CHARLES FOX (1897–1977)

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Charles Fox died on 30 April, 1977, in Montreal, at the age of 80 as a result of cardiac arrest. He had been a member of the Society since 1919.

He was born on 17 March, 1897, in London, England, the son of Morris and Fanny Fox. He began his education at Coopers Company School in Bow (London) and then at the City of London School, where he was a Scholar. He went up to Cambridge in 1915 as a Scholar of Sidney Sussex College and obtained First Class Honours in Part I of the Mathematical Tripos in 1916 and in Part II the following year. That same year he joined the British Expeditionary Forces in France and was wounded in action in 1918. He then returned to Cambridge and completed his B.A.

Fox began his professional career in 1919 as a Demonstrator and Lecturer in Mathematics at the Imperial College of Science in London. The following year he joined Birkbeck College of the University of London as a Lecturer in Mathematics, where he remained until 1948. In 1925 he published his first paper in the *Proceedings of the London Mathematical Society*. Three years later, in 1928, he was awarded the D.Sc. degree of the University of London.

In 1932 he married Eileen Kaye in London, and in 1949 they moved to Montreal, Canada, where he joined the faculty of McGill University as an Associate Professor of Mathematics. He was very happy in his family life. Two children were born; a son, Edward, and a daughter, Frances, and there are seven grandchildren.

Within a year after his arrival at McGill University, Fox finished his only book: *An Introduction to the Calculus of Variations*, which was published by Oxford University Press. His motivation for writing this book is fairly well reflected in the second paragraph of its preface, which reads: "... *During my many years of teaching at London University I felt that none of the existing texts covered the subject as I would like to teach it and so I undertook the task of writing one of my own* ...".

Fox was promoted to the rank of Professor of Mathematics at McGill University in 1956 and five years later he was elected a Fellow of the Royal Society of Canada. Upon his retirement from McGill University in 1967, Fox accepted a visiting professorship at Sir George Williams University (now Sir George Williams Campus of Concordia University), also in Montreal, so he remained an active teacher of mathematics until 1975, when he reluctantly gave up his lectureship at Concordia University. The following year he was awarded an honorary LL.D. degree by Concordia University.

Fox's publications span a period of half a century, his first paper having appeared in 1925, while the last one was published a couple of years before his death. His papers may be divided into six main (not mutually exclusive) groups:

(i) theory of null series and null integrals (*cf.* **[1]** and **[2]**);

(ii) hypergeometric functions and their generalizations (*cf.* **[3]**, **[5]**, **[7]** and **[32]**);

(iii) integral transforms and integral equations (*cf.* **[4]**, **[8]**, **[12]**, **[14]**, **[16]**, **[18]**, **[22]** to **[25]**, **[27]** to **[29]**, **[31]** to **[39]**, **[41]** and **[42]**);

(iv) mathematics of navigation (*cf.* **[20]** and **[43]**);

(v) theory of statistical distributions (*cf.* [29] and [40]);

(vi) notes on miscellaneous topics (*cf.* [6], [9] to [11], [13], [15], [17], [19], [21], [26], [28] and [30]).

One of Fox's major contributions to the theory of hypergeometric functions is his systematic study of the asymptotic expansion of the generalized hypergeometric function defined by (*cf.* [5], p. 389 *et seq.*)

$$ {}_pF_q^*[(\alpha_1, A_1), \ldots, (\alpha_p, A_p);\ (\beta_1, B_1), \ldots, (\beta_q, B_q);\ z] = \sum_{n=0}^{\infty} \frac{f(n)z^n}{n!}, \tag{1} $$

where, for convenience,

$$ f(t) = \left\{ \prod_{j=1}^{p} \Gamma(\alpha_j + A_j t) \right\} \left\{ \prod_{j=1}^{q} \Gamma(\beta_j + B_j t) \right\}^{-1}, \tag{2} $$

and the coefficients $A_1, \ldots, A_p$, $B_1, \ldots, B_q$ are positive real numbers such that

