Thus $$[e_\beta(m)\, be_\alpha(n)][ne_\alpha(n)]^* = [me_\beta(m)]^*\, [e_\beta(m)\, be_\alpha(n)],$$

so $$e_\beta(m)\, bn^*\, e_\alpha(n) f = e_\beta(m)\, m^*\, be_\alpha(n) f$$

for all $f \varepsilon D_{n^*}$. Let $\alpha \to \infty$ and then $\beta \to \infty$ (strong limits) and we obtain $bn^* f = m^* bf$. Hence $bn^* \subseteq m^* b$ and the proof is complete.

References.

1. N. Dunford, " Spectral operators ", *Pacific Journal of Math.*, 4 (1954), 321–354.
2. B. Fuglede, " A commutativity theorem for normal operators ", *Proc. Nat. Acad. U.S.A.*, 36 (1950), 35.
3. P. R. Halmos, " Commutativity and spectral properties of normal operators ", *Acta Scientiarum Mathematicarum*, 12 (1950), 153–156.
4. E. Hille, *Functional analysis and semi-groups* (American Math. Soc. Colloquium Publications, vol. 31, 1948).
5. C. R. Putnam, " On normal operators in Hilbert space ", *American J. of Math.*, 73 (1951), 357–362.
6. J. v. Neumann, " Approximative properties of matrices of high finite order ", *Portugaliae Mathematica*, 3 (1942), 1–62.

University of Virginia,
Charlottesville, Virginia, U.S.A.

HANS LUDWIG HAMBURGER

M. E. GRIMSHAW.

Hans Ludwig Hamburger was born in Berlin on 5 August, 1889. From 1907 to 1914 he studied at the Universities of Berlin, Lausanne, Göttingen and Munich. He obtained his doctorate at the University of Munich in 1914 and his " Habilitation " at the University of Berlin in 1919. He became Ausserordentlicher Professor of the University of Berlin in 1922, and Ordentlicher öffentlicher Professor and Director of the Mathematical Institute of the University of Cologne in 1924. In 1927 he married Malla Jessen. This marriage was later dissolved.

Hamburger left Cologne in 1935 and was living with his mother in Berlin until he came to England in 1939. In 1941 he became Lecturer in Mathematics at the University College, now the University, of Southampton. He enjoyed working in this country and often expressed his admiration of the English way of life. Nevertheless, he welcomed the opportunity to take a Chair again when the invitation came in 1947 from the University of Ankara. He remained there until 1953, when he resumed his Professorship in Cologne. The return to Cologne gave him great pleasure, for his mathematical loyalty had remained centred in Germany. He paid two visits to the U.S.A. between 1950 and 1956 from which there resulted the publications in the *Proceedings of the Stillwater Symposium* ([43], [44]) and a technical report (not yet published as far as I know) with the cooperation of Arlen Brown and Shlomo Sternberg, on primitive operators of deficiency (m, m), for a research project contract between the Office of Scientific Research and the University of Chicago Department of Mathematics.

Hamburger had not been in good health since his last years in Ankara. He died in Cologne, after a short severe illness, at the age of 67, on 14 August, 1956, only two months after his marriage to Vera Schereschewsky.

In carrying out his mathematical research, Hamburger formed the habit of working far into the early morning hours and of sleeping in the afternoon. He claimed that this made possible a more intense concentration. In his relaxed and non-mathematical moments he sought society and enjoyed conversation. He was an enthusiastic dancer, skier and climber; he was knowledgeable about modern art, he enjoyed the theatre and, rather late in life, he came to have a great interest in music. As a colleague he was stimulating though exacting; as a companion he was gay and entertaining.

Throughout Hamburger's published work there recurs evidence of the effect of his early interest in differential geometry and mathematical methods. He had easy control over complicated analytical manipulations and long-sustained detailed investigations, and he turned again and again to geometrical interpretations to provide both motivating ideas and illustrative examples. The course of his work on each of the main problems that he tackled follows the same pattern; a series of shorter preparatory papers on different aspects of the problem, followed by a very detailed complete account, usually spread over two or three papers. There are two distinct periods of activity, before and after 1942.

