OBITUARY NOTICES

PAUL GORDAN.

The death of Paul Gordan on December 21st, 1912, in the seventy-sixth year of his age, removed one who for over thirty-four years had been an honorary member of the London Mathematical Society. He was born at Breslau on April 27th, 1837 ; he studied at the University of Giessen, and there took the degree of Doctor of Philosophy in the year 1862. The following year saw the publication of the first of his writings, which were to raise him very rapidly to a position of eminence. From that time there flowed from his pen a steady stream of work, which ceased only three years before his death. Altogether we owe over eighty publications to him—including two books and the famous *Programm.*

The coming of Clebsch to the University of Giessen in the year 1862 was an event of the greatest importance in determining the direction in which Gordan's genius should be applied. His interest at that time centred round the theory of Abelian functions ; and he introduced Clebsch to Riemann's work in this subject. The two entered into a fruitful partnership, producing among other things their treatise *Abel'sche Functionen.* At a time when Riemann's ideas were only imperfectly understood, and the accuracy of his reasoning open to question, this little volume, based on familiar geometrical reasoning, but informed with a rare insight into the principles of the subject, was able to exert a great influence. What are the precise contributions of Gordan and Clebsch perhaps the authors themselves would not say. But that between them, in addition to Clebsch's undoubted geometric genius, they were in touch with the work of Weierstrass and Kronecker, as well as intent on bringing Riemann's results into common use, no reader can fail to see. The book is of permanent value.

Clebsch on his side made known to Gordan the new theory of modern algebra which he was presently to make his own in so remarkable a manner ; and to this theory Gordan made his first contributions in the year 1867.

It was natural that these should have in view applications to his old study of Abelian functions, in the first place. But very soon Gordan

applied himself to the elucidation of the theory of forms for its own sake. Two years later (1869) he published* the first proof of his great theorem of the "Finiteness":—*i.e.*, "Every invariant or covariant of a binary quantic can be expressed rationally and integrally in terms of a finite number of them." A theorem which at once placed the theory of modern algebra on an entirely fresh basis. The proof was built up by an elaborate and somewhat difficult system of inductions; it possessed the great merit that it gave a practical method of constructing the "complete system" in any given case. The original proof was accompanied by the construction of the complete system for the cases of the quintic and the sextic, by way of illustration—this had not been accomplished before, in fact it was generally accepted at the time that no *finite* complete system existed in either case. A variety of other proofs of this theorem have since been put forward by various writers; and while these bring into prominence important and interesting facts, there still remains no other proof which actually provides the means of constructing the finite system.

Gordan next devoted his energies to the simplification of his method of proof; and to the extension of his theorem to other cases. Thus he shewed that it was true for any finite system of binary quantics as well as for a single quantic; he proved it for combinants, also for "types," and finally for ternary forms of the lowest orders. The most notable advance in this direction was the publication of his *Programm* at Erlangen in 1875, entitled "Ueber das Formensystem Binaerer Formen," which contained his original proof of the finiteness, modified and extended, and further developments in the theory of binary forms.

The proof of the finiteness produced no small sensation amongst the other mathematicians who were engaged in building up the theory of modern algebra. In England, Cayley, to whom the foundations of the subject were so largely due, had stated his conclusion in the second Memoir on Quantics (1855) that the number of covariants of a binary quantic is infinite first in the case of the quintic; and that the number of invariants is first infinite in the case of the septic. This statement coming from so great an authority, given with apparently adequate reasons, was accepted as correct, until Gordan's paper was published. In 1870, Cayley published his tenth Memoir on Quantics as the result of Gordan's theorem; in this he shewed the flaw in his own reasoning by which he arrived at the above false conclusion, and gave a *résumé* of Gordan's proof of his theorem, "the importance of which," he went on to state, "in reference to the whole theory of forms it is impossible to estimate too highly."

* *Crelle*; Vol. LXIX, pp. 323–354.

A friendly rivalry ensued between the English mathematicians and Gordan and his followers. To one party was due the honour of laying the foundations of the subject, to Gordan the honour of proving the fundamental theorem. The methods of attack were different; in fact, the one was the complement of the other. For while Gordan and his school obtained a system in terms of which all other invariants and covariants could be rationally and integrally expressed; there remained some uncertainty as to whether some members of the system might not be reducible. They, in fact, obtained an upper limit to the irreducible system, while the methods of the English school gave a lower limit to this system. Thus when the two came to an agreement an absolute certainty was established that the system in question was complete and irreducible. Sylvester, in particular, applied himself to attacking the same systems by the method of generating functions which Gordan had discussed by his symbolical methods: and in several instances succeeded in reducing covariants which had been included in the systems. He speaks of the matter thus in discussing the system for two biquadratics:* "J'ajouterai seulement que cette preuve éclatante de l'insuffisance de la méthode de M. Gordan et de son école, pour séparer les formes véritablement élémentaires des formes superflues qui s'y rattachent (insuffisance reconnue par M. Gordan lui-même de la manière la plus loyale dans son discours inaugural prononcé à Erlangen), n'ôte rien à la valeur immense du service qu'il a rendu à l'Algèbre, en ayant le premier démontré l'existence d'une limite au nombre de ces formes."

The untimely death of Clebsch, in the year 1872, brought to an end a partnership which had been most fruitful. The last of the memoirs† published under the joint names of Clebsch and Gordan did not appear until the following year.

In 1873, Gordan was placed on the staff of the *Mathematische Annalen*, which he had already enriched with a good many contributions; and which continued to be the principal means by which he made the results of his labours known to the world.

The year 1875 saw Gordan appointed to a professorship of mathematics at Erlangen; and at Erlangen he remained for the rest of his life. His arrival there was marked by one of his most memorable publications, the *Programm*, "Ueber des Formensystem Binaerer Formen," which has already been mentioned.

In 1885–1887 there appeared his book on *Invarianten-Theorie*, in two

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* *Comptes Rendus*, LXXXIV, 1877; *Collected Papers*, Vol. III, p. 66.

† "Ueber cubische ternäre Formen," *Math. Ann.*, Vol. VI.

volumes; in writing which he had the assistance of Dr. G. Kerchensteiner. The second volume contained an exposition of his theorem of the finiteness and of his own work in that region; an exposition of the greatest value to those who wished to learn the subject. A third volume was announced and eagerly anticipated by many, but it has never appeared.

It is not possible in a short obituary notice to give a full account of all the writings of Prof. Gordan. Naturally most of his researches followed on the foundation of the "finiteness." Many of his papers dealt with resultants and discriminants, and also with the solution of quintic and sextic equations. He made important contributions to the theory of ternary forms, particularly to the cases of the ternary cubic and biquadratic. He also wrote valuable expositions of already known theorems, notably that on the transcendency of e and π.*

A. Y.

The above notes are written by one who had not the privilege of knowing the man, but only his writings. Prof. M. Noether has courteously informed the writer that an account of the life and work of Prof. Gordan is about to appear in the Mathematische Annalen.

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* *Math. Ann.*, Vol. XLIII.