THOMAS HENRY HAVELOCK

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Thomas Henry Havelock was born in Newcastle upon Tyne on 24 June, 1877, to Michael Havelock, an engineer, and Elizabeth Barras Havelock (*née* Bell). Some signs of his talent became clear during his early education at a private school, Singleton House, in Newcastle; he was, for example, the only candidate in the entire country to achieve in the Cambridge University Junior Local Examinations of 1891 a distinction in Botany, a like class being also awarded to his French and his Mathematics papers. After further specialisation in Science, he left Singleton House in 1893 at the age of 16, with ambitions of becoming an apprentice draftsman at the Neptune Works on Tyneside. It was decided that, while waiting for such a vacancy, he could profitably fill his time by pursuing his scientific studies at the Durham College of Science in Newcastle. Here his gifts were swiftly recognised and, encouraged by his successes in, and his growing love for, Mathematics and Physics, thoughts of the shipyard faded: when the once hoped for opening at the Neptune works did finally occur in March 1894, he declined it. Nevertheless, his early interest in ships and their motion remained with him throughout his life, and ultimately brought him international recognition.

In 1894, Havelock was awarded his Associateship in Physical Science and the following year, at the age of 18, he completed his Honours B.Sc. course, with a distinction in Physics. He stayed in Newcastle for a further two years of postgraduate study. In 1897, he entered St. John's College, Cambridge, as a Pensioner, becoming a Scholar the following year. On 24 November, 1898, an event occurred which was to have serious repercussions on his life. Major-General H. H. Kitchener visited Cambridge to receive the Freedom of the Borough, an honorary LL.D. from the University, and honorary membership of the University Union Society. In the evening the undergraduate community celebrated less formally in the market place by discharging fireworks, and by lighting a huge bonfire which they fed in every manner their wit could devise. Trees lost boughs, the Pieces their seats, tradesmen their stalls, householders in Sussex Street their front doors, and Trinity College a large garden shed, reduced to manageable components by a student-propelled horse-roller. Earlier in the day, undergraduates had similarly substituted their power for that of horses in the shafts of the Sirdar's carriage, and had triumphantly drawn it and its distinguished occupant through the streets. The excitement of the day was, however, marred by a dangerous incident. At about 2 o'clock, when Kitchener was to receive his degree at the Senate House, the cordon of policemen holding back the dense crowd that had gathered there broke, and " . . . a general scramble to get upon the railings ostensibly for a good view of the Sirdar ensued. Some of the bolder spirits among the undergraduates attempted to climb over . . . " (*Cambridge Weekly*

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News, 25 November, 1898). Local reports continue: " Just as the Sirdar's carriage was moving off, the mouldings of the railings with a loud crack gave way. Instantly everyone clinging to them jumped down, and those persons in the near vicinity quickly moved away. Hardly had the last person reached the ground ere over sixty yards of the iron palings began to fall. Involuntarily every person within reach grasped them, with the result that the force was broken. So great was the number of people underneath that by much exertion they were enabled to safely lower the ponderous weight of iron that threatened to crush them, but a few persons were hurt, one or two badly. . . . Mr. H. Havelock, of St. John's, fared very badly. He fell under the stonework and railings. His injuries are not exactly known, but it is feared that he was badly crushed." (*Cambridge Express*, 26 November, 1898.)

The fears were justified: Havelock's health was permanently impaired and for the remainder of his life he was forced to avoid all unnecessary physical exertions. More immediately, he was unable to keep the Lent Terms of 1899 and 1900, and in Part I of the Mathematics Tripos in 1900, the year in which J. E. Wright became Senior Wrangler, he was placed a disappointing fifteenth wrangler (out of 16). But now his mental power began to wax, perhaps partly because his physical stamina had waned. In 1901 he alone in Part II of the Tripos was placed in Class 1, Division 2, with Wright alone above him in Division 1. In 1902 for an essay " On the distribution of energy in the continuous spectrum " he shared the Smith's Prize with Wright. Also in this year he was elected to a three-year Isaac Newton Studentship for an essay " On the general theory of wave propagation ". In 1903, his college elected him to a six-year Gregson Fellowship.

