rho, \theta+t)|^2 \, dt \right\}^{1/2}$$

and applies them to the study of Cesaro means for power series. For a function

$h: [0, \infty) \to [0, \infty)$, $0 < k \leqslant \infty$, $0 < \mu < \infty$, Flett considers in **[57]** the transfotms

$$I^*_{k,\mu}(h) = \left\{ \int_0^\infty \left(t^\mu \int_0^\infty e^{-st} h(s)\, ds \right)^k t^{-1}\, dt \right\}^{1/k}$$

$$I^*_{k,\mu}(h) = \left\{ \int_0^\infty \left(t^{-\mu} \int_0^\infty e^{-s/t} h(s)\, ds \right)^k t^{-1}\, dt \right\}^{1/k}$$

$$J_{k,\mu}(h) = \left\{ \int_0^\infty \left(t^{-\mu} \int_0^t h(s)\, ds \right)^k t^{-1}\, dt \right\}^{1/k}$$

$$J^*_{k,\mu}(h) = \left\{ \int_0^\infty \left(t^{-\mu} \int_t^{2t} h(s)\, ds \right)^k t^{-1}\, dt \right\}^{1/k}$$

He shows that $I_{k,\mu}(h) = I^*_{k,\mu}(h)$ and that there are absolute constants depending only on k and μ bounding the ratio between $I_{k,\mu}(h)$ and each of $J_{k,\mu}(h)$, $J^*_{k,\mu}(h)$. These results are applied to the theory of n-dimensional Fourier transforms.

4. *Geometric Analysis* **[3, 37, 39, 42, 49, 54, 58]**.

When Flett came to write his undergraduate text book [3] on Analysis, he formulated the theory in a context more general than is normally used for teaching undergraduates. For example his definition of a derivative is given in essentially geometric terms to relate it to both linear algebra and topology. It is clear that the careful thought which went into the book led to a number of useful research contributions. For example, in [39], he considers a function $f: A \to Y$ where A and Y are Banach spaces and, using the Dieudonné definition for the differential of f, he extends Lagrange's theorem on conditional extrema to Banach spaces. Again, for Banach spaces X, Y and a C^1 function $f: X \to Y$, $c \in Y$, $S = f^{-1}(\{c\})$ is a level surface for f; Flett **[42]** uses the tangent cone to S to obtain the existence of points in S of minimum norm.

When E and F are Euclidean spaces, $A \subset E$, $f: A \to F$ is a Lipshitz function with constant K, the celebrated theorem of Kirszbaum shows that f has an extension to all of E with the same Lipshitz constant. Minty showed that this theorem remains true when E, F are Hilbert spaces, but is false for arbitrary normed complete spaces. Flett **[53]** considers the case when A is a bounded closed convex set, with non-empty interior, in a complete normed vector space. He gives an explicit construction for an extension of f to E with Lip constant αK, where $\alpha = \rho/\delta$ and ρ is the diameter of A, δ the radius of an inscribed ball.

5. *Complex analysis, harmonic functions* **[9, 10, 14, 32, 43, 44, 45]**.

If $w = f(z)$ defines a conformal map from the interior G of a Jordan curve onto $|w| < 1$, it is known that the map extends to one which is continuous on $\bar{G}$. In **[9]**, Flett produced a new proof of this result using the idea that G is "uniformly locally connected", because $\bar{G}$ is compact. In **[14]** Flett uses a topological argument to shorten a proof of Tsuji of the important Lindelöf theorem that if $w = f(z)$ is a conformal map of a simply connected domain D onto $|w| < 1$ and $\{z_n\}$ is a sequence in D such that $w_n = f(z_n)$ converges to α, $|\alpha| = 1$, in a Stolz angle, then every condensation point of $\{z_n\}$ is a principal point of the prime end of D which corresponds to α. He obtains an elementary proof, in [32] for the Denjoy–Seidel–Walsh theorem to the effect that a complex function $f(z)$ which is regular and univalent in $|z| < 1$ has the property that, for almost all θ, $f'(z) = O[(1-z)^{-1/2}]$ uniformly as $z \to e^{i\theta}$ in a Stolz angle.

For the three papers [**43**, **44**, **45**] Flett uses his expertise to obtain properties of harmonic functions. In [**43**] he studied the integral $M(w; t)$ of a non-negative subharmonic function w in the Euclidean half-space Ω on hyperplanes parallel to the boundary $\partial\Omega$, at a distance t away from it. The main results are sufficient conditions, involving the behaviour of w at infinity, for M to be decreasing and convex as a function of t. The main result in [**44**] is, most unexpectedly, the subharmonic-type behaviour in the unit ball of the function $|u|^p$, where u is harmonic in B and $p < 1$. He showed that, although the volume mean-value inequality cannot possibly hold, it does hold provided one introduces a multiplicative constant independent of u. The corresponding result for functions harmonic in Ω is also obtained. The Marcinkiewicz interpolation theorem, which he cherished and about which he gave many lectures, is the starting point of an easy proof, in [**45**], for a theorem of Hardy and Littlewood on fractional integrals of functions holomorphic in the unit disc.