$$ \omega = 1 + \sum_{j=1}^{q} B_j - \sum_{j=1}^{p} A_j > 0. \tag{3} $$

Although a substantially more general integral function than (1) was studied earlier by G. N. Watson in 1913, Fox's methods were an improvement over that of E. W. Barnes (who, in 1907, had discussed the asymptotic expansions of the generalized hypergeometric function (1) in the familiar special case when $A_j = 1$, $j = 1, \ldots, p$, and $B_j = 1, j = 1, \ldots, q$) and differed from those of Watson mainly in that no appeal was made to the properties of certain inverse factorial series. It should be mentioned that Fox's methods were further generalized by E. M. Wright in 1935 (and again in 1940) in order to cover the case of the integral function (1) when

$$ |\arg(-z)| \leqslant \pi - \tfrac{1}{2}\pi\varepsilon, \tag{4} $$

where $0 < \varepsilon \leqslant 2$ and ε need not be rational; in the subsequent literature, therefore, the integral function (1) is quite often referred to as Wright's generalized hypergeometric function.

The contribution to the theory of special functions for which Fox will always be remembered by workers in these areas of applicable analysis is, beyond any manner of doubt, his paper [32] in which he formally introduced the H-function defined by (*op. cit.*, p. 408)

$$ H_{p,q}^{m,n}\left[z \left| \begin{matrix} (\alpha_1, A_1), \ldots, (\alpha_p, A_p) \\ (\beta_1, B_1), \ldots, (\beta_q, B_q) \end{matrix} \right. \right] = \frac{1}{2\pi i} \int_L \theta(\zeta) z^{\zeta} d\zeta, \tag{5} $$

where

$$ \theta(\zeta) = \frac{\prod_{j=1}^{m} \Gamma(\beta_j - B_j \zeta) \prod_{j=1}^{n} \Gamma(1 - \alpha_j + A_j \zeta)}{\prod_{j=m+1}^{q} \Gamma(1 - \beta_j + B_j \zeta) \prod_{j=n+1}^{p} \Gamma(\alpha_j - A_j \zeta)}, \quad 0 \leqslant m \leqslant q, \ 0 \leqslant n \leqslant p, \tag{6} $$

and L is a suitable contour of the Mellin–Barnes type (in the complex ζ-plane) which separates the poles of one product from those of the other. If the positive coefficients $A_1, \ldots, A_p$ and $B_1, \ldots, B_q$ are constrained by the inequality

$$\Omega = \sum_{j=1}^{n} A_j - \sum_{j=n+1}^{p} A_j + \sum_{j=1}^{m} B_j - \sum_{j=m+1}^{q} B_j > 0, \tag{7}$$

then, under certain additional conditions, the integral in (5) is absolutely convergent and defines the H-function, analytic in the sector

$$|\arg(z)| < \tfrac{1}{2}\Omega\pi, \tag{8}$$

the point $z = 0$ being tacitly excluded.

The H-function may be looked upon as an appropriate further extension of the generalized hypergeometric function defined by (1); it also provides an elegant generalization of T. M. MacRobert's E-function and C. S. Meijer's G-function, both of which evidently correspond to the special case of (5) when $A_j = 1$, $j = 1, \ldots, p$, and $B_j = 1, j = 1, \ldots, q$. It may be remarked in passing that a study of one form or the other of the H-function, which was initiated as long ago as 1888 by S. Pincherle, appeared in the works of E. W. Barnes in 1908, H. Mellin in 1910, A. L. Dixon and W. L. Ferrar in 1936, S. Bochner in 1958, and several others. Nonetheless, a first systematic presentation of the properties of the H-function as a symmetrical Fourier kernel was made in the aforementioned 1961 paper by Fox whose name has naturally been associated with this function in the literature ever since.

Fox did not pursue his H-function beyond the invaluable discovery of its properties (as a symmetrical Fourier kernel) incorporated in his paper **[32]**; instead, he turned to the solution of certain classes of integral equations by operational techniques involving integral transformations. Nevertheless, a large number of research workers have since been engaged in the investigation of the H-function and its natural extensions in two and more complex variables; until his death Fox encouraged and was appreciative of some of these developments especially in his correspondence and long discussions with the present author.

Charles Fox leaves behind him the memory of a quiet family man, an effective teacher of mathematics who made significant contributions to his field of expertise, and who inspired many of his colleagues and students to carry out independent researches for themselves. He will be remembered for his great intellectual gifts and research contributions, for his courtesy and kindliness and, above all, for his qualities of honesty and integrity.

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