In his early career, Havelock developed the mathematical theory of wave motions. In addition to generalizations to p-dimensions of the work of Poisson, Kirchhoff and Whittaker, he was one of the first [3]† to clarify the distinctive difference between the cases of even and odd p. He also [4] presented the work of Coulon, Hadamard, and Hedrick in a fresh light. These authors had supposed the partial derivatives were everywhere continuous. Extending a paper by Love (1903), Havelock showed [4] that, for the simple linear wave equation or for Maxwell's equations in free space, some partial derivatives might be allowed to be discontinuous at a wave-front or other characteristic.

During this period at St. John's, Havelock gradually drifted away from the study of mathematical points for their own intrinsic interest, and towards the application of mathematics, first to optics as the title of his Adam's Prize essay suggests, and later to water waves. He was, then, given the title of Special Lecturer in Applied Mathematics when he returned in 1906 to Armstrong College, as the Durham College of Science at Newcastle had since 1904 been known, as its first teacher of that subject. The following year he received its D.Sc. degree (by examination) in preparation for which he had already obtained an *ad eundem* M.Sc.

Soon Havelock began to take an interest in the ship wave problem, that is, the

† Numbers refer to the order of papers in the bibliography below.

determination of the surface waves created by a boat in motion. In 1887, Kelvin (1891, 1905) had proposed a simple model in which the vessel was represented by a δ-function of pressure moving uniformly over an infinitely deep ocean. Apparently inventing his method of stationary phase for the purpose, he showed that, for a point lying within two straight lines radiating from the point of disturbance, each making an angle of $\sin^{-1}(1/3) \approx 19° 28'$ with the path the point had previously described, the Fourier integral representing the surface elevation of the waves possessed two points of stationary phase, corresponding to the so-called transverse and diverging wave systems. Outside this region, the disturbance was exponentially small. For the two " caustics ", i.e. the lines mentioned above separating the two regions, the points of stationary phase merged, and the amplitude of each wave system became infinite. Kelvin (1905) had recognised the cause of this unrealistic feature, but it was left to Havelock [**11**, see also **59**] to refine the analysis, and to show that, although the amplitude of the disturbance on the caustic was large compared to its magnitude elsewhere, it was not infinite. (For a recent and deeper discussion, see Ursell, 1960.) Havelock also generalised Kelvin's results to include the effects of surface tension, and finite ocean depth, h. The latter case is quite interesting: both phase and group velocities of the waves are then bounded by the critical speed $c = \sqrt{(gh)}$, where g is the acceleration due to gravity. Havelock found that, if the velocity, v, of the ship exceeded c, the wave pattern it generated was quite different from the case $v < c$ (or $h \to \infty$). In fact, because of the analogy between waves on shallow water and two-dimensional compressible flow (c then being the speed of sound), it was later realised that Havelock had created insight into the shock-wave pattern produced by a body moving at supersonic speeds.

The surface waves generated by a moving ship are of great practical importance, for they radiate its motion " to infinity " and, unless a motor, sail or tow-rope replenishes this energy, the boat will come to rest, even in the absence of frictional effects. This phenomenon is known as " wave resistance " (W). Strictly it is not a well defined concept, since in addition to its direct " skin friction " (S), viscosity causes separation of the streamlines from the hull. Nevertheless, since S and W " scale " in different ways (S depending on Reynolds number and W on Froude number, F), this division of the total resistance (T) experienced by a boat in motion is of fundamental importance in predicting its behaviour from model experiments. Crude estimates of S can be obtained from Froude's experiments on thin planks [**30**], and it is found that W is an important fraction of T; indeed, for high performance (e.g. naval) vessels with streamlined hulls, W may exceed S. The magnitude of W depends crucially on the shape of the hull. It can be reasonably calculated, in a first approximation, by neglecting viscosity and computing the radiation of energy by the surface waves in the resulting " ideal " flow. Kelvin's δ-function of pressure is now too gross a model, and Havelock [**14, 29, 34**] at first replaced it by an assigned continuous function of pressure over a finite surface area. The care with which he attempted [e.g. **17, 18, 25**] to relate the resulting theory to observation is exemplary.