6. *Miscellaneous* [**18**, **19**, **21**, **28**, **34**, **35**, **36**, **52**, **55**, **56**].

Some of these publications show the care with which Flett did his teaching and examining. For example [**18**] contains a new formulation for the remainder term in a polynomial expansion in terms of the $(n-1)$th derivative of the slope of the chord. The paper [**19**] shows his continued interest in a precise geometrical understanding of the calculus, while [**35**, **36**] contain simple and precise solutions to problems which are elementary in nature.

Towards the end of his life Tom Flett began to take a deeper interest in differential analysis. In [**52**], he extends some classical results to differential equations defined on a Banach space. At the time of his death he had essentially completed the manuscript of a text-book [**5**] on differential analysis—which we must hope will now appear posthumously. It is a pity that the chapter Tom intended to include in this book on the historical development of the subject was never written. His failing health prevent a planned expedition to Cambridge to consult original sources in the library.

Tom's work as a mathematician will live on: it will continue to affect those who read it and build on it. However those of us who knew Tom Flett personally have a double sense of loss—we have not only lost a colleague with good judgement who was contributing actively in many ways, but we have lost a personal friend who was both caring and kind. We extend our sympathy and best wishes to his wife Joan, and their two daughters, and to Tom's younger sister Mrs. Jean Smith Cottington.

References

[S] D. B. Scott, " Sydney Rex Tims ", *Bull. London Math. Soc.*, 4 (1972), 100–101.
[L] J. E. Littlewood, *Lectures on the theory of functions*, (Oxford, 1944).

Publications

8. Ph.D. thesis, (University of Cambridge, 1950).

Books

2. *Traditional Dancing in Scotland* (written jointly with his wife) (Routledge, Kegan Paul 1964).
3. *Mathematical analysis*, (McGraw Hill, London 1966).
4. (Joint editor with W. W. Rogosinski) *Collected papers of G. H. Hardy, Vol. III*, Trigonometric series. Mean values of power series—Notes and comments on the papers by T. M. Flett. (Cambridge, 1968).
5. *Text-book on Differential Analysis* to be published by Cambridge University Press.

