

The Journal of The London Mathematical Society.

VOL. 25. PART 2.

APRIL, 1950.

No. 98.

GODFREY HAROLD HARDY

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Godfrey Harold Hardy was born on 7 February, 1877, at Cranleigh, Surrey. He was the only son of Isaac Hardy, Art Master, Bursar and House Master of the preparatory branch of Cranleigh School. His mother, Sophia Hardy, had been Senior Mistress at the Lincoln Training College. Both parents were extremely able people and mathematically minded, but want of funds had prevented them from having a university training.

The future professor's interest in numbers showed itself early. By the time he was two years old he had persuaded his parents to show him how to write down numbers up to millions. When he was taken to church he occupied the time in factorizing the numbers of the hymns, and all through his life he amused himself by playing about with the numbers of railway carriages, taxi-cabs and the like.

He and his sister were brought up by enlightened parents in a typical Victorian nursery, and, as clever children do, he agonized his nurse with long arguments about the efficacy of prayer and the existence of Santa Claus: "Why, if he gives me things, does he put the price on? My box of tools is marked 3s. 6d." The Hardy parents had many theories about education. Their children had few books, but they had to be good literature. In the nursery G. H., who was slightly older than his sister, read to her such books as *Don Quixote*, *Gulliver's Travels* and *Robinson Crusoe*. They were never allowed to play with any toy that was broken and past repair. The nurse gave them some instruction in reading and writing, but they never had a governess, and on the whole were left to find things out for themselves.

A minute newspaper written by G. H. at the age of eight was unfortunately lost in the London blitz. It contained a leading article, a speech by Mr. Gladstone, various tradesmen's advertisements, and a full report of a cricket match with complete scores and bowling analysis. He also embarked on writing a history of England for himself, but with so much detail that he never got beyond the Anglo-Saxons. Two exquisite little illustrations for this work have survived. He inherited artistic ability from his father, but it was crushed by bad teaching at Winchester. He had no interest in music.

As soon as he was old enough G. H. went to Cranleigh School, and by the time he was twelve he had passed his first public examination with distinctions in mathematics, Latin and drawing. By this time too he had reached the sixth form—the Cranleigh standard was at that time very low—so some of his work was sent to Winchester. He was offered a scholarship there on his mathematics alone, but was considered too young to go that year, and went the following year.

Apparently he was never taught mathematics in a class. Mr. Clarke, Second Master at Cranleigh, and Dr. Richardson, Head of "College", always coached him privately. He was never enamoured of public school life. He was grateful to Winchester for the education it gave him, but the Spartan life in "College" at that time was a great hardship, and he had one very bad illness.

There was some question of his going up to New College, but his mind was turned in the direction of Cambridge by a curious incident, which he has related in *A Mathematician's Apology*. He happened to read a highly coloured novel of Cambridge life called *A Fellow of Trinity*, by "Alan St. Aubin" (Mrs. Frances Marshall), and was fired with the ambition to become, like its hero, a fellow of Trinity. He went up to Trinity College, Cambridge, as an entrance scholar in 1896, his tutor being Dr. Verrall. He was first coached by Dr. Webb, the stock producer of Senior Wranglers. He was so annoyed by Webb's methods that he even considered turning over to history, a love of which had been implanted in him by Dr. Fearon, Headmaster of Winchester. However, his Director of Studies sent him to A. E. H. Love, and this, he considered, was one of the turning points of his life, and the beginning of his career as a "real mathematician". Love was, of course, primarily an applied mathematician; but he introduced Hardy to Jordan's *Cours d'Analyse*, the first volume of which had been published in 1882, and the third and last in 1887. This must have been Hardy's first contact with analysis in the modern sense, and he has described in *A Mathematician's Apology* how it opened his eyes to what mathematics really was.

Hardy was fourth wrangler in 1898, R. W. H. T. Hudson being Senior Wrangler, with J. H. Jeans and J. F. Cameron, later Master of Gonville and Caius, bracketed next. He took Part II of the tripos in 1900, being placed in the first division of the first class, Jeans being then below him in the second division of the first class. In the same year he was elected to a Prize Fellowship at Trinity, and his early ambition was thus fulfilled. Hardy and Jeans, in that order, were awarded Smith's prizes in 1901.