He succeeded in obtaining a semi-empirical expression for W in terms of three parameters dependent on the form of the hull, and it did appear that, when these parameters were correctly chosen, the value for W for given F was given with reasonable accuracy (better than $\pm 50\%$, according to figures indicated in **18**). The primary difficulty was that of predicting the correct values of the three crucial parameters in advance of observation, for it was not obvious how the assigned pressure distribution should be related to the hull form, and this limitation appears to have brought him increasing disillusionment with the practical usefulness of this method. Nevertheless, the experience he gained in relating pressure patterns to wave resistance clearly influenced his later thought.

While these developments were taking place, Havelock had not lost his early interest [**1, 2, 6**] in optics. His efforts became increasingly focussed on understanding the scattering of light, particularly at infra-red wavelengths, by matter, particularly crystalline solids. Rayleigh (1892), in attempting to explain early results by Lorentz (1880) and Lorenz (1880) on the variation of refractive index, n, with density, ρ, had proposed a simple model of a crystal in which spherical particles of one material were centred on a cubic lattice of another. He computed n as a function of ρ and of the wave-length λ, assumed large compared with the interparticle spacing. Havelock [**7**] realised that a mechanical stress, either directly applied to the solid or indirectly through the magnetostrictive action of an applied field, would distort Rayleigh's cubical structure and create (artificial) birefringence. He examined the effect by generalising Rayleigh's model to a rectangular lattice.

A more fundamental way of examining the interaction of light with matter is to return to the motion of the individual ions recognising that the displacement of the charges by the electric field, **E**, of the wave will affect the polarisation, **P**, which therefore depends on the frequency, ω, of the wave. In the absence of the wave, the force opposing the displacement of an ion from its mean position in the lattice depends, in the simplest treatment, linearly on the vector changes in its separations from the remaining ions. Under these forces, the lattice possesses normal modes of oscillation (frequency ω_0, say). Of these, the optic modes, i.e. those in which the ions of each species (in a diatomic molecule) move against a similar motion in the reverse direction by those of the other, are most important. When light of infra-red frequency, ω, passes through such a crystal it sets up a disturbance of this type (by an Umklapp process), but this becomes, on the present model, infinite in amplitude as ω approaches the resonance frequency ω_0. This failure is exemplified by the behaviour of the refractive index, n, which on this theory is

$$n^2 = n_\infty{}^2 + \frac{n_0{}^2 - n_\infty{}^2}{1-(\omega/\omega_0)^2}, \tag{a}$$

