WILLIAM VALLANCE DOUGLAS HODGE

M. F. ATIYAH†

Sir William Hodge, formerly Lowndean Professor of Astronomy and Geometry in the University of Cambridge, Master of Pembroke College, Cambridge, and Physical Secretary of the Royal Society, was for over forty years a leading figure in British mathematical life. He died in July 1975 at the age of 72.

Early Life

Hodge came from a moderately prosperous Edinburgh background, but it appears that on both sides of the family this prosperity only went back a couple of generations and that his two grandfathers had been responsible for raising the social level, from rather humble origins, to a modest middle-class standing in the last quarter of the nineteenth century.

The paternal grandfather, Archibald Hodge, spent most of his life in a firm, Douglas and Company, of Searchers of Records. This business, peculiar to Scotland, involved the examination of land titles and its fortunes were closely linked with those of the property market. In due course Archibald Hodge became senior partner of the firm and his only son Archibald James Hodge joined as junior partner.

The other grandfather, William Vallance, built up a flourishing confectionery business which eventually had valuable premises in the centre of Edinburgh. His wife Janet (born Horsburgh) appears to have been the dominating partner both in the business and in the family. In spite of having eight children she remained in command of the business until her husband's death in 1903. Her eldest son then took over but was required to inform her daily on how things were going. In later years she became a devoted grandmother and lived to the ripe age of 104.

Hodge's parents, Archibald James Hodge and Janet Vallance, were married in 1900. At first the Vallance family objected strongly to the marriage. Although both families were by this time comfortably off the Vallances, having carefully built up their savings, were distinctly wealthier and they suspected young Archibald of being after their daughter's money (which was quite untrue). In addition Archibald, being the only son of well-off parents, had led a gay life and acquired a taste for drink. Later on, however, after the death of Archibald Hodge senior, the families were reconciled.

There were three children of this marriage. The eldest Archibald Vallance, born in April 1901, William Vallance Douglas, born 17 June, 1903 and their younger sister Janet, who later married Professor T. A. A. Broadbent. The family lived in a comfortable suburban house in Granby Road, Edinburgh, and this remained William's house until he went to Cambridge many years later.

At the age of six, after two years in kindergarten, young William entered George Watson's Boys College in Edinburgh where he was to have his entire schooling. The education he received there was sound, of the traditional type of the period, and his academic record was good without being outstanding. He was nearly always in the

† This obituary has previously appeared in the Biographical Notices of the Royal Society. It was prepared by Professor M. F. Atiyah, and we are grateful for permission to reproduce it with modifications.

[Bull. London Math. Soc., 9 (1977), 99–118]

top third of his class and sometimes higher. The success he achieved here, and also in later life, he attributed in part to a desire to compensate for his deficiencies on the golf course. Where his father had been a first class player and his brother competent, William was always a "rabbit". By contrast the classroom was where he could hold his own, and he determined to make the most of it.

The early years at school were uneventful but, for a variety of reasons, 1916 marked a turning point in William's life. The effect of the war was beginning to be felt, but more important was the death of his uncle William Vallance. The family decided that brother Archie, now fifteen, should leave school and be apprenticed to the Vallance firm with a view to becoming manager in due course. With Archie away from school, and making his own living, William developed more of his own life at school. His friends were intellectually inclined and this had a considerable influence on his activities both inside and outside the classroom. From this period on his academic performance improved and his literary horizons were widened. Taking the Scottish Leaving Certificate at the early age of fourteen he moved into the upper school where he was confronted with a choice of courses. English and mathematics were compulsory for everyone but there was a basic choice between classics, languages and science. Without proper guidance he opted for a middle course of Latin and French. Since he was not a great linguist it seems likely that a science course would have been much more useful to him, and he regretted this gap in his education.

In 1918 the end of the war brought a boom in the housing market and a consequent increase in the work of Douglas and Company. With many of his staff still in the army Archibald Hodge senior was in need of assistance. At Easter 1918 it was therefore decided that William should leave school and join his father's firm. The work, of a routine clerical kind, was not onerous but neither was it stimulating. After six months there was a change of plan and William returned to school. Except for a brief emergency due to the influenza epidemic this was the end of William's office career and he spent two further enjoyable and fruitful years at George Watson's.

During his years in the senior school there was frequent family discussion concerning his future career and the idea gradually emerged that he should try to enter the administrative branch of the Civil Service. For this purpose it was agreed that he should first go to university and take a degree. By this time it was quite clear that his talents lay in the direction of mathematics and that he should read for honours in mathematics and philosophy. Accordingly he sat for the Edinburgh University Bursary Competition in June 1920 and came high on the list without being outstanding, but in the special paper for the John Welsh Mathematical Bursary he came top of the list. His entry to the University was therefore assured financially but more important, his success was noted by Professor Whittaker who was therefore on the lookout for him when he entered the classroom in October 1920.

Before leaving this period it is perhaps appropriate to mention the influence of Peter Ramsey, the mathematics teacher at George Watson's. He had the gift of being able to stimulate any mathematical ability which could be found in his pupils in a manner which, as described by Hodge, was a remarkable combination of sarcasm and kindliness.

Edinburgh and Cambridge, 1920–26

When he entered Edinburgh University in 1920 Hodge took to the student life there at once. Perhaps the most significant fact was that, whereas at school he had been high in the class but by no means top, at Edinburgh he found that his place as first in the mathematical honours school was virtually unchallenged, and that he could maintain it without undue effort. He was therefore able to take a full part in extra-curricular activities, became prominent in a number of student societies and in general terms was much more socially inclined than he had been in school.

The natural philosophy (physics) half of the course did not cause him any undue difficulty despite the fact that he had had no science in his last three years at school. He also had to take two other subjects to ordinary degree standard and, with the Civil Service still as ultimate objective, he elected to take economics and English. While he managed to achieve a reasonable standard in these subjects, his performance in them forced him to realize that his whole success depended entirely on mathematics, and that he might well find himself in real difficulty if he entered the Civil Service with no wider qualification for a career there. It was at this point that his thoughts began to turn to an academic career.

The influence of Whittaker had much to do with the shaping of Hodge's subsequent career. Personal contact with Whittaker began early through his son Jack (J. M. Whittaker, F.R.S., later Vice-Chancellor of Sheffield University) who had entered Edinburgh University at the same time although only fifteen years old. It was on one of Hodge's visits to the Whittaker home in George Square that the first serious discussions of an academic career for him took place. From the beginning Whittaker stressed the necessity of going up to Cambridge and taking a degree there. Fortunately Hodge's parents readily agreed to put up the money and Whittaker took great trouble to make his admission to Cambridge as simple as possible. St. John's College had exhibitions not restricted to candidates under nineteen and with the prospect of a £100 scholarship from Edinburgh the idea of going to Cambridge became a practical possibility. The only snag was the timetable: the St. John's examination was perilously close to the date of the Edinburgh degree examinations. By carefully selecting the papers for his finals (and with some helpful rearrangement of the examination timetable by Whittaker) Hodge was able to sit the Cambridge examination then return overnight to Edinburgh to appear in the examination room at 9 a.m. the next morning. Despite this strenuous programme all turned out well; he was awarded a £60 exhibition at St. John's, first class honours in Edinburgh and the van Dunlop scholarship in mathematics and so was able to enter St. John's in October 1923.