Articles

6. " On the function $\sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{t}{n}$ ", *J. London Math. Soc.*, 25 (1950), 5–19.
7. " On a coefficient problem of Littlewood and some trigonometrical sums ", *Quart. J. Math.*, 2 (1951), 26–52.
8. " Note on a function-theoretic identity ", *J.. London Math Soc.*, 29 (1954), 115–118.
9. " A note on conformal mapping ", *J. London Math. Soc.*, 29 (1954), 118–121.
10. " Some remarks on schlicht functions and harmonic functions of uniformly bounded variation ", *Quart. J. Math. (Oxford)*, 6 (1955), 59–72.
11. " Some remarks on a maximal theorem of Hardy and Littlewood ", *Quart. J. Math. (Oxford)*, 6 (1955), 275–282.
12. " On the degree of approximation to a function by the Cesàro means of its Fourier series ", *Quart. J. Math. (Oxford)*, 7 (1965), 81–95.
13. " On some theorems of Littlewood and Paley ", *J. London Math. Soc.*, 31 (1956), 336–344.
14. " On a theorem of Lindelöf concerning prime ends ", *Tohoku Math. J.* (2), 8 (1956), 273–274.
15. " On an extension of absolute summability and some theorems of Littlewood and Paley ", *Proc. London Math. Soc.* (3), 7 (1957), 113–141.
16. " A high-indices theorem ", *Proc. London Math. Soc.* (3), 7 (1957), 142–149.
17. " Some theorems on power series ", *Proc. London Math. Soc.* (3), 7 (1957), 211–218.
18. " A note on Taylor's theorem ", *Math. Gazette*, 41 (1957), 131–133.
19. " The definition of a tangent to a curve ", *Edinburgh Math. Notes*, No. 41 (1957), 1–9.
20. " Some theorems on odd and even functions ", *Proc. London Math. Soc.* (3), 8 (1958), 135–148.
21. " A mean value theorem ", *Math. Gazette*, 42 (1958), 38–39.
22. " On the absolute summability of a Fourier series and its conjugate series ", *Proc. London Math. Soc.* (3), 8 (1958), 258–311.
23. " Some more theorems concerning the absolute summability of Fourier series and power series ", *Proc. London Math. Soc.* (3), 8 (1958), 357–387.
24. " On the strong summability of Fourier series ", *J. London Math. Soc.*, 33 (1958), 311–326.
25. " A local property of Fourier series ", *J. London Math. Soc.*, 33 (1958), 450–454.
26. " Some theorems on fractional integrals ", *Proc. Camb. Philos. Soc.*, 55 (1959), 31–50.
27. " A note on some inequalities ", *Proc. Glasgow Math. Assoc.*, 4 (1958), 7-15.
28. " An example on term-by-term differentiation of infinite series ", *Math. Gazette*, 43 (1959), 278–279.
29. " Some generalisations of Tauber's second theorem ", *Quart. J. Math. (Oxford)*, 10 (1959), 70–80.
30. " Some remarks on strong summability ", *Quart. J. Math. (Oxford)*, 10 (1959), 115–139.
31. " On the summability of a power series on its circle of convergence ", *Quart. J. Math. (Oxford)*, 10 (1959), 179–201.
32. " On the radial order of a univalent function ", *J. Math. Soc. Japan*, 11 (1959), 1–3.
33. " A note on a maximal function ", *Tohoku Math. J.*, 12 (1960), 34–36.
34. " An early nineteenth century arithmetic exercise book ", *Math. Gazette*, 45 (1961), 1–8.
35. " The evaluation of definite integrals as the limit of sums ", *Math. Gazette*, 46 (1962), 6–13.
36. " Continuous solutions of the functional equation $f(x+y)+f(x-y)=2f(x)f(y)$ ", *Amer. Math. Monthly*, 70 (1963), 392–7.
37. " On transformations in R^n and a theorem of Sard ", *Amer. Math. Monthly*, 71 (1964), 623-9.
38. " A theorem on functions of class H^λ ", *Proc. London Math. Soc.* (3), 14 A (1965), 86–92.
39. " On differentiation in normed vector spaces ", *J. London Math. Soc.*, 42 (1967), 523–33.
40. (with W. C. Carefoot), "A note on Rademacher functions", *J. London Math. Soc.*, 42 (1967), 542–44.
41. " Mean values of power series ", *Pacific J. Math.*, 25 (1968), 463–94.
42. " Points of minimum or maximum norm on smooth surfaces in Banach spaces ", *J. London Math. Soc.*, 44 (1969), 583–86.
43. " Mean values of subharmonic functions on half spaces ", *J. London Math. Soc.* (2), 1 (1969), 375–83.

44. " Inequalities for the *p*th mean values of harmonic and subharmonic functions with $p \leqslant 1$ ", *Proc. London Math. Soc.* (3), 20 (1970), 249–75.
45. " On the rate of growth of mean values of holomorphic and harmonic functions ", *Proc. London Math. Soc.* (3), 20 (1970), 749–68.
46. " A theorem concerning the real part of a power series ", Mathematical Essays dedicated to A. J. MacIntyre, (Ohio University Press, 1970), 135–43.
47. " Temperatures, Bessel potentials and Lipschitz spaces ", *Proc. London Math. Soc.* (3), 22 (1971), 385–451.
48. " The dual of an inequality of Hardy and Littlewood and some related inequalities ", *J. Mathematical Analysis and Applications*, 38 (1972), 746–765.
49. " Mean value theorems for vector valued functions ", *Tohoku Math. J.* 2nd series, 24 (1972), 141–151. Dedicated to Professor Gen-ichiro Sunouchi.
50. " Some inequalities for a hypergeometric integral ", *Proc. Edinburgh Math. Soc.*, 18 (1972), 31–34.
51. " Lipschitz spaces of functions on the circle and the disc ", *J. Math. Analysis and Applications*, 39 (1972), 125–158.
52. " Some applications of Zygmund's lemma to non-linear differential equations in Banach and Hilbert spaces ", *Studia Math.*, 44 (1972), 335–344 + Addendum and corrigendum to the paper, 649–650.
53. " On a theorem of Pitt ", *J. London Math. Soc.* (2), 7 (1973), 376–384.
54. " Extensions of Lipschitz functions ", *J. London Math. Soc.* (2), 7 (1974), 604–608.
55. " Aims and purposes of examining ", *Bull. I.M.A.*, 9 (1973), 126–133.
56. " Some historical notes and speculations concerning the mean value theorems of the differential calculus ", *Bull. I.M.A.*, 10 (1974), 66–72.
57. " Some elementary inequalities for integrals with application to Fourier transforms " *Proc. London Math. Soc.* (3), 29 (1974), 538–556.
58. " Ampere and the horizontal chord theorem ", *Bull. I.M.A.*, 11 (1975), 34.