His life's work of research had now begun, his first paper apparently being that in the *Messenger of Mathematics*, 29, 1900. It is about the evaluation of some definite integrals, a subject which turned out to be one of his permanent minor interests, and on which he was still writing in the last year of his life.

In 1906, when his Prize Fellowship was due to expire, he was put on the Trinity staff as lecturer in mathematics, a position he continued to hold until 1919. This meant that he had to give six lectures a week. He usually gave two courses, one on elementary analysis and the other on the theory of functions. The former included such topics as the implicit function theorem, the theory of unicursal curves, and the integration of functions of one variable. This was doubtless the origin of his first Cambridge tract, *The Integration of Functions of a Single Variable*. This work is so well known now that it is often forgotten that its systematization was due to Hardy. He also sometimes took small informal classes on elementary subjects, but he was never a "tutor" in the Oxford sense.

In 1908 Hardy made a contribution to genetics which seems to be little known by mathematicians, but which has found its way into textbooks as "Hardy's Law". There had been some debate about the proportions in which dominant and recessive Mendelian characters would be transmitted in a large mixed population. The point was settled by Hardy in a letter to *Science*. It involves only some simple algebra, and no doubt he attached little weight to it. As it happens, the law is of central importance in the study of Rh-blood-groups and the treatment of haemolytic disease of the newborn. In the *Apology* Hardy wrote, "I have never done anything 'useful'. No discovery of mine has made, or is likely to make, directly or indirectly, for good or ill, the least difference to the amenity of the world." It seems that there was at least one exception to this statement.

He was elected a Fellow of the Royal Society in 1910, and in 1914 the University of Cambridge recognized his reputation for research, already world-wide, by giving him the honorary title of Cayley Lecturer.

To this period belongs his well-known book *A Course of Pure Mathematics*, first published in 1908, which has since gone through numerous editions and been translated into several languages. The standard of

mathematical rigour in England at that time was not high, and Hardy set himself to give the ordinary student a course in which elementary analysis was for the first time done properly. *A Course of Pure Mathematics* is hardly a *Cours d'Analyse* in the sense of the great French treatises, but so far as it goes it serves a similar purpose. It is to Hardy and his book that the outlook of present-day English analysts is very largely due.

He also played a large part in the reform of the old Cambridge Mathematical Tripos Part I, and in the abolition of the publication of the results in order of merit.

Another turning point in Hardy's career was reached about 1912, when he began his long collaboration with J. E. Littlewood. There have been other pairs of mathematicians, such as Phragmén and Lindelöf, or Whittaker and Watson, who have joined forces for a particular object, but there is no other case of such a long and fruitful partnership. They wrote nearly a hundred papers together, besides (with G. Pólya) the book *Inequalities*.

Soon afterwards came his equally successful collaboration with the Indian mathematician Ramanujan, though this was cut short six years later by Ramanujan's early death. An account of this association is given by Hardy in the introductions to Ramanujan's collected works and to the book *Ramanujan*. In a letter to Hardy in 1913, Ramanujan sent specimens of his work, which showed that he was a mathematician of the first rank. He came to England in 1914 and remained until 1919. He was largely self-taught, with no knowledge of modern rigour, but his "profound and invincible originality" called out Hardy's equal but quite different powers. Hardy said, "I owe more to him than to any one else in the world with one exception, and my association with him is the one romantic incident in my life".

Hardy was a disciple of Bertrand Russell, not only in his interest in mathematical philosophy, but in his political views. He sympathized with Russell's anti-war attitude, though he did not go to the lengths which brought Russell into collision with the authorities. In a little book *Bertrand Russell and Trinity*, which he had printed for private circulation in 1942, Hardy has described the Russell case and the storms that raged over it in Trinity. It was an unhappy time for those concerned, and one may think that it all would have been better forgotten. It must have been with some relief that, in 1919, he heard of his election to the Savilian Chair of Geometry at Oxford, and migrated to New College.