While in Edinburgh Hodge did not fully appreciate how important it was that he should take the Cambridge Tripos: many thought an Edinburgh degree was a sufficient passport to an academic career, but at Cambridge Hodge soon learnt to appreciate the value of Whittaker's advice. The Edinburgh mathematics course in 1920 was still of the old-fashioned non-rigorous type so common in the nineteenth century. This was not the fault of Whittaker, who was aware of the department's deficiencies, but had not yet had the opportunity, because of the war, of recruiting new staff. Thus when Hodge went to Cambridge he had little idea of what modern mathematics was about.

The Cambridge Tripos in 1923 had been substantially modernized. The old-fashioned trick question of fiendish ingenuity which had characterized the Tripos in the nineteenth and early twentieth centuries, if not completely banished, had been reduced to size and formed the subject of part I, from which in any case Hodge, as an affiliated student, was exempt. The two years leading to part II were devoted to giving students a proper grounding in the modern ideas of mathematics, and for the specialist there was the optional schedule B of part II, in which advanced courses could be offered. The reform brought about by this part II was not complete: it depended to a certain extent on the interests of those available to teach and on their ability to stake a share of the timetable for their speciality.

In pure mathematics the dominating subject was analysis and the leading figure was J. E. Littlewood. Most of those staying on for research chose to work in this field. Geometry also bulked large in the part II curriculum, but here the situation was peculiar. In 1914 H. F. Baker had been appointed Lowndean Professor of Astronomy and Geometry in succession to Sir Robert Ball, the astronomer. Baker was an analyst of the Germanic school, his principal publication being "Abel's theorem and allied theory". This, of course, had many connections with geometry and on his appointment to the chair Baker's conscience made him devote all his efforts to geometry. Curiously he abandoned, for a time, the type of geometry most closely related to his interests in analysis, only returning to this in later years. In 1923 geometry was strictly projective geometry and in this his lieutenant was F. P. White of St. John's. The other important branches of mathematics received little attention. Algebra was entirely neglected and only revived in Cambridge about 1930 under Philip Hall, who in fact went up to King's in 1923. There was no topology taught; this was not surprising as it was a "new"subject and the deficiency was soon remedied by M. H. A. Newman who became a Fellow of St. John's in 1923.

On the applied side mathematical physics dominated the scene. This no doubt was due to the tremendous influence of Rutherford in the Cavendish Laboratory, and there were many followers both in physics and in mathematics. Among the mathematicians R. H. Fowler was the leader in Cambridge but activity and enthusiasm, was widespread. 1923 was also the year in which P. A. M. Dirac came from Bristol as a research student. Finally, in classical applied mathematics, very important work was being done by G. I. Taylor and Harold Jeffreys.

All this presented a somewhat bewildering picture to Hodge whose only acquaintance was with mathematical subjects regarded as completely passé. Although he did not despair he found he was in for the hardest period of his life, working incredibly long hours and severely restricting his social activities.

Fairly soon after arriving in Cambridge Hodge had to decide what subject he intended to specialize in for schedule B. In the event this decision turned out to be extremely easy. From the moment he joined F. P. White's course on projective geometry he never gave a thought to any other topic. This may seem surprising since White, though an enthusiastic geometer and a charming man, made no claims to being a brilliant lecturer. The explanation lies in the contrast between the way projective geometry was treated in Edinburgh and Cambridge. Whereas in Edinburgh it was an appendage to Euclidean geometry, White's course gave a comprehensive account of the synthetic approach, based on the axioms of incidence, and its relation to coordinate geometry. This opened up a whole new world to Hodge and, in his own words, he was promptly "hooked". Many years later he repaid his debt to White by expounding this material in volume I of his joint book with Pedoe [55].

When Hodge's attraction to geometry became clear White took him under his wing, giving advice on what courses to attend, introducing him to Baker and helping whenever he was in difficulty. The result was that things went well and in 1925 Hodge graduated with first class honours in part II and distinction in schedule B.

Hodge's financial position allowed him to return to Cambridge for a third year; the van Dunlop scholarship from Edinburgh had one year to run, the St. John's exhibition was raised to a scholarship and in 1924 he had won the Ferguson scholarship for mathematics, open to graduates of Scottish universities, which provided a further £80 per annum for two years. He had no intention of taking a Ph.D. degree—at that time it was not considered essential for an academic career. His object was simply to make a start on research before taking up an appointment in 1926, by which time he would have completed six years as a university student. He had regular informal contact with H. F. Baker and tried his hand at various geometrical problems but it was not until the summer of 1926 that he found a line in which he could make some progress. But although the year was not very profitable mathematically it was of great value in other ways. After two gruelling years working for the Tripos the pressure was off and, as he now moved into rooms in College, Hodge was able to enter much more fully into the life of the College. His companion of this year was T. A. A. Broadbent who eventually married Hodge's sister, became Professor of Mathematics at the Royal Naval College, Greenwich, and remained a lifelong friend.

Bristol

In 1926 Hodge took up his first teaching appointment, as an assistant lecturer at Bristol. The move to Bristol turned out to be a great success from all points of view. The atmosphere was much more relaxed than in Cambridge and Hodge found himself a member of a small but happy mathematical community. The head of the department was Professor H. R. Hassé, an applied mathematician who had been a Fellow of St. John's Cambridge and then a lecturer in Manchester before going to Bristol about 1919. He was an excellent head, supervising in a kindly way the work of his staff and, with his wife, looking after their general welfare. Hodge felt that he owed a great deal to Hassé who did everything possible to lighten Hodge's teaching duties so that he could concentrate on his research.

Next in seniority was Peter Fraser, reader in geometry, who was a remarkable person and one who greatly influenced Hodge's mathematical development at this stage. Fraser was a Scot, educated at Aberdeen University and Queens', Cambridge, who settled in Bristol in 1908 and spent the rest of his life there. He genuinely loved mathematics, especially geometry, and read widely but always refused to publish any original work. His knowledge was encyclopaedic and mathematicians from all over the country, including even H. F. Baker, would consult him from time to time.

About twice a week Hodge, Fraser and a few other colleagues including Hassé, when his duties permitted, would proceed after lunch to Long Ashton Golf Club to play a round of golf. The standard of play was atrocious but the conversation, which ranged widely, more than compensated for the inferior golf. On numerous occasions Fraser would expound for Hodge's benefit some interesting point in geometry while simultaneously puffing at his pipe and struggling to extricate his ball from an impossible position. Hodge's education as a geometer owed much to these expeditions and to similar discussions in Fraser's room in the department.