and which evidently is infinite for $\omega = \omega_0$. Here n_0 is the static refractive index, good for $\omega \ll \omega_0$, while n_∞ ($< n_0$) is the high frequency refractive index valid for $\omega_0 \ll \omega \ll \omega_e$, where ω_e is a typical electronic frequency beyond which the present theory totally fails. Again, if $\omega_0 < \omega < \omega_l \equiv \omega_0\, n_0/n_\infty$, light incident on the crystal should be totally reflected ($n^2 < 0$), which, in fact, is not the case, although the crystal often exhibits an absorption maximum in that frequency range. Indeed, the linear equations are not adequate for $\omega \approx \omega_0$. The large primary oscillations which then occur cascade the incident energy, by non-linear interactions which are then important, down the wavelength spectrum to lattice vibrations of ever smaller scale until the energy supplied appears as heat. A proper treatment of this process would be as formidable as the turbulence problem, but a simple *ad hoc* procedure (akin to the notion of eddy viscosity in turbulence) is to add, to the equation of motion of an ion, a frictional force proportional and opposed to its velocity. By this device, the linearity of the equations is preserved, at the minor expense of a complex refractive index $(1 \pm i\kappa)\, n$, where $n\kappa$ is known usually as " the index of extinction ". Havelock made several comparisons **[19, 21, 39]** of (a), now using complex n, with experiments, and gave a method **[21]** for extracting κ from the data with the minimum of assumptions. Many of his results, concerning the location of the maxima of the absorption and of the reflective power $R \equiv |(n-1)/(n+1)|^2$ were in good agreement with observations (see, for example, his table on p. 498 of **[39]**). The prevailing view today seems to be that significant departures from the theory occur in crystals (see, for example, Born and Huang, 1954, §10). It appears, however, that Havelock's own opinion **[19**, p. 522], that the theory is often adequate to describe dispersion in gases and liquids, is tenable. Havelock **[36]** also used the theory leading to (a) to compute the Verdet constant ($\propto \omega dn/d\omega$) of the Faraday effect. Also, as before, he generalised (a) to the non-isotropic case **[10]**, and applied his results **[13, 51]** to birefringence in solids and liquids (e.g. carbon disulphide, rock salt, quartz, etc.).

Scattering created by a diffuse cloud of particles illuminated by a pencil of radiation whose wavelength is long compared with the grain size is known as " Rayleigh scattering " (Rayleigh, 1871). (The word " diffuse " implies that only an insignificant fraction of the beam encounters a grain more than once before leaving the cloud.) If the particles are optically spherical, the component of linear polarisation of the incident beam which lies in the plane defined by the incident and emergent rays is seriously curtailed by the particles. Indeed, when the angle between the rays is a right-angle, its intensity is zero. When the particles are not optically spherical this is no longer the case because since the orientation of the particle is random, only a small percentage will fail to scatter, at least to some extent, in the required direction. Rayleigh (1918) computed the ratio of the intensity of the resulting (weak) component of linear polarisation to that of the (strong) component perpendicular to both incident and emergent rays. This picture was, to some extent, akin to his model for the lattice mentioned above and, in the same way, Havelock **[35]** carried out a generalisation analogous to his development [7] of the Rayleigh theory for a solid, obtaining the dispersion of each component of polarisation. He applied the theory to molecular hydrogen. The frequencies with which he was now concerned were in the ultraviolet and the corresponding motions in the gas were electronic. Each atom and its surrounding electrons was considered as a resonator whose properties (e.g. ω_0) depended on the direction of polarisation of the incident light. This work led him on to examine dispersion in various diatomic (H Cl, H Br) and triatomic (H_2O, H_2S, CO_2, CS_2) molecules [**45**], and later [**46**] to more complicated molecules (NH_3, CH_4, C Cl_4) of four or five molecules, regarding each as a rigid framework connecting resonators. (He even considered the mathematician's molecule XH_n, for some X!) For a five atom molecule such as methane, he could distinguish between (say) the four H in a square coplanar with the C, and a tetrahedral structure of H with C at its centre. He did so [**47**], finding in favour of the latter, so confirming a view long held by the chemists (Van't Hoff (1874) and Le Bel (1874)). He also concluded that a methane molecule should be considered as a C^+ ion surrounded by four H^- ions rather than a C^- ion surrounded by H^+.

As always, Havelock was tireless in attempting to marry theory to experiment (e.g. [**13, 19, 51**]), and undoubtedly his success contributed to his election on 7 May, 1914, to a Fellowship of the Royal Society at the early age of 37, the first member of Armstrong College to be so honoured. The following year a second chair of (applied) mathematics was created for him.

During World War I, Havelock served as a lieutenant in the Officers Training Corps, and the University Library at Newcastle still contains notes on squad drill taken by him when he attended a course at Chelsea. His damaged health prevented, of course, a more active participation in the struggle.