In the informality and friendliness of New College Hardy always felt completely at home. He was an entertaining talker on a great variety of subjects, and one sometimes noticed everyone in common room waiting to see what he was going to talk about. Conversation was one of the games

that he loved to play, and it was not always easy to make out what his real opinions were.

He played several games well, particularly real tennis, but his great passion was for cricket. He would read anything on this subject, and talk about it endlessly. His highest compliment was "it is in the Hobbs class". Even until 1939 he captained the New College Senior Common Room side against the Choir School and other opponents. He liked to recall the only occasion in the history of the Savilian chairs when one Savilian professor (himself) took the wicket of the other (H. H. Turner). The paper "A maximal theorem with function-theoretic applications", published in *Acta Math.* 54, and presumably addressed to European mathematicians in general, contains the sentences "The problem is most easily grasped when stated in the language of cricket . . . Suppose that a batsman plays, in a given season, a given 'stock' of innings . . .".

A vivid account of Hardy's affection for cricket and of his life in his later Cambridge years is given by C. P. Snow, in an article entitled "A mathematician and cricket", in *The Saturday Book*, 8th Year.

He liked lecturing, and was an admirable lecturer. His matter, delivery, and hand-writing (a specimen of which appears on the dust-cover of *A Mathematician's Apology*) were alike fascinating. Though no original geometer, he fulfilled the conditions of his Oxford chair by lecturing on geometry as well as on his own subjects. He also lectured occasionally on Mathematics for Philosophers, and drew large audiences of Oxford philosophers to whom ordinary mathematics made no appeal. His Rouse Ball lecture on this subject, delivered at Cambridge in 1928, entitled *Mathematical Proof*, was published in *Mind*, 38.

Hardy had singularly little appreciation of science, for one who was sufficiently nearly a scientist to be a Fellow of the Royal Society. In *A Mathematician's Apology* he is at some pains to show that real mathematics is useless, or at any rate harmless. He says, "It is true that there are branches of applied mathematics, such as ballistics and aerodynamics, which have been developed deliberately for war . . . but none of them has any claim to rank as 'real'. They are indeed repulsively ugly and intolerably dull; even Littlewood could not make ballistics respectable, and if he could not, who can?". His views on this subject were obviously coloured by his hatred of war, but in any case his whole instinct was for the purest of mathematics. I worked on the theory of Fourier integrals under his guidance for a good many years before I discovered for myself that this theory has applications in applied mathematics, if the solution of certain differential equations can be called "applied". I never heard him refer to these applications.

Nevertheless, he was a Fellow of the Royal Astronomical Society, which he joined in 1918 in order that he might attend the meetings at which the theory of relativity was debated by Eddington and Jeans. He even once, in 1930, took part in a debate on stellar structure, which involved R. H. Fowler's work on Emden's and allied differential equations. On this he made the characteristic remark that Fowler's work, being pure mathematics, would still be of interest long after all the physical theories which had been discussed had become obsolete. This prophecy has since been very largely fulfilled.

I first came into contact with him when I attended his advanced class at Oxford in 1920. The subjects which I remember specially as having been discussed at this class are Fourier series, continued fractions, and differential geometry, a commentary on R. H. Fowler's Cambridge tract. Whatever the subject was, he pursued it with an eager single-mindedness which the audience found irresistible. One felt, temporarily at any rate, that nothing else in the world but the proof of these theorems really mattered. There could have been no more inspiring director of the work of others.

He was always at the head of a team of researchers, both colleagues and students, whom he provided with an inexhaustible stock of ideas on which to work. He was an extremely kind-hearted man, who could not bear any of his pupils to fail in their researches. Many Oxford D.Phil. dissertations must have owed much to his supervision.

Hardy always referred to God as his personal enemy. This was of course a joke, but there was something real behind it. He took his disbelief in the doctrines of religion more seriously than most people seem to do. He would not enter a religious building, even for such a purpose as the election of a Warden of New College. The clause in the New College by-laws, enabling a fellow with a conscientious objection to being present in Chapel to send his vote to the scrutineers, was put in on his behalf.

He has been described as absent-minded, but I never saw any sign of this. If he dined at high table in tennis clothes it was because he liked to do so, not because he had forgotten what he was wearing. He had a way of passing in the street people whom he knew well without any sign of recognition, but this was due to a sort of shyness, or a feeling of the slight absurdity of a repeated conventional greeting.