When Hodge arrived in Bristol he was in the process of writing up his Smith's Prize essay, so there was no immediate problem of looking for a research problem and the only difficulty was to find the necessary time. Here Hassé's thoughtfulness made things easy: he simply decreed that Hodge should give exactly two lectures a week for the first term, and these were fixed at the end of the week so that Hodge could really devote most of his time to the essay. This made all the difference and, when the Smith's Prize was eventually awarded to Hodge, Hassé was delighted.

With the essay completed and then written up for publication [1] Hodge began looking round for a new topic and came across a paper of Baker on the Newton polygon. This led to [2] which, though fairly routine in itself, is significant because it represents the beginning of Hodge's interest in integrals on an algebraic variety. But he was still in the position of scraping round for problems to work on, without a general framework to fit them in. After many discussions with Peter Fraser he eventually made strenuous efforts to master the work of the Italian geometers, Castelnuovo, Enriques and Severi, on the general birational geometry of surfaces. In the course of his reading he soon came across a problem which was to determine the course of his work for years to come.

Severi, in one of his papers, mentioned the importance of knowing whether a non-zero double integral of the first kind could have all its periods zero. By a stroke of luck Hodge saw a paper of Lefschetz in the 1929 *Annals of Mathematics* in which purely topological methods were used to obtain the period relations and inequalities for integrals on a curve. It was clear to Hodge that Lefschetz's methods could be extended to surfaces to solve Severi's problem and he found it incredible that this should have escaped Lefschetz's notice. In any case it did not take Hodge long to work out the details and write [3] for publication. This was the big turning point in Hodge's career and it led on to many things, but before following these up it is perhaps appropriate to describe the essence of Hodge's proof and its relation to Lefschetz's paper.

If $\omega_1, \ldots, \omega_g$ are a basis for the holomorphic differentials on a curve X of genus g we have $\omega_i \wedge \omega_j = 0$ for simple reasons of complex dimension. If $[\omega_i]$ denotes the 1-dimensional cohomology class defined by ω_i (i.e. given by its periods) it follows that the cup-product $[\omega_i] \cup [\omega_j] = 0$. If we spell this out in terms of the periods we obtain the Riemann bilinear relations for the ω_i. This is the modern approach and it is the essential content of Lefschetz's 1929 paper though one has to bear in mind that cohomology had not yet appeared and that the argument had to be expressed in terms of cycles and intersection theory. Of course it was Lefschetz himself who led the developments in algebraic topology which enable us now to express his original proof so succinctly.

Similarly, the Riemann inequalities arise from the fact that, for any holomorphic differential $\omega \neq 0$, $i/i\omega \wedge \bar{\omega}$ is a positive volume and so $i/i \int_X \omega \wedge \bar{\omega} > 0$. In cohomological terms this implies in particular that $[\omega] \cup [\bar{\omega}] \neq 0$ and hence that $[\omega] \neq 0$, in other words ω cannot have all its periods zero. If we now replace X by an algebraic surface and ω by a non-zero holomorphic 2-form (a "double integral" in the classical terminology) exactly the same argument applies and disposes of Severi's question.

In addition to the primitive state of topology at this time we have to recall that complex manifolds (other than Riemann surfaces) were not conceived of in the modern sense, and the simplicity of the proof indicated above owes much to Hodge's work in later years which made complex manifolds familiar to the present generation of geometers. This must have had something to do with Lefschetz's surprising failure to see what Hodge saw. In fact this was no simple omission on Lefschetz's part and he took a great deal of convincing on this point. At first he insisted publicly that Hodge was wrong and he wrote to him demanding that the paper should be withdrawn. Eventually Lefschetz and Hodge had a meeting in May 1931 in Max Newman's rooms in Cambridge. There was a lengthy discussion leading to a state of armed neutrality and an invitation to Hodge to spend the next academic year at Princeton. Hodge took up the invitation and after a month there Lefschetz conceded defeat and then, with typical generosity, publicly retracted his criticisms of Hodge's paper. After this dramatic episode Lefschetz became one of Hodge's strongest supporters and fully made up for his initial scepticism.

The publication of [3] opened many doors for Hodge. In November 1930 he was elected to a Research Fellowship at St. John's College, Cambridge, and shortly afterwards he was awarded an 1851 Exhibition Studentship. He was thus in a position to take up Lefschetz's invitation to spend a year in Princeton. Hassé, helpful as ever, arranged a two-year leave of absence for Hodge so that his job in Bristol would not be jeopardized. This was of considerable importance because it was the time of the great depression and secure jobs were hard to find. More importantly Hodge had in July 1929 married Kathleen Anne Cameron, also of Edinburgh, and so had acquired family responsibilities. However, with everything satisfactorily arranged, the Hodges set sail for New York on Sunday, 20 September, 1931.

PRINCETON

In 1931 Princeton was a relatively small university. The Institute for Advanced Study was not founded until a few years later but the academic staff of the University were very distinguished. In mathematics pioneering work was being done in the new field of topology by Oswald Veblen, J. W. Alexander and Lefschetz. While Hodge never became a real expert in topology he regularly attended the Princeton seminars and picked up enough general background for his subsequent work.

Lefschetz was, without doubt, the mathematician who exercised the strongest influence on Hodge's work. Already in Bristol, after his early encounter with Lefschetz's Annals paper, Hodge had proceeded to read Lefschetz's Borel Tract "L'Analyse Situs et la Géometrie Algébrique", and was completely won over to the use of topological methods in the study of algebraic integrals. In Princeton Lefschetz's dominant personality propelled Hodge further along his already chosen path. They had frequent mathematical discussions together in which Lefschetz's fertile imagination would throw up innumerable ideas, most of which would turn out to be false, but the remaining ones would be invaluable. In return Hodge became in due course Lefschetz's true successor in algebraic geometry. Whereas the Princeton school inherited and developed Lefschetz's contributions in topology, his earlier fundamental work on the homology of algebraic varieties was somewhat neglected, probably because it was ahead of its time. Hodge's work was to be complementary or dual to that of Lefschetz, providing an algebraic description of the homology instead of a geometric one. The fact that Lefschetz's theory has now been restored to a central place in modern algebraic geometry is entirely due to the interest aroused by its interactions with Hodge's theory.

Lefschetz, recognizing that he was no longer an expert on algebraic geometry, persuaded Hodge to spend a couple of months at Johns Hopkins where Oscar Zariski was the leading light. This visit had a great impact on Hodge's future. In the first place he and his wife formed a close friendship with the Zariskis which lasted for the rest of his life. He also became impressed with the new algebraic techniques which Zariski was developing, and in later years Hodge devoted much time and effort to mastering these. Although their technical involvement with algebraic geometry was different, Zariski and Hodge felt a common love for the subject and had serious mathematical discussions whenever they met. They also kept up an intermittent correspondence for over forty years.