The work of the first period contains important contributions in two fields; that of differential geometry and partial differential equations, as well as that of continued fractions and the Stieltjes moment problem with which his name is probably most frequently associated. The work on the moment problem incorporated earlier results of Stieltjes, Perron and others, generalized them and gave new proofs and presentations. The moment problem formulated by Stieltjes is that of finding, for a given sequence of real constants c_ν, a real non-decreasing function $\phi(u)$ such that

$$c_\nu = \int_0^\infty u^\nu \, d\phi(u).$$

A necessary condition for a solution is that two sets of Hankel determinants formed from the constants should be positive and, moreover, if they are positive there is at least one solution. Stieltjes called the problem determined if there is precisely one solution and undetermined if there is more than one. Hamburger's advance on the earlier results consisted of dropping half the sufficiency condition for the existence of a solution (he took one set of determinants positive and the other set not all zero) and of replacing the integral requirement by the more general form

$$c_\nu = \int_{-\infty}^{\infty} u^\nu \, d\phi(u).$$

Further, he gave necessary and sufficient conditions for the problem to be determined in terms of what he called the "complete" convergence of an associated continued fraction. Finally, he also gave the criterion in terms of the properties of a pair of associated quadratic forms of which the given constants are the coefficients. The work contained considerable detail about the convergence of continued fractions and the behaviour of associated power series. As a by-product there were results on the completeness of certain systems of orthogonal functions in $[0, \infty)$ which included the Laguerre polynomials.

The work of the first period culminated in three papers concerned with the proof of Carathéodory's conjecture that on every closed (analytic) surface of genus 0 there are at least two umbilics. The truth of the conjecture was first established, in a paper of 23 pages, under certain regularity conditions for the surface, and then, in two further papers of 156 pages, for the general case for which a long and detailed study of the associated differential equation had to be undertaken. The presence of exceptional cases made the investigation difficult and somewhat tedious and the staying power of the reader is severely taxed by the length of the argument. I recently turned up a letter of Hamburger's in which he expressed his disappointment "to be condemned to the writing of ever so long papers . . . but I can't help it". Such was the type of problem that challenged his interest.

Belonging to the first period of Hamburger's work there were also series of papers on spherical representation of two-parameter surfaces and on the functional equation of the Riemann ζ-function.

After 1942 his work was confined to the investigation of linear transformations in Hilbert space. The starting point of his work on closed Hermitian transformations of deficiency index (m, m) was J. v. Neumann's theory of these transformations and their self-adjoint extensions, and the observation that the two best known examples, the boundary problem of the self-adjoint ordinary linear differential equation of second order and the infinite Jacobi matrix of deficiency index $(1, 1)$ associated with an undetermined moment problem, each lead to a self-adjoint extension whose spectrum consists of an infinite number of isolated eigenvalues.

Hamburger obtained necessary and sufficient conditions for this situation for a type of transformation which he called " prime " (later " primitive ") and he went on to construct all such transformations of which a given self-adjoint transformation, which might have continuous spectrum, was an extension. (A prime Hermitian transformation H is such that there is no subspace which reduces H and with regard to which H is self-adjoint.) A refinement of this work was contained in the Chicago report. Collateral work in this field obtained general properties characterizing the infinite Jacobi matrices of deficiency index (1, 1) and the formally self-adjoint differential operators within the class of closed Hermitian transformations of deficiency index (m, m).

The last main topic of Hamburger's recent research concerned the possibility of extending the theory of the Jordan canonical decomposition to bounded non-symmetrical linear transformations in Hilbert space. This work was introduced by the paper on the general linear transformation in finite-dimensional space [**38**] for which the canonical representation is based on the Jordan form for a nilpotent linear transformation, and this in turn on the decomposition of the space into subspaces spanned by Jordan chains of finite order. (A Jordan chain of order k for a linear transformation A in either infinite dimensional Hilbert space or finite-dimensional space consists of k elements ξ_ν such that

$$A\xi_1 = 0, \quad A\xi_\nu = \xi_{\nu-1} \text{ for } 2 \leqslant \nu \leqslant k$$

and such that ξ_k does not lie in the range of A. When k is infinite and the last condition is removed we have an infinite Jordan chain for A.)