Returning to his mathematical career, I may refer here to the founding of the *Quarterly Journal of Mathematics* (Oxford series). Glaisher, the editor of the *Messenger of Mathematics* and the old *Quarterly Journal*, had died in 1928, and these two periodicals had come to an end. There was an obvious need for something to replace them, and it was largely due to Hardy that a new series of the *Quarterly Journal* was started in Oxford.

The London Mathematical Society occupied a leading place in his affections, and by his death the Society has lost its most loyal as well as its most distinguished member. He served on the Council from 1905-8, joined it again in 1914, and from that time, except for two absences of a year each, in 1928-9 (when he went to America) and 1934-5, he was on it continuously until his final retirement in 1945. He was one of the secretaries from 1917 to 1926, President in 1926-8 and again for a second term in 1939-41, and secretary again from then until 1945. In his Presidential Address (1928), *Prolegomena to a Chapter on Inequalities*, he boasted that he had been at every meeting both of the Council and of the Society, and sat through every word of every paper, since he became secretary in 1917. He was awarded the Society's De Morgan medal in 1929.

In 1928-9 he was Visiting Professor at Princeton and at the California Institute of Technology, O. Veblen coming to Oxford in his place. In 1931 E. W. Hobson died, and Hardy returned to Cambridge as his successor in the Sadleirian Chair of Pure Mathematics, becoming again a Fellow of Trinity.

Perhaps the most memorable feature of this period was the Littlewood-Hardy seminar or "conversation class". This was a model of what such a thing should be. Mathematicians of all nationalities and ages were encouraged to hold forth on their own work, and the whole thing was conducted with a delightful informality that gave ample scope for free discussion after each paper. The topics dealt with were very varied, and the audience was always amazed by the sure instinct with which Hardy put his finger on the central point and started the discussion with some illuminating comment, even when the subject seemed remote from his own interests.

He also lectured on the Calculus of Variations, a subject to which he had been drawn by his work on Inequalities.

After his return to Cambridge he was elected to an honorary fellowship at New College. He held honorary degrees from Athens, Harvard, Manchester, Sofia, Birmingham, Edinburgh, Marburg and Oslo. He was awarded a Royal Medal of the Royal Society in 1920, its Sylvester Medal in 1940, and the Copley Medal, its highest award, in 1947. He was President of Section A of the British Association at its Hull meeting in 1922, and of the National Union of Scientific Workers in 1924-6. He was an honorary member of many of the leading foreign scientific academies.

Some months before his death he was elected "associé étranger" of the Paris Academy of Sciences, a particular honour, since there are only ten of these from all nations and scientific subjects. He retired from the Sadleirian chair in 1942, and died on 1 December, 1947, the day on which the Copley Medal was due to be presented to him.

He was unmarried. He owed much to his sister, who provided him throughout his life with the unobtrusive support which such a man needs. Miss Hardy has supplied most of the personal information contained in this notice.

In addition to the books mentioned above, Hardy wrote three more Cambridge tracts, *Orders of Infinity* (1910), *The General Theory of Dirichlet's Series*, with M. Riesz (1915), and *Fourier Series*, with W. W. Rogosinski (1944). In 1934 he published *Inequalities* with J. E. Littlewood and G. Pólya, and in 1938 *The Theory of Numbers* with E. M. Wright. In 1940 followed *Ramanujan*, a collection of lectures or essays suggested by Ramanujan's work. His last book was on Divergent Series, and was completed but not published at the time of his death. His inaugural lecture at Oxford, *Some famous problems of the theory of numbers, and in particular Waring's problem*, was published in 1920. He was also one of the editors of the collected papers of Ramanujan, which were published in 1927.