By the time Hodge came to Princeton his mathematical ideas, arising from [3], had already progressed very significantly. In studying integrals in higher dimensions he soon put his finger on the crucial point. Whereas for the Riemann surface of a curve the number of holomorphic 1-forms is half the first Betti number, there is no corresponding relation in higher dimensions for holomorphic p-forms with $p > 1$. He discussed this point on a number of occasions with Peter Fraser until one day Fraser pointed out de Rham's thesis which had just arrived in the Bristol library. In later years Hodge described this as a stroke of good fortune, and although it did not solve his problem it helped him to see what was involved. In de Rham's theory, valid on any real differentiable manifold, the main result is that there always exists a closed p-form ω with prescribed periods, and that ω is unique modulo derived forms. On a Riemann surface there are natural choices given by the real and imaginary parts of the holomorphic differentials and Hodge was looking for an appropriate generalization to higher-dimensional algebraic varieties. He saw that the real and imaginary parts of a holomorphic 1-form on a Riemann surface are in some sense duals of one another and he had a hunch that there should be an analogous duality in general. More precisely for each p-form ω there should be an $(n-p)$-form $*\omega$ (n being the dimension of the manifold) and the preferred forms, later to be called harmonic, would be those satisfying $d\omega = 0$ and $d(*\omega) = 0$. The main theorem to be proved would be the existence of a unique harmonic form with prescribed periods.

Once established in Princeton Hodge tried to clarify and develop these vague ideas. He soon realized that the relationship of ω to $*\omega$ was a kind of orthogonality and he was able to make this precise in Euclidean space and, more generally, on a conformally flat manifold. He then attempted to prove the existence theorem by generalizing the classical Dirichlet methods, and this led to [5]. The next stage was to try to remove the restriction of conformal flatness, but for this Hodge needed to become familiar with classical Riemannian geometry. In fact this was his major preoccupation and achievement during his stay in Baltimore. At the same time he also came across a paper by the Dutch mathematician Mannoury in which an explicit and convenient metric was introduced on complex projective space, and hence on any projective algebraic manifold. This metric was to prove of fundamental importance for all Hodge's subsequent work.

Cambridge, 1932–39

Hodge returned to Cambridge in July 1932 and spent the next academic year there as a research Fellow of St. John's. In 1933 he was appointed a University Lecturer and a college post fortunately fell vacant at Pembroke with the departure of Jack Whittaker to a chair at Liverpool. Hodge became Director of Studies at Pembroke and, in January 1935, he was elected to a Fellowship.

The years 1933–36 thus saw Hodge settled in Cambridge on a permanent basis but he found the teaching burden extremely heavy. In addition to a hundred hours of lecturing per year he also did twelve hours a week of College teaching. As a result he decided to move to a chair at the first suitable opportunity. Fortunately, Baker decided at this time to retire from the Lowndean chair and Hodge was elected as his successor in March 1936. Although he was only thirty-two his work had attracted world-wide attention and he had the strong backing of such major figures as Lefschetz and Hermann Weyl.

As Lowndean Professor Hodge had of course to give up his teaching Fellowship at Pembroke but the College elected him to a Professorial Fellowship and he was thus able to continue what was to become a long and happy association with the College.

On returning from Princeton Hodge continued with his efforts to prove the existence theorem for harmonic forms on a general Riemannian manifold. His first version [6] was in his words "crude in the extreme" and Hermann Weyl found it hard to judge whether the proof was complete or rather how much effort would be needed to make it complete. Nevertheless, Hodge was now convinced that he was on the right track and his next step was to apply his theory in detail to algebraic surfaces. Using the Mannoury metric he proceeded to study the harmonic forms and found the calculations much simpler than he had expected. Finally, to his great surprise, he discovered that his results gave a purely topological interpretation of the geometric genus (the number of independent holomorphic 2-forms). This was a totally unexpected result and, when published [8], it created quite a stir in the world of algebraic geometers. In particular it convinced even the most sceptical of the importance of Hodge's theory, and it became justly famous as "Hodge's signature theorem". Twenty years later it played a key role in Hirzebruch's work on the Riemann–Roch theorem and it remains one of the highlights of the theory of harmonic forms.

After this success Hodge worked steadily, polishing his theory and developing its applications to algebraic geometry. He began also to organize a connected account of all his work as an essay for the Adams Prize. He was awarded the Prize in 1937 but the magnum opus took another three years to complete and finally appeared in book form [54] in 1941. In the meantime he published in [11] another approach to the existence theorem which had been suggested to him by H. Kneser. This involved the use of the parametrix method of Levi and Hilbert and was, as Hodge said, superior in all respects to his first attempt. Unfortunately this version, reproduced in his book, contained a serious error which was pointed out by Bohnenblust. The necessary modifications to provide a correct proof were made by Hermann Weyl in Princeton and independently by Kodaira in (wartime) Japan.

Hodge himself freely admitted that he did not have the technical analytical background necessary to deal adequately with his existence theorem. He was only too pleased when others, better qualified analysts than himself, completed the task. This left him free to devote himself to the applications in algebraic geometry which was what really interested him.

In retrospect it is clear that the technical difficulties in the existence theorem did not really require any significant new ideas, but merely a careful extension of classical methods. The real novelty, which was Hodge's major contribution, was in the conception of harmonic integrals and their relevance to algebraic geometry. This triumph of concept over technique is reminiscent of a similar episode in the work of Hodge's great predecessor Bernhard Riemann.

In 1938 Hodge was elected a Fellow of the Royal Society, his work was in full flood and he looked forward to many years of active research. Alas, his visions were rudely shattered in September 1939.

Wartime Cambridge

Although Hodge was not directly involved in war work, his life was very much affected by the alterations which the war brought to Cambridge. With so many of his colleagues away on war duty he took on an increasing number of administrative tasks. Of these the first and most onerous was that of Bursar of Pembroke, a post vacated in April 1940 by Gordon Sutherland (later Master of Emmanuel College). By University Statute a Professor could not be Bursar of a College but this was circumvented (with official connivance) by the expedient of calling Hodge Steward, a post which he held till the end of the war.

As Steward Hodge was responsible for the domestic side of College affairs. It was his first taste of administration and he soon found, to his surprise and amusement. that he was coping very efficiently with his new responsibilities. This proved a turning point in Hodge's life. Once it was discovered that he both enjoyed and was efficient at administration he was constantly called on to serve in various capacities both within and outside the University.