The next stage in Hamburger's work was the proof that for a certain kind of completely continuous transformation C in Hilbert space, with simple non-zero eigenvalues, the space can be decomposed into subspaces $\mathfrak{S}$ and $\mathfrak{N}$, each invariant for C, the subspace $\mathfrak{S}$ being spanned by all eigen-elements corresponding to non-zero eigenvalues, $C(\mathfrak{S})$ being dense in $\mathfrak{S}$, and C being quasi-nilpotent in $\mathfrak{N}$. There followed an attempt to extend the ideas to more general linear transformations, which took the form of the definition of N-transformations generalizing the quasi-nilpotent transformations ([**43**] and [**45**]). The theory is not in a final form. To quote Hamburger, " These notes . . . by establishing a large set of definitions and dealing with some significant special cases . . . give a large frame which has still to be filled by the picture of a complete theory. When this is done, I expect that some of the definitions which are still of a rather complicated nature may be simplified. I hope further that there will arise . . . a set of clear-cut problems which may stimulate research on this difficult subject, neglected so far by mathematicians. In the later notes we shall be involved in questions dealing with the residual spectrum, another subject on which very little mathematical research has hitherto been done."

Certain highly specialized types of what Hamburger called perfect N-transformations were shown by him to provide a natural generalization of the finite dimensional situation in that, for them, the space can be spanned by an infinite set of finite Jordan chains. He investigated more general transformations with closed ranges and gave various conditions under which the space can be spanned by their Jordan chains. He also tackled the problem from the other end by an investigation of a transformation defined by an infinite Jordan chain which is a stationary sequence. In the course of the work results were obtained concerning those eigenvalues of the bounded linear transformation whose complex conjugates belong to the residual spectrum of the adjoint. A number of unsolved problems were formulated and it is to be hoped that others will be able to take them up where Hamburger has left them. The work on the residual spectrum was of particular interest to him and he believed that important results were within reach.

Hamburger had very definite views on the best ways of presenting new material to a reader and on the value of an appeal to the imagination through geometrical ideas. These views are well illustrated in the presentation adopted in our book [47] where there is a deliberate attempt to prepare for later abstractions through an easy familiarity with all aspects of a concrete case, even at the price of repetition. The same method was followed in his lecture courses on Hilbert space. The spaces $\mathscr{L}_2$ and $\mathfrak{H}_0$ were thoroughly handled before the abstract space $\mathfrak{H}$ was introduced. This point of view which seemed natural to his generation has been challenged by later generations. It was, at any rate, much appreciated as an introductory method by one who had the pleasure and privilege of working with him.

I am greatly indebted to Frau Vera Hamburger, Mr. Otto Hamburger and Prof. Dr. Dörge for their kind help in the preparation of this notice.

Bibliography.

1. "Über eine Erweiterung des Stieltjesschen Momentenproblems", *Sitzungsberichte der Kgl. Bayer. Ak. d. Wiss. zu München, Math. Phys. Klasse* (1919), 381–393.
2. "Beiträge zur Konvergenztheorie der Stieltjesschen Kettenbrüche", *Math. Zeitschrift*, 4 (1919), 186–222.
3. "Über eine Riemannsche Formel aus der Theorie der Dirichletschen Reihen", *Math. Zeitschrift*, 6 (1920), 1–10.
4. "Bemerkung zu einer Fragestellung des Herrn Pólya", *Math. Zeitschrift*, 7 (1920), 302–322.
5. "Über die Konvergenz eines mit einer Potenzreihe assozierten Kettenbruche", *Math. Annalen*, 81 (1920), 31–45.
6. "Über die Riemannsche Funktionalgleichung der ζ-Funktion", *Sitzungsberichte der Berliner Math. Ges.*, XX (1920), 4–9.
7. "Über die Funktionalgleichung der L-Reihen", *Sitzungsberichte der Berliner Math. Ges.*, XX (1920), 10–13.
8. "Über eine Erweiterung des Stieltjesschen Momentenproblems I", *Math. Annalen*, 81 (1920), 235–319.