The student of Hardy's style should also read his obituary notices of Ramanujan (*P.L.M.S.* (2) 19, 1921), Mittag-Leffler (*J.L.M.S.* 3, 1928), Bromwich (*J.L.M.S.* 5, 1930), Paley (*J.L.M.S.* 9, 1934), Hobson (*J.L.M.S.* 9, 1934), Landau (*J.L.M.S.* 13, 1938), W. H. Young (*J.L.M.S.* 17, 1942), J. R. Wilton (*J.L.M.S.* 20, 1945), and Glaisher at the end of the *Messenger of Mathematics*. These tributes to his late colleagues must have made every mathematician wish that he could have seen his own career described in the same generous terms.

Hardy was the author, or part author, of more than 300 original papers, covering almost every kind of analysis, which by their originality and quantity marked him as one of the leading mathematicians of his time. It is rarely possible to disentangle his own contributions from those of others. He liked collaboration, and much of his best work is to be found in joint papers, particularly those written with Littlewood and with Ramanujan. He used to say that each author of a joint paper gets much more than half the credit for it. No doubt both the scope and the depth of his work was increased by his collaboration with mathematicians of such varied interests, but he was certainly the prime mover in a great deal of the joint work.

Hardy's work has had a profound influence throughout the whole of analysis. It has resulted in the complete remodelling of some parts of the subject, and has enriched other parts with new methods and theories of fundamental importance.

The following articles deal with the most important aspects of his work.

*List of papers by G. H. Hardy.***Abbreviations*

- M.M.* *Messenger of Mathematics* (2nd series).
Q.J. *Quarterly Journal of Mathematics*.
P.L.M.S. *Proceedings of the London Mathematical Society*.
J.L.M.S. *Journal of the London Mathematical Society*.
P.C.P.S. *Proceedings of the Cambridge Philosophical Society*.
T.C.P.S. *Transactions of the Cambridge Philosophical Society*.
M.A. *Mathematische Annalen*.
M.Z. *Mathematische Zeitschrift*.
Q.J.O. *Quarterly Journal of Mathematics* (Oxford Series).
N.I.C. Notes on some points in the integral calculus.
D.A. Some problems of Diophantine approximation.
P.N. Some problems of "Partitio Numerorum".
N.S. Notes on the theory of series.
J.E.L. J. E. Littlewood.
S.R. S. Ramanujan.
E.C.T. E. C. Titchmarsh.

1899

1. Question 13848, *Educ. Times* 70, 43.
2. Question 13917, *Educ. Times* 70, 78-9.
3. Question 14124, *Educ. Times* 71, 100-1.
4. Question 14005, *Educ. Times* 71, 111-2.

1900

1. On a class of definite integrals containing hyperbolic functions, *M.M.* 29, 25-42.
2. Question 14243, *Educ. Times* 72, 80-1.
3. Question 14271, *Educ. Times* 73, 36-7.
4. Question 14179, *Educ. Times* 73, 53-4.
5. Question 14317, *Educ. Times* 73, 61-3.

1901

1. On differentiation and integration under the integral sign, *Q.J.* 32, 66-140.
2. General theorems in contour integration, with some applications, *Q.J.* 32, 369-84.
3. N.I.C. I: On the formula for integration by parts, *M.M.* 30, 185-7.
4. N.I.C. II: Two general convergence theorems, *M.M.* 30, 187-90.
5. Question 14496, *Educ. Times* 74, 37-8.
6. Question 14447, *Educ. Times* 74, 98-100.
7. Question 14467, *Educ. Times* 74, 111-2.
8. Question 14028, *Educ. Times* 74, 122-3.
9. Question 14369, *Educ. Times* 75, 135-6.

1902

1. The elementary theory of Cauchy's principal values, *P.L.M.S.* (1) 34, 16-40.
2. The theory of Cauchy's principal values (Second Paper: the use of principal values in some of the double limit problems of the integral calculus), *P.L.M.S.* (1) 34, 55-91.

* Compiled by Prof. E. C. Titchmarsh. Valuable assistance in checking the references has been given by Dr. A. F. Ruston and Miss Mary E. Earl.

1902 (cont.)

3. On the Frullanian integral

$$\int_0^{\infty} \frac{\phi(ax^m) - \psi(bx^n)}{x} (\log x)^n dx,$$

Q.J. 33, 113-4.