Of his contributions to University administration Harold Taylor, formerly Treasurer of the University and Secretary General of the Faculties has written:

> "Not many active teachers and research workers on University staffs are as a rule willing to take on continuing and heavy commitments in connection with the administration that is needed to keep a University running efficiently and happily. Professor Hodge was a notable exception to this general rule. He was an active and most helpful member over long periods not only of the Faculty Board of Mathematics which is responsible for the detailed administration of his own subject, but also on central University administrative bodies such as the Council of the Senate and the General Board of the Faculties.
>
> "Many members of central bodies such as these may be willing to attend their meetings and to offer opinions from their own store of knowledge on such matters as come under discussion. But Professor Hodge was outstanding in his willingness always to devote the time and thought that were necessary to ensure that he came to meetings fully briefed with the reactions of the bodies whose interests he represented on the central board. Perhaps only someone such as myself who has had long experience of central administration (as well as of teaching and research in earlier years!) can adequately express my admiration for the energy and devotion which Sir William showed in his handling of these duties. His work certainly did a great deal to help me in my duty of ensuring the efficient running of the University."

By the spring of 1940 Hodge had completed the manuscript of his book on harmonic integrals and felt he had exhausted his ideas in that direction for the time being. He was therefore looking round for a new field to enter. On the other hand, his increasing administrative commitment to the College and University left him less time and energy to devote to mathematics. These two factors help to explain the shift in his interests which took place over the next decade. For some time he had been aware of the powerful algebraic techniques which had been introduced into algebraic geometry by van der Waerden and Zariski. These ideas had had as yet little impact on British geometers and Hodge felt a duty to interpret and explain the new material to his colleagues. In this he was motivated by a desire to make amends to the Baker school of geometry for the sharp change of direction which his work on harmonic integrals had produced. He thus conceived the idea of writing a book which would replace Baker's *Principles of geometry*. Although not as demanding as original research this task soon proved too much for his unaided effort and he was very glad therefore to enlist the assistance of Daniel Pedoe. Thus began a collaborative enterprise which was to last for ten years and led to their three volume work *Methods of algebraic geometry* [**55**].

While this book, when it appeared, discharged Hodge's obligations to classical algebraic geometry and contained much useful material it did not achieve its main objective of converting British geometers to modern methods. The principal reason is that it was overtaken by events. By the time it appeared algebraic geometry was exploding with new ideas and entirely new foundations were being laid. In addition it must be confessed that Hodge did not have the elegance and fluency of style which alone makes algebra palatable. Hodge himself recognized his limitations as an algebraist and despite his admiration for, and interest in, Zariski's work he eventually returned to his " first love ", the transcendental theory.

Postwar Cambridge

At the end of the war Hodge was able to relinquish his stewardship of Pembroke and in due course his other University commitments, but by now he was a senior established figure on the national scene and acquired further responsibilities. Thus he served on the Council of the Royal Society from 1944 to 1946 and, from 1947 to 1949, he was President both of the London Mathematical Society and of the Cambridge Philosophical Society at a time when both these societies were being reactivated after the war. Together with his continued involvement with the later volumes of his book **[55]** these activities kept him fully occupied during the period 1945–49.

There was, however, one notable event in 1946 when Hodge was invited to take part in the Bicentennial Conference of Princeton University. This was a select gathering at which Hodge met, for the first time, many of the world's leading mathematicians. He found that there was much more interest in his work on harmonic integrals than he had imagined, and in the course of his visit, which included a brief lecture tour, he made several lasting contacts which meant much to him.

He was impressed with the stimulating effect of these mathematical meetings and, on his return to England, he joined forces with Max Newman and Henry Whitehead to organize the British Mathematical Colloquium. This took firm root and has now grown to a major annual event, attended by over four hundred mathematicians.

His next and longer visit to the United States came in 1950 on the occasion of the International Congress of Mathematicians at Harvard. Hodge was invited to give one of the main lectures and, through the assistance of Zariski, he was invited to Harvard as a visiting lecturer from January 1950. This stay did much to re-kindle his enthusiasm for mathematics but it also led to further involvement in other directions. At this time moves were afoot to form an International Mathematical Union and Hodge was involved in the negotiations at an early stage. In due course the Union was formally established at a meeting in Rome in 1952 and Hodge found himself on the Executive Committee for the next six years, serving as Vice-President from 1954 to 1958. He also became Chairman of the British National Committee for Mathematics. Professor K. Chandrasekharan, formerly Secretary and later President of the I.M.U., has written:

> " Hodge played a crucial role in reviving the International Mathematical Union in 1950, and in enlisting support for it from the established international scientific organizations, like Unesco and I.C.S.U., in addition to the leading national scientific bodies, notably the Royal Society. The early years of the Union

happened to be the years of the 'cold war', and neither the U.S.S.R. nor China were represented at the Foundation Meeting in New York in 1950. The careful stewardship of the Union during those years owed not a little to Hodge's wise counsel. He laid great stress on the international character of the Union, and on the need to be non-political, while being realistic, in dealing with member countries of varying political hues".

On his return to Cambridge in October 1950 Hodge looked forward to an active period of lecturing and research on the pattern of the prewar years. But this was not to be. In the first place he was now approaching fifty and he no longer felt able to put in the long hours of intensive original work that he had done before the war. The main reason, however, had to do with the plans for the next International Congress. Even before the Harvard congress Hodge had discussed the possibility of having the 1954 congress in the United Kingdom, but there were many difficulties and he was relieved when the Dutch mathematicians arranged the congress in Amsterdam. This only postponed the problem as far as Britain was concerned, and in due course Hodge was responsible for the 1958 congress in Edinburgh. While many others carried the burden of organizing the congress, Hodge initiated and supervised the whole operation, from the early discussions in 1950 until the successful conclusion in 1958. Throughout this period but particularly in 1956–58 it occupied a significant amount of his energy and time.

Hodge's stay at Harvard and his discussions with Zariski had convinced him that he was more likely to make significant contributions on the transcendental side of algebraic geometry. He still had to finish volume III of his joint book with Pedoe but his research interests were directed back to harmonic integrals. He wrote a number of further papers which though not sensational were a steady development of his original ideas. In particular his paper **[32]** on Kähler manifolds of restricted type led a few years later, to Kodaira's final characterization of projective algebraic manifolds. The manifolds singled out by Hodge in **[32]** were, for a few years, known as 'Hodge manifolds' and it is ironic that Kodaira's proof that Hodge manifolds are algebraic should have led to their disappearance.

Hodge also took an interest in the theory of characteristic classes and wrote a paper **[33]** to bridge the gap between the algebro-geometric classes of J. A. Todd and the topological classes introduced by Chern. As a research student I well remember being advised by Hodge to study the papers of Chern and others in this area. He clearly saw the significance of this work at an early stage and subsequent developments have fully justified him.