9. "Über eine Erweiterung des Stieltjesschen Momentenproblems II", *Math. Annalen*, 82 (1921), 120–164.

10. "Über eine Erweiterung des Stieltjesschen Momentenproblems III", *Math. Annalen*, 82 (1921), 168–187.

11. "Über die Riemannsche Funktionalgleichung der ζ-Funktion (Erste Mitteilung)", *Math. Zeitschrift*, 10 (1921), 240–254.

12. "Über die Riemannsche Funktionalgleichung der ζ-Funktion (Zweite Mitteilung)", *Math. Zeitschrift*, 11 (1921), 224–245.

13. "Über die Riemannsche Funktionalgleichung der ζ-Funktion (Dritte Mitteilung)", *Math. Zeitschrift*, 13 (1922), 283–311.

14. "Bemerkung zu einem Satze über die Riemannsche ζ-Funktion", *Sitzungsberichte der Kgl. Bayer. Ak. d. Wiss. Math. Phys. Klasse* (1922), 151–156.

15. "Über einige Beziehungen die mit der Funktionalgleichung der Riemannschen ζ-Funktion äquivalent sind", *Math. Annalen*, 85 (1922), 129–140.

16. "Ein Satz über Kurvennetze auf geschlossenen Flächen", *Sitzungsberichte d. preuss. Ak. d. Wiss. Berlin*, 21 (1922), 258–262.

17. "Über Kurvennetze mit isolierten Singularitäten auf geschlossenen Flächen, Teil I", *Math. Zeitschrift*, 19 (1924), 50–66.

18. "Über sphärische Abbildung im Grossen", *Jahresberichte der Deutsch. Math. Vereinigung*, 38 (1929), 2 *Abteilung*, 1–2.

19. Aufgaben **83**, 1, 2, 3. *Jahresberichte der Deutsch. Math. Vereinigung*, 39 (1930), 2 *Abteilung*, 85–86.

20. "Zur Theorie der sphärischen Abbildung im Grossen I. Konvexe Flächen mit zwei Nabelpunkten", *Math. Zeitschrift*, 31 (1930), 629–708.

21. "Über die Laplacesche Kaskadenmethode", *Math. Annalen*, 104 (1931), 96–138.

22. "Über die partielle linear homogene Differentialgleichung zweiter Ordnung vom hyperbolischen Typus, deren Koeffizienten in einer Veränderlichen periodisch sind. Teil I. Das Integrationsproblem", *Math. Annalen*, 105 (1931), 437–498.

23. "Über die partielle linear homogene Differentialgleichung zweiter Ordnung vom hyperbolischen Typus, deren Koeffizienten in einer Veränderlichen periodisch sind. Teil II. Das Umkehrungsproblem und der Approximationssatz", *Math. Annalen*, 106 (1932), 503–535.

24. "Bericht über Untersuchungen, welche auf die Differentialgleichung . . . bezeihen", *Jahresberichte der Deutsch. Math. Vereinigung*, 41 (1932), 229–245.

25. "La transformation de Ribaucour et la représentation sphérique, I. Remarques sur la théorie générale de la transformation de Ribaucour", *Rend. d. R. Acc. N. dei Lincei XV*, série 6a (1932), 936–941.

26. "La transformation de Ribaucour et la représentation sphérique, II. Applications de la transformation de Ribaucour à la représentation sphérique", *Rend. d. R. Acc. N. dei Lincei XVI*, série 6a (1932), 200–205.

27. "La transformation de Ribaucour et la représentation sphérique, III. Les systèmes cycliques de Ribaucour", *Rend. d. R. Acc. N. dei Lincei XVI*, série 6a (1932), 296–298.

28. "Ribaucourtransformation und sphärische Abbildung", *Verhandlungen des intern. Math. Kongr. Zürich* (1932), II, 153–154.