4. N.I.C. III: On the logarithmic criteria for the absolute convergence of an integral whose upper limit is ∞ , *M.M.* 31, 1-6.
5. N.I.C. IV: On the integral $\int_0^{\infty} \sin x \psi(x) dx$, *M.M.* 31, 6-8.
6. A new proof of Kummer's series for $\log \Gamma(a)$, *M.M.* 31, 31-3.
7. N.I.C. V: On absolutely convergent integrals of functions which are infinitely often infinite, *M.M.* 31, 73-6.
8. N.I.C. VI: Absolute convergence of infinite multiple integrals, *M.M.* 31, 125-8.
9. N.I.C. VII: On differentiation under the integral sign, *M.M.* 31, 132-4.
10. N.I.C. VIII: Absolutely convergent integrals of irregular types, *M.M.* 31, 177-83.
11. On the zeroes of the integral function

$$x - \sin x = \sum_1^{\infty} (-)^{n-1} \frac{x^{2n+1}}{2n+1!},$$

M.M. 31, 161-5.

12. Questions 1423, 2316, 3941, 4794, *Educ. Times* (2) 1, 25.
13. Question 14851, *Educ. Times* (2) 1, 58-9.
14. Question 14055, *Educ. Times* (2) 2, 41-2.

1903

1. The theory of Cauchy's principal values (Third Paper: differentiation and integration of principal values), *P.L.M.S.* (1) 35, 81-107.
2. On the continuity and discontinuity of definite integrals which contain a continuous parameter, *Q.J.* 34, 28-53.
3. Note on the limiting values of the elliptic modular-functions, *Q.J.* 34, 76-86.
4. N.I.C. IX: On the integral $\int_0^{\infty} \{A - a(\sin^2 x)\} \psi(x) dx$, *M.M.* 32, 1-3.
5. On the zeroes of certain integral functions, *M.M.* 32, 36-45.
6. On the integral $\int_{-\infty}^{\infty} \frac{\log(ax^2 + 2bx + c)^2}{ax^2 + 2bx + \gamma} dx$, *M.M.* 32, 45-50.
7. N.I.C. X: On conditionally convergent infinite multiple integrals, *M.M.* 32, 92-7.
8. N.I.C. XI: Some conditionally convergent infinite double integrals, *M.M.* 32, 159-65.
9. N.I.C. XII: On the operation which is the inverse of double integration, *M.M.* 32, 187-92.
10. Question 14988, *Educ. Times* (2) 3, 94-5.
11. Question 14989, *Educ. Times* (2) 4, 69-70.
12. Question 15019, *Educ. Times* (2) 4, 75.
13. Question 15265, *Educ. Times* (2) 4, 109-10.

1904

1. On the convergence of certain multiple series, *P.L.M.S.* (2) 1, 124-8.
2. A general theorem concerning absolutely convergent series, *P.L.M.S.* (2) 1, 285-90.
3. On differentiation and integration of divergent series, *T.C.P.S.* 19, 297-321.
4. Researches in the theory of divergent series and divergent integrals, *Q.J.* 35, 22-66.
5. A theorem concerning the infinite cardinal numbers, *Q.J.* 35, 87-94.
6. Note on the function $\int_x^{\infty} e^{b^2 x^2 - t^2} dt$, *Q.J.* 35, 193-207.
7. The asymptotic solution of certain transcendental equations, *Q.J.* 35, 261-82.
8. N.I.C. XIII: On differentiation under the integral sign (continued), *M.M.* 33, 62-7,

1904 (cont.)

9. The cardinal number of a closed set of points, *M.M.* 33, 67-9.
10. N.I.C. XIV: Integrals whose discontinuities are everywhere dense, *M.M.* 33, 80-5.
11. Note on divergent Fourier series, *M.M.* 33, 137-44.
12. On the zeroes of two classes of Taylor series, *Brit. Ass. Rep.*, 441-3.
13. Question 15300, *Educ. Times* (2) 5, 61.
14. Additional note on Question 15282, *Educ. Times* (2) 5, 113-4.
15. Question 15361, *Educ. Times* (2) 5, 118.
16. Question 15125, *Educ. Times* (2) 6, 31.