The early 1950s saw a remarkable influx of new topological ideas into algebraic geometry. In the hands of Cartan, Serre, Kodaira, Spencer and Hirzebruch these led in a few years to spectacular successes, and the solution of many classical problems such as the Riemann–Roch theorem in higher dimensions. It was my good fortune to have been Hodge's research student just at this time. The great revival of transcendental methods provided by sheaf theory and its intimate connection with harmonic forms naturally aroused Hodge's interest. He made strenuous efforts to understand the new methods and eventually saw that they could be applied to the study of integrals of the second kind. At this time (early 1954) he was busy preparing a talk to be delivered in Princeton in honour of Lefschetz's seventieth birthday so he suggested I might try to develop the ideas further and see if they led to a complete treatment of integrals of the second kind. In fact it did not take long to see that one obtained a very elegant and satisfactory theory in this way and I was able to give Hodge a complete manuscript a few days before his departure for Princeton. He was thus able to describe our results (subsequently published in [**41**], [**42**]) at the Princeton conference and we were both gratified by the stir they created.

Hodge himself greatly enjoyed the exciting discussions and the great ferment of ideas at this conference, which he found the most stimulating he had ever attended. At the same time it helped to bring home to him that he was no longer able to keep up with agile minds of people much younger than himself. From this point on he felt that his creative ability was drying up and this feeling no doubt played an important part in his decision to take on the important offices which came to him in subsequent years.

Pembroke and the Royal Society

Hodge's role in the International Mathematical Union and his service on the Council of the Royal Society had kept him in close contact with Royal Society affairs, and in 1957 he succeeded Sir David Brunt as Physical Secretary. Although the post was distinctly time-consuming—he spent three days a week in London—he found the experience enjoyable and worthwhile. He also felt that he was cementing the position of mathematics in the world of science, which he regarded as quite essential. Throughout his time as Physical Secretary he was on excellent terms with his colleagues, particularly with Lord Florey whose position as Provost of Queen's College, Oxford (the sister college of Pembroke, Cambridge) brought him into contact with Hodge in a dual capacity.

In 1958 Hodge was elected Master of Pembroke in succession to S. C. Roberts. Coming as it did just before the Edinburgh congress and on top of his duties at the Royal Society this made 1958 an eventful year, and one which altered the whole tenor of the Hodges' life. They gave up the house in Barrow Road which had been their home for twenty-three years and moved into the Master's Lodge. Here for the next twelve years they led a busy social life entertaining students, Fellows and distinguished visitors. Much of the burden necessarily fell on Lady Hodge whose kindly and unassuming presence helped to put all visitors to the Lodge at their ease.

In College affairs Hodge, as Master, exerted an unassertive but constructive influence; and on occasion he had to take critical decisions. He put much effort into raising funds for the College Appeal but this and other aspects of his life as Master have been well described, on the occasion of his retirement, in the following extract from the *Pembroke College Gazette* of October 1970:

> "Sir William Hodge, who retired this summer, presided over Pembroke's fortunes for twelve years of rapid change. To him it must have seemed almost daily.
>
> "More and more undergraduates, more and more graduates, perhaps this is no longer news; but when the very instrument of government—the Fellowship—more than doubles in twelve years, it surely is. In 1958 there were 24 Fellows: this summer there were 50, of whom Sir William had personally admitted 36. He had also presided over the addition of eight distinguished Honorary Fellows to an already illustrious list, a list which it is good to note is now lengthened and strengthened by Sir William's own name.

"In Cambridge terms he inherited a small College and has handed over a large one; not only large, but in a true sense a College which he himself did much to shape, guide and hold together. The government of an academic community is no sinecure—as has become more public of recent years; and if it can be said that a governing body of half a hundred opinionated academics, of nearly as many disciplines and temperaments, is as nearly united and harmonious and friendly as any in the land, the reason is not far to seek.

"This benign reign was not free of storm clouds; threats to the character of Pembroke were not lacking at University and national level. It is probable that the names Bridges, Robbins, and perhaps a few more, are graven on Sir William's heart; the more honour to him for his steadfastness in identifying and preserving Pembroke's true interests and individuality.

"Crumbling buildings posed a more tangible threat. For only one year of the Master's twelve has he been able to see Pembroke's fair face free of scaffolding. The Old Library, the inside of Wren's Chapel, the Cloisters, some of Old Court, all of Ivy Court and all of Red Buildings were re-made; while a new Junior Parlour, an enlarged Senior Parlour, and more than a 100 extra living rooms outside the walls were added to the College's domain. It is to be hoped that the Master feels that all the dust and rust, the hammering and shouting have been worth while, for he certainly hands over a large College, largely rehoused.

"On his initiative it was decided in 1968, but with some diffidence, to ask past members of the College to help with the funds needed for part of this restoration. Seldom can diffidence have been so quickly dispelled by results—the target reached with ease and nearly half as much again: surely a fitting crown to Sir William's Mastership. His tireless journeyings over the length and breadth of the country; his long, happy evenings with groups of Pembroke men, talking with them and answering questions; his unvarying warmth and optimism must all be remembered with pleasure by those he visited and with gratitude by his colleagues.

"Warmth, good-humour, friendship, and boundless hospitality—in these terms will many generations of grateful undergraduates, Fellows, and past members of the College think of the Pembroke Master's Lodge in the years 1958–70."

One aspect of his joint role as Master of Pembroke, Professor of Mathematics and Secretary of the Royal Society that gave Hodge much satisfaction was the feeling that he was, in all these posts, the successor to Sir George Stokes. A portrait of Stokes hangs in the Pembroke hall and Hodge would frequently point it out with visible pride.

In 1965 Hodge's term as Physical Secretary expired but this was offset by his becoming Head of the newly formed Department of Pure Mathematics and Mathematical Statistics in Cambridge, and in this capacity he was responsible for the move into new premises. In fact the new Department owed its existence and its physical home very much to Hodge's efforts. Almost alone among the older generation of Cambridge pure mathematicians he recognized that the traditional College-based approach was altering and that a University mathematical centre was essential. Ideally he would have preferred all mathematicians, pure and applied, under one roof but his initial efforts in this direction were frustrated by the notorious planning difficulties facing an ancient University.

Not surprisingly the conjunction of all these various responsibilities was taking its toll and Hodge found himself mentally exhausted. In 1970 therefore, on reaching retiring age for the Lowndean chair, he decided to retire also from the Mastership.

In 1973 an International Colloquium on Algebraic Geometry was held in Cambridge in honour of Hodge's seventieth birthday. This gave him great pleasure as did the subsequent award of the Copley Medal of the Royal Society in 1974. Only a few months later he had a severe heart attack and although he appeared to have made a good recovery he suffered a second and fatal attack a short while later. He died on 7 July, 1975.

A Mathematical Assessment

Hodge's mathematical work centred so much round the one basic topic of harmonic integrals that it is easy to assess the importance of his contributions and to measure the impact they made. The theory of harmonic integrals can be roughly divided into two parts, the first dealing with real Riemannian manifolds and the second dealing with complex, and particularly algebraic, manifolds. I shall consider these separately.