Apart from a teaching text on Mechanics [**15**], Havelock wrote only one short book [**24**]. It expounded the theory of linear dispersive waves in one dimension, and carefully clarified the pitfalls and paradoxes that can arise when the concepts of phase velocity, group velocity, wavefront, and energy propagation are too carelessly used. He showed a gift for the felicitously chosen example (see also [**16**]), and in this connection it is worth noting in passing that his solution [**38**] to an example of adiabatic invariance (proposed by Levi-Civita, 1912) is still useful in teaching this topical subject.

Havelock's activities in theoretical optics were maintained until 1929, but gradually diminished as his preoccupation with hydrodynamics grew. (Volume 84 of the *Proceedings of the Royal Society* contains, interestingly enough, a contribution from each field.) Perhaps this increasing dedication to naval architecture dates from his recognition of the potential importance of a neglected paper by J. H. Michell (1898). Michell postulated a ship whose sides were at a small angle to the vertical and to the (bow to stern) plane of symmetry. Without introducing any of Havelock's empirical parameters (or any others), Michell was able to compute W directly from the plan form of the ship, and he performed such a calculation in one particular case. After testing Michell's theory and applying it to four further models [**37**], Havelock recast it in a form in which the disturbance was excited along the symmetry plane by an equivalent distribution of dipolar source-sinks, the theory of which he had previously considered [**29**]. An energetic extension to include discrete and continuous sources followed [**48, 58**], and this allowed him to investigate all the major factors relating hull form to wave resistance such as, the effects of finite draught, of straight or hollow bow lines, of blunt or fine sterns, of parallel middle bodies, of variations of entrance and beam for constant displacement, of variations in wave profiles with systematic changes in ship form, and so on [**37, 40, 41, 42**].

An analytically more tractable example of wave resistance is provided by the submerged body in motion, and it is not surprising that Havelock studied this theory, often in parallel with his development of Michell's method, for which it provided valuable points of comparison [**27, 42, 44, 48, 49, 50, 53, 54, 62, 76, 77, 79, 81**].

The form of W as a function of F is often quite complicated, showing maxima at speeds for which the bow and stern waves interfere. Provided the plan form of the ship is described by a relatively simple function and provided the Michell theory is applicable, Havelock found a good agreement between the theoretical and experimental values of $W(F)$, except at small values of F. He attributed [**42**] this discrepancy to the separation of the boundary layers from the sides of the hull, a process which he felt would depress the wave production from the stern below the value predicted on the earlier ideal theory. He first, therefore, reduced semi-empirically the source strength representing the wave generating power of the stern [**57, 61**]. Encouraged by the improved agreement, he redoubled his efforts, ultimately effecting a considerable improvement in the theory [**74**] which reduced the conflict between the measured and theoretical $W(F)$ for small F without destroying the satisfactory agreement at large F. It is probably fair to say, however, that much still remains to be done in this area.

Many other problems of ship hydrodynamics occupied Havelock's later years. He became intrigued by the influence of the weather, first considering [**64**; see also **81**] the effect on W of free waves generated at infinity. Later he examined the phenomena of trim [**73**], sinkage [**66**], virtual mass and damping [**70**], drifting force [**69**], accelerated motion [**76**], and heaving and pitching [**72, 82, 85, 97**]. For many of these, a rational theory is even more difficult than for wave resistance, and the importance of his contributions has, in some cases, still to be evaluated. It is widely believed, however, that his insight into these problems will speed their ultimate solution. Throughout his career in naval architecture his faith in the applicability of linear theory outside its apparent range of validity was unshaken, and on the whole has been well vindicated. (His single attempt [**28**] in non-linear theory concerned the infinite train of finite amplitude surface waves over an infinitely deep ocean, and seems to have been overshadowed by a parallel study by Rayleigh (1917).) From time to time he published valuable progress reports [**43, 59, 78**] of his theoretical advances.