1905

1. On the roots of the equation $\Gamma\left(\frac{1}{x+1}\right) = c$, *P.L.M.S.* (2) 2, 1-7.
2. (With T. J. I'A. Bromwich.) Some extensions to multiple series of Abel's theorem on the continuity of power series, *P.L.M.S.* (2) 2, 161-89.
3. Note in addition to a former paper on conditionally convergent multiple series, *P.L.M.S.* (2) 2, 190-1.
4. On the zeroes of certain classes of integral Taylor series. Part I: On the integral function $\sum_{n=0}^{\infty} \frac{x^n}{\{\phi(n)\}!}$, *P.L.M.S.* (2) 2, 332-9.
5. On the zeroes of certain classes of integral Taylor series. Part II: On the integral function $\sum_{n=0}^{\infty} \frac{x^n}{(n+\alpha)^s n!}$ and other similar functions, *P.L.M.S.* (2) 2, 401-31.
6. A method for determining the behaviour of certain classes of power series near a singular point on the circle of convergence, *P.L.M.S.* (2) 3, 381-9.
7. On a class of analytic functions, *P.L.M.S.* (2) 3, 441-60.
8. On certain series of discontinuous functions connected with the modular functions, *Q.J.* 36, 93-123.
9. Note on an integral function, *M.M.* 34, 1-2.
10. N.I.C. XV: On upper and lower integration, *M.M.* 34, 3-6.
11. N.I.C. XVI: A class of conditionally convergent infinite multiple integrals, *M.M.* 34, 6-10.
12. A generalization of Frullani's integral, *M.M.* 34, 11-18, and note, p. 102.
13. On the zeroes of a class of integral functions, *M.M.* 34, 97-101.
14. On certain conditionally convergent multiple series connected with the elliptic functions, *M.M.* 34, 146-53.
15. Question 15686, *Educ. Times* (2) 8, 74.
16. The expression of the double zeta function and double gamma function in terms of elliptic functions, *T.C.P.S.* 20, 1-35.

1906

1. On Kummer's series for $\log \Gamma(a)$, *Q.J.* 37, 49-53.
2. On double Fourier series, and especially those which represent the double zeta-function with real and incommensurable parameters, *Q.J.* 37, 53-79.
3. On the function $P_s(x)$, *Q.J.* 37, 146-72 (correction p. 378).
4. On certain double integrals, *Q.J.* 37, 360-9.
5. On the integral function $\Phi_{n, n+\alpha}(x) = \sum_{n=0}^{\infty} \frac{x^n}{(n+\alpha)^{n+\alpha}}$, *Q.J.* 37, 369-78.
6. N.I.C. XVII: On the integration of series, *M.M.* 35, 126-30.
7. A formula for the prime factors of any number, *M.M.* 35, 145-6.
8. N.I.C. XVIII: On some discontinuous integrals, *M.M.* 35, 158-66.
9. Some notes on certain theorems in higher trigonometry, *Math. Gazette* 3, 284-8.

1907

1. The continuum and the second number class, *P.L.M.S.* (2) 4, 10-17.
2. Some theorems connected with Abel's theorem on the continuity of power series, *P.L.M.S.* (2) 4, 247-65.
3. On the singularities of functions defined by Taylor's series (Remarks in addition to a former paper), *P.L.M.S.* (2) 5, 197-205.
4. The singular points of certain classes of functions of several variables, *P.L.M.S.* (2) 5, 342-60.
5. On certain oscillating series, *Q.J.* 38, 269-88.
6. Some theorems concerning infinite series, *M.A.* 64, 77-94.
7. N.I.C. XIX: On Abel's lemma and the second theorem of the mean, *M.M.* 36, 10-13.
8. Higher trigonometry, *Math. Gazette* 4, 13-4.
9. A curious imaginary curve, *Math. Gazette* 4, 14.
10. The line at infinity, etc., *Math. Gazette* 4, 14-5.

1908

1. Generalization of a theorem in the theory of divergent series, *P.L.M.S.* (2) 6, 255-64.
2. The multiplication of conditionally convergent series, *P.L.M.S.* (2) 6, 410-23.
3. Further researches in the theory of divergent series and integrals, *T.C.P.S.* 21, 1-48.
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