For a compact Riemannian manifold (without boundary) Hodge defined a harmonic form as one satisfying the two equations $d\phi = 0$ and $d^*\phi = 0$ where d is the exterior derivative and d^* its adjoint with respect to the Riemannian metric. An equivalent definition proposed later by André Weil is that $\Delta\phi = 0$ where $\Delta = dd^* + d^*d$. Hodge's basic theorem asserts that the space $\mathscr{H}^q$ of harmonic q-forms is naturally isomorphic to the q-dimensional cohomology of X (or dual to the q-dimensional homology). The beauty and simplicity of this theorem made a deep impression. Henry Whitehead once jocularly remarked that he would have sold his soul to the devil for such a theorem. As mentioned earlier, Hermann Weyl, the foremost mathematician of the time, was so impressed that he assisted Hodge with the technicalities of the proof. At the International Congress of Mathematicians in 1954 Weyl said that, in his opinion, Hodge's book on *Harmonic integrals* was "one of the great landmarks in the history of science in the present century".

As an analytical result in differential geometry one might have expected Hodge's theorem to have been discovered by an analyst, a differential geometer or even a mathematical physicist (since in Minkowski space the equations $d\omega = d^*\omega = 0$ are just Maxwell's equations). In fact Hodge knew little of the relevant analysis, no Riemannian geometry and only a modicum of physics. His insight came entirely from algebraic geometry, where many other factors enter to complicate the picture.

The long term impact of Hodge's theory on differential geometry and analysis was substantial. In both cases it helped to shift the focus from purely local problems to global problems of geometry and analysis " in the large ". Together with Marston Morse's work on the calculus of variations it set the stage for the new and more ambitious global approach which has dominated much of mathematics ever since.

One of the most attractive applications of Hodge's theory is to compact Lie groups where, as Hodge himself showed in his book, the harmonic forms can be identified with the bi-invariant forms. Another significant application due to Salomon Bochner showed that suitable curvature hypotheses implied the vanishing of appropriate homology groups, the point being that the corresponding Hodge–Laplacian Δ was positive definite.

If we turn now to a complex manifold X with a Hermitian metric we can decompose any differential r-form ω in the form

$$\phi = \sum_{p+q=r} \phi^{p,q},$$

where $\phi^{p,q}$ involves in local coordinates $(z_1, \ldots, z_n)$ p of the differentials dz_i and q of the conjugate differentials $d\bar{z}_i$ (and is said to be of type (p,q)). In general if $\Delta\phi = 0$, so that ω is harmonic, the components $\phi^{p,q}$ need not be harmonic. It was one of Hodge's remarkable discoveries that for the Mannoury metric on a projective algebraic manifold (induced by the standard metric on projective space) the $\phi^{p,q}$ are in fact harmonic. This property of the Mannoury metric is a consequence of what is now known as the Kähler condition, namely that the 2-form (of type $(1, 1)$) associated to the metric ds^2 by the formula

$$ds^2 = \sum g_{ij} dz_i d\bar{z}_j, \quad \omega = \frac{i}{2\pi} \sum g_{ij} dz_i \wedge d\bar{z}_j$$

is closed (i.e. $d\omega = 0$). As a consequence we obtain a direct sum decomposition

$$\mathscr{H}^r = \sum_{p+q=r} \mathscr{H}^{p,q}$$

of the space of harmonic forms and consequently, by Hodge's main theorem, a corresponding decomposition of the cohomology groups. In particular the Betti numbers $h^r = \dim \ \mathscr{H}^r(X, C)$ are given by

$$h^r = \sum_{p+q=r} h^{p,q}$$

where $h^{p,q} = \dim \mathscr{H}^{p,q}$. Moreover, as Hodge showed, these numbers $h^{p,q}$ depend only on the complex structure of X and not on the particular projective embedding (which defines the metric). In this way Hodge obtained new numerical invariants of algebraic manifolds. As the $h^{p,q}$ satisfy certain symmetries, namely

$$h^{p,q} = h^{q,p} \quad \text{and} \quad h^{p,q} = h^{n-p,n-q}$$

we immediately deduce various simple consequences for the Betti numbers. Thus h^{2k+1} is always even, generalizing the well known property of a Riemann surface, but showing also that many even-dimensional real manifolds cannot carry a complex algebraic structure.

The fact that one obtains in this way the new intrinsic invariants $h^{p,q}$ is a first vindication of the Hodge theory as applied to algebraic manifolds. It shows that the apparently strange idea of introducing an auxiliary metric into algebraic geometry does in fact produce significant new information. For Riemann surfaces the complex structure defines a conformal structure and hence the Riemannian metric is not far away, but in higher dimensions this relation with conformal structures breaks down and makes Hodge's success all the more surprising. Only in the 1950s, with the introduction of sheaf theory, was an alternative and more intrinsic definition given for the Hodge numbers, namely

$$h^{p,q} = \dim H^q(X, \Omega^p),$$

where Ω^p is the sheaf of holomorphic p-forms.

A further refinement of Hodge's theory involved the use of the basic 2-form ω which is itself harmonic. Hodge showed that every harmonic r-form ϕ has a further decomposition of the form

$$\phi = \sum_s \phi_s \omega^s$$

where ϕ_s is an " effective " harmonic $(r-2s)$-form. This decomposition is the analogue of the results of Lefschetz which relate the homology of X to the homology of its hyperplane sections, ω playing the role of a hyperplane.

The theory of harmonic forms thus provides a remarkably rich and detailed structure for the cohomology of algebraic manifolds. This " Hodge structure " has been at the basis of a vast amount of work over the past forty years, and it has become abundantly clear that it will in particular play a key role in future work on the theory of " moduli ".

One problem which Hodge recognized as of fundamental importance is that of characterizing the homology classes carried by algebraic sub-varieties of X. For divisors, i.e. varieties of dimension $n-1$, this problem had been settled by Picard and Lefschetz, and Hodge saw what the appropriate generalization should be. For many years he attempted to establish his conjecture, and it was the one problem which he carried with him into his retirement, but it eluded all his efforts. The conjecture is that a rational cohomology class in $H^{2q}(X, Q)$ is represented by an algebraic sub-variety if and only if its harmonic form is of type (q, q). The necessity of this condition is easy, the difficulty lying in the converse which asserts the existence of a suitable algebraic sub-variety. This " Hodge conjecture " has by now achieved a considerable status, almost on a par with the Riemann hypothesis or the Poincaré conjecture. Its central importance is fully recognized but no solution is in sight, and we may have to wait many years for the answer.

As a lecturer Hodge, like his mentor White, relied more on the intrinsic interest of his material than on the mode of presentation. This no doubt accounts for the comparatively small number of research students he attracted in comparison with such a virtuoso performer as Philip Hall. For those more persistent students, however, who were prepared to ignore the lack of polish, there was a wealth of solid mathematics to be found in Hodge's lectures on differential and algebraic geometry. The diligent student also needed to be an early riser, because Hodge's favourite hour for lecturing was always nine o'clock. This early start, which left the rest of the day free for other matters, was typical of Hodge's energetic approach to life.

