## GEORGE WILLIAM HILL.

GEORGE WILLIAM HILL was born in New York City on March 13, 1838, of English-Huguenot stock. Having shown at school some signs of that mathematical ability which was afterwards so conspicuous in him, at the age of seventeen he was sent to Rutger's College, New Brunswick, N.J. His professor there was Dr. Theodore Strong, a friend of Nathaniel Bowditch, the translator of Laplace's *Mécanique Céleste;* while still an undergraduate Hill read in his professor's library this and other works by the great French mathematicians—Laplace, Lagrange, Poisson, Legendre, and others. Strong was keenly interested in applied mathematics of the older school, and his influence helped to turn the thoughts of his pupil Hill in the same direction. Before he was twenty-one Hill began original research, and his first published paper, which contains an attempt to extend Laplace's investigation on the figure of the earth, already showed that mastery of analytical resources and facility in exposition which distinguished his later work.

In 1861, soon after leaving college he accepted a position in the *Nautical Almanac* Office, then situated at Cambridge, Mass., and subsequently at Washington. Here, under the successive directorships of Runkle and Newcomb, he laboured for thirty years on the great problems of dynamical astronomy. In his earlier years, indeed, he published a few papers on mathematics unconnected with astronomy, but as time went on he became exclusively absorbed in the latter subject; here also he gradually tended to leave the side more directly connected with observations and their computation, and to devote himself to the distinctively theoretical branches of gravitational astronomy.

After retiring from the *Nautical Almanac* Office in 1892, he was able to give his full time to those favourite studies which had formerly been prosecuted largely in the hours of leisure from official duties. These twenty-two years, during which he was "out of harness," were spent for the most part in much seclusion at his quiet home at West Nyack, where he died on April 16, 1914, at the age of seventy-six. During this interval, however, he gave considerable time and service to the American Mathematical Society, of which he was President in 1894–96; and he served as lecturer on celestial mechanics in Columbia University during the academic years 1898 to 1901.

Hill never married, and he entered very little into society of any kind.

His mode of living was of great simplicity, and he avoided any undertakings in ordinary spheres of life which might interfere with his astronomical investigations. His labours were broken only by long walks around his home, by occasional visits to his more intimate friends, and by infrequent vacation tours to distant parts of the world. Those who knew him well describe him as being, in the privacy of his own circle, a delightful companion and singularly attractive in personality. He had a strong liking for geographical exploration, and made two trips into Canada for this purpose: one into the Hudson Bay region and one into the North-west. It was during the latter journey that he worked out his famous solution of the problem involving an infinite determinant, a solution (in Poincaré's words) *aussi originale que hardie*. The value of the work was immediately recognized in this country through the notice of J. C. Adams; and it presently appeared that Adams had himself in lectures at Cambridge treated the similar problem presented by the progression of the lunar node substantially in the same manner, though with less elaboration of analytical development. Hill visited this country in 1892, and received the honorary doctorate of Cambridge. This was one of many honours bestowed upon him, in Europe as well as in his own country. In this country he received the Copley medal and foreign membership of the Royal Society, the Gold Medal and associateship of the Royal Astronomical Society, and in 1907 he was elected to honorary membership of our own Society.

Over the long period from his entry in the *Nautical Almanac* Office until his death his output of original work remained at a high level both of quality and quantity. His papers appeared mainly in the *Nautical Almanac* Office publications and in American journals; those written up to about 1905 have been republished in four volumes of his *Collected Mathematical Works* by the Carnegie Institution of Washington, and his subsequent papers will suffice to make a fifth volume. Among his early writings were a few which appeared in *The Analyst*, which are of some general interest to mathematicians; they consist of occasional work on such topics as the theory of numbers, the theory of equations, finite differences, and the theory of gravitation. But the works by virtue of which his name will endure are on subjects which must always remain little known to most, even among mathematicians, and the number in a single generation who will study his works is never likely to be anything but small. Nevertheless, in his chief work, the theory of the moon's motion, he was led to a new development in mathematics, the infinite determinant already mentioned, which is doubtless destined to prove as useful in other spheres as in that for which it was invented, and which is of considerable interest to the pure mathematician.

In this new lunar theory, by the introduction of several ideas of considerable novelty, he laid down the lines of a new method of calculating the moon's motion, which bids fair to hold the field against the works of all his predecessors in the attack on this great problem. The treatment differed profoundly in many respects from that of Delaunay, Hansen, and the earlier dynamical astronomers. Instead of polar coordinates referred to fixed axes, rectangular coordinates referred to moving axes were introduced, leading to differential equations in a simple algebraic form suitable for solution by infinite series, and the convergence was increased by the use of a new parameter. A new orbit of reference, called the variational curve, obviated the difficulties met with in using the usual reference orbit (the Keplerian ellipse), on account of the troublesome motion of the perigee of the latter; this is because the variational curve contains a most important part of the solar perturbation, called the "variation." Apart from its value in the lunar theory, moreover, the variational curve was the starting point of a general theory of periodic orbits in the problem of three bodies, which has been developed by Poincaré, Darwin, Brown, Moulton, and others. As regards the lunar theory itself, one of the best tributes to it is the enormous labour, extending over years, devoted to its development and application by Prof. E. W. Brown; subsequent observations have shown, however, that our refractory satellite has even now not yet yielded up all her secrets to the mathematicians.

Extensive as was Hill's work on the moon, both in the development of his own theory and in the discussion of Delaunay's researches, his labours were not confined to this branch of gravitational astronomy. The orbits of Venus, the minor planets, and especially of Jupiter and Saturn, received much of his attention, and in his later years he interested himself greatly in the investigations in periodic and other orbits by Poincaré and Gyldén. His last paper was published in January 1914 in the *Astronomical Journal*, of which he was an associate editor.

At the *Nautical Almanac* Office, Hill, while he occupied only a humble post, lived contentedly in the liberty it afforded him, and never sought to better his position materially; all he desired was peace and freedom to prosecute his work. Newcomb did his best to secure official recognition for his illustrious assistant, but Hill seemed to take little interest in the matter. In Newcomb's words, "here was perhaps the greatest living master in the highest and most difficult branch of astronomy, winning world-wide recognition for his country in the science, and receiving the salary of a department clerk." Newcomb showed the respect in which he held Hill by leaving to him the most difficult section of his scheme for the revision of all the tables of the planets on a uniform basis. Hill's share, the theories of Jupiter and Saturn, occupied him nearly fifteen years, and the investigation takes up the whole of one volume of his *Collected Works*. Its aim was to form theories of these planets such as would be "practically serviceable for a space of three hundred years on each side of the central epoch, taken near the centre of gravity of all the times of observation; theories whose errors in this interval would result, not from neglected terms in the developments, but from the unavoidable imperfections in the values of the arbitrary constant and the masses, adopted from the indications of observation" (Hill). All the original calculations he made himself, and though Newcomb pressed him to accept assistance, he consented only so far as duplication was concerned. In this and all his work he manifested not only the greatest clarity of mind and high mathematical genius, but also the sincerest altruism in the cause of science, labouring in the pursuit of knowledge in its most abstruse branches without a thought of the personal reward which he might fairly have claimed, but to which the simplicity of his life and the innate modesty of his character rendered him almost wholly indifferent.

S. C.