In all his work, the physical explanation, the mathematical style, and the comparison of theory with available experimental information is admirably thorough and clear. During much of his active life there were few experiments that bore directly on his work, and he was very skilful in extracting useful knowledge from unpromising material. Havelock's combination of talents was indeed unusual, but was happily fully recognised in his lifetime. He was knighted in 1951 for his work in naval hydrodynamics. Other honours included (1956) the award of the first William Froude Gold Medal of the Royal Institution of Naval Architects (RINA), election (1945) to an Honorary Fellowship of St. John's College Cambridge, and (1943) the unusual distinction of Honorary Membership of the RINA; the only scientists who have been similarly recognised are Kelvin (1892) and Rayleigh (1911). He also became an associate member of the council of the RINA, and the 1967 volume of its *Transactions* contains a tribute to him. Durham University conferred an honorary D.C.L. on him in 1958, and Hamburg University an honorary D.Sc. in 1960. In 1947, the French Academy of Sciences elected him Corresponding Member for geography and navigation. The Third Symposium of Naval Hydrodynamics, held at the Hague in 1960, dedicated its activities to him. His work was widely appreciated in the United States. He was the featured guest at a special meeting of The Society of Naval Architects and Marine Engineers in 1950 (see [78]), and in 1963 the U.S. Office of Naval Research published a collection of his papers on hydrodynamics (Wigley, 1963).

He maintained an active interest in the running of his University, taking his positions on Senate, Council, Court and the Boards of Faculty very seriously. When Jessop retired in 1928, Havelock became the Head of both Pure and Applied Mathematics Departments which were then merged. From 1933 to 1937 he acted as Vice-Principal of the College and played a leading role in the deliberations leading to its coalescence in 1937 with the College of Medicine to form King's College (of which he became first sub-Rector) in a newly reorganised University of Durham. In addition to these numerous administrative chores and to regular teaching and research in his department, Havelock became in 1943 for three years Honorary Acting Head of the Department of Naval Architecture in succession to Dr. F. H. Todd, who had been seconded from the National Physical Laboratory to the department after Westcott Abell's retirement in 1941. Although Havelock too reached retiring age in 1942, he was invited to remain on the staff for a further three years, and even after 1945 he continued to give special lectures on ship hydrodynamics to Honours students of Naval Architecture.

Although, perhaps because of his early injury, Havelock was ever shy and retiring, he was not a recluse, and as a young man was not above the occasional game of billiards. His interest in music was lifelong. He is said to have run the Mathematics Department at Newcastle with unobtrusive efficiency, and with a sympathetic understanding of his students and younger colleagues. As a mathematician he clearly enjoyed his mastery of nineteenth century analysis. Dr. Mitchell of the department recalls a colloquium given by Havelock in which the argument had apparently reached an unsuperable hurdle, "but," said Havelock triumphantly, "recalling a result given on page N of Watson . . ."! Despite this technical expertise, students of naval architecture found him, as Dr. Muckle remembers, "a very great teacher of what to us engineers was very difficult mathematics". Dr. Townsin of the same department who also as a student received lectures from Havelock recalls him "as a kindly person, tolerant of youth, and as an outstanding teacher lucid in exposition and an authority without doubt; yet withall a man of humility. He was an inspiring academic and an example of what I think we might all strive for. He would have been a good model for us even if he had not such a high international academic status." His colleagues in both departments clearly regarded him with great awe and affection, although perhaps the former a little outweighed the latter, for it is otherwise remarkable that, apart from the elementary text-book on mechanics [15] already mentioned, none of his work was co-authored. His over-eighty research papers, published mainly by the Royal Society in their *Proceedings*, were all solo efforts.

Although Havelock's manner was quiet, he was a shrewd judge of human nature, on which his commentary was often humorous, occasionally pungent, but never malicious. As a University Statesman his influence was almost invariably beneficial, and this was recognised when, after King's College became in 1963 the University of Newcastle upon Tyne, it was decided in 1968 to name a new Hall of Residence after him. Havelock did not marry; he lived with a sister to whose devoted care his longevity must have been due, and who survived his death on 1 August, 1968, by only a few weeks.

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