29. "Ribaucour transformationen und sphärische Abbildung", *Acta. Math.*, 62 (1933), 1–90.

30. "Beweis einer Carathéodoryschen Vermutung I", *Annals of Math.* (2), 41 (1940), 63–86.

31. "Beweis einer Carathéodoryschen Vermutung II", *Acta Math.*, 73 (1941), 175–228.

32. "Beweis einer Carathéodoryschen Vermutung III", *Acta Math.*, 73 (1941), 229–332.

33. "Contributions to the theory of closed Hermitian transformations of deficiency index (m, m)", *Quart. J. of Math* (Oxford), 13 (1942), 117–128.

34. "Contributions to the theory of closed Hermitian transformations of deficiency index (m, m), Part I", *Annals of Math.* (2), 45 (1944), 59–99.

35. "Hermitian transformations of deficiency index (1, 1), Jacobi matrices and undetermined moment problems", *American J. of Math.*, 66 (1944), 489–522.

36. "On a class of Hermitian transformations containing self-adjoint differential operators", *Proc. Nat. Acad. Sci. U.S.A.*, 31 (1945), 185–189.
37. "On a class of Hermitian transformations containing self-adjoint differential operators", *Annals of Math.* (2), 47 (1946), 667–687.
38. "Remarks on the reduction of a linear transformation to Jordan's normal form", *Journal London Math. Soc.*, 22 (1947), 173–179.
39. "A theorem on commutative matrices", *Journal London Math. Soc.*, 24 (1949), 200–206.
40. "Remarks on the Dirac δ-operator", *Proc. Cambridge Phil. Soc.*, 45 (1949), 489–494.
41. "On the reduction of a completely continuous linear transformation in Hilbert space", *International Congress of Mathematicians, Cambridge, U.S.A.*, I (1950), 454.
42. "Über die Zerlegung des Hilbertschen Raumes durch vollstetige lineare Transformationen", *Math. Nachrichten*, 4 (1951), 56–69.
43. "Non-symmetric operators in Hilbert space", *Proceedings of the Symposium on Spectral Theory and Differential Problems* (Oklahoma Agricultural and Mechanical College, Stillwater, Oklahoma, 1951), 67–112.
44. "On a new characterization of self-adjoint differential operators in the Hilbert space $\mathscr{L}_2$", *Proceedings of the Symposium on Spectral Theory and Differential Problems* (Oklahoma Agricultural and Mechanical College, Stillwater, Oklahoma, 1951), 229–247.
45. "Five notes on a generalization of quasi-nilpotent transformations in Hilbert space", *Proc. London Math. Soc.* (3), 1 (1951), 494–512.
46. "Zur Theorie der Singularitäten beschrankten nicht-Hermitischen linearer Transformationen im Hilbertschen Raum", *Jahresberichte der Deutsch. Math. Vereinigung*, 54 (1951), 2 *Abteilung*, 36.
47. *Linear transformations in n-dimensional vector space* (Cambridge, 1951) (with M. E. Grimshaw).
48. "Remarks on self-adjoint differential operators", *Proc. London Math. Soc.* (3), 3 (1953), 446–463.

HUNG CHING CHOW

L. S. BOSANQUET.

Hung Ching Chow was born on 11 January, 1902, at Tung Shan in the Kiangsu province of China. He graduated from the South Eastern University (later the National Central University), Nanking, in 1926. After holding teaching posts at Nanking Middle School, Amoy University and the National Ching Hwa University, he spent three years, 1934-7, in London, at University College. He obtained the M.Sc. degree, with a mark of distinction, and began working for a Ph.D., but when the Japanese invaded China in 1937 he returned home to look after his family. Joining the National Central University, which was evacuated to Chungking, he taught from 1938 to 1944, first in the Mathematics Department of the School of Science, then as Head of the Mathematics Department in the Teachers' School of the Central University, and from 1942 was Dean of student affairs. In December 1944 he was granted a year's leave to do research at the Tata Institute, Bombay, but after five months he was invited to join the Ministry of Education in Chiang Kai-shek's Government, as Head of the Department for Higher Education. After