A Personal Appreciation

Despite the high offices he held Hodge was always modest and unassuming. He got on easily with people in all walks of life and was far removed from the traditional picture of the eccentric mathematician. His easy good-nature was no doubt one of the reasons why he was so much in demand in College and University affairs, though beneath the genial exterior there lurked the shrewd Scot whose business acumen was much respected.

Hodge was one of those who thrive on a busy and varied life. He was rarely weighed down by his responsibilities but instead enjoyed the challenge which they presented. Fortunately, his health was good until his very last years and he had a happy and stable domestic life. Lady Hodge took good care of him in a quiet but efficient way and kept his exuberance under control. He took a close interest in his two children of whom the elder, Michael, became vicar of Cobham, Kent, and his daughter Gillian became Superintendent of Physiotherapy in Newmarket General Hospital. In his later years he derived much pleasure from his four grandchildren.

Hodge's enjoyment of life had a certain boyish streak which is illustrated by the following anecdotes, which I owe to Wilkie Rowe. After a dinner party in the Master's Lodge a grand new dishwasher, which had just been installed, was started up. It was going well until Hodge's curiosity was aroused and he lifted the lid to see how it was working—the results can be imagined, but fortunately no one was hurt. On another occasion, on the spur of the moment, he bought a large photographic screen and marched proudly back to the Master's Lodge with the five-foot tube carried prominently over his shoulder.

His devotion to mathematics was never allowed to intrude too far into his personal and family life. This was made clear during the stay at Harvard in 1950, when the Hodges used to dine every Sunday at the Zariski's house. After dinner Professor Zariski would suggest to Hodge that they should adjourn to the study to discuss mathematics, but Hodge resisted these attempts saying that he had come to see the Zariski family and not just Oscar. This attitude was much appreciated by Mrs. Zariski and the rest of the family.

His relation with his research students was friendly and informal, and he was always accessible, however busy he might be in other ways. When the time came for my Ph.D. oral examination, an occasion which can produce tremors in the hardiest of breasts, he suggested we should move into the Pembroke garden, and the examination was conducted in amicable style with the two examiners and myself sharing a garden bench.

Honours

Hodge's standing in mathematics is indicated by the many honours he received both in this country and abroad. He was knighted in 1959 and received an honorary LL.D. from his home University of Edinburgh at the time of the 1958 International Congress. He also received honorary degrees from the Universities of Bristol, Leicester, Sheffield, Exeter, Wales and Liverpool. He was elected an honorary member of the following:

U.S.A. National Academy of Sciences

American Academy of Arts and Sciences

American Philosophical Society

Akademie der Wissenschaften in Göttingen

Kongelige Danske Videnskabernes.

The London Mathematical Society awarded him the Berwick Prize and the de Morgan Medal, while the Royal Society of Edinburgh, which elected him to a Fellowship in 1928, presented him with the Gunning Victoria Jubilee Prize some thirty years later.

Hodge received the Royal Medal of the Royal Society in 1957 and the Copley Medal in 1974. Both of his Cambridge Colleges, Pembroke and St. John's, elected him to an honorary Fellowship.

In preparing this memoir I have been greatly assisted by the extensive personal record left by Sir William. In addition I am indebted to Lady Hodge, the Master of Pembroke, Dr. Harold Taylor, Mr. W. Rowe, Sir David Martin, Professor Zariski and Professor Chandrasekharan.

The photograph is by Elliott and Fry.

Bibliography

Articles

1. " Linear systems of algebraic curves on a plane and on a cone ", *Proc. Lond. Math. Soc.*, (2), 29 (1928), 145–167.
2. " The isolated singularities of an algebraic surface ", *Proc. Lond. Math. Soc.*, (2), 30 (1929), 134–143.
3. " On multiple integrals attached to an algebraic variety ", *J. Lond. Math. Soc.*, 5 (1930), 283–290.
4. " Further properties of abelian integrals attached to algebraic varieties ", *Proc. natn. Acad. Sci. U.S.A.*, 17 (1931), 643–650.
5. "A Dirichlet problem for harmonic functionals, with applications to analytic varieties ", *Proc. Lond. Math. Soc.*, (2), 36 (1932), 257–303.
6. " Harmonic functionals in a Riemannian space ", *Proc. Lond. Math. Soc.*, (2), 38 (1933), 72–95.
7. "Abelian integrals attached to algebraic varieties ", *Math. Gaz.*, 17 (1933), 303–306.
8. " The geometric genus of a surface as a topological invariant ", *J. Lond. Math. Soc.*, 8 (1933), 312–319.
9. " Harmonic integrals associated with algebraic varieties ", *Proc. Lond. Math. Soc.*, (2), 39 (1935), 249–271.
10. " Some theorems on abelian integrals associated with an algebraic variety ", *Proc. Camb. Phil. Soc.*, 31 (1935), 18–25.
11. " The existence theorem for harmonic integrals ", *Proc. Lond. Math. Soc.*, (2), 41 (1936), 483–496.
12. "A theorem on algebraic correspondences ", *Proc. Camb. Phil. Soc.*, 32 (1936), 337–341.
13. " Note on the theory of the base for curves on an algebraic surface ", *J. Lond. Math. Soc.*, 12 (1937), 59–63.
14. " On the stationary points of integrals attached to algebraic varieties ", *J. Lond. Math. Soc.*, 12 (1937), 280–290.
15. "An extension of Poincaré's theorem on defective integrals ", *Proc. Lond. Math. Soc.*, (2), 44 (1938), 216–225.
16. "Algebraic correspondences between surfaces ", *Proc. Lond. Math. Soc.*, (2), 44 (1938), 226–242.
17. " A new set of relative birational invariants of algebraic varieties ", Reale Accademia d'Italia (1939).
18. " The base for algebraic varieties of given dimension on a Grassmannian variety ", *J. Lond. Math. Soc.*, (1941), 245–255.
19. "A note on *k*-connexes ", *Proc. Camb. Phil. Soc.*, 38 (1942), 129–143.
20. " Note on the degeneration of algebraic varieties ", *Proc. Camb. Phil. Soc.*, 38 (1942), 231–233.
21. " The intersection formulae for a Grassmannian variety ", *J. Lond. Math. Soc.*, 17 (1942), 48–64.
22. " Some enumerative results in the theory of forms ", *Proc. Camb. Phil. Soc.*, 39 (1943), 22–30.
23. Tullio Levi-Civita (obituary notice). *Obit. Not. Fell. R. Soc. Lond.* 4 (1942), 151–165. (Reprinted with modifications, in *J. Lond. Math. Soc.* 18 (1943), 107–114.)
24. " On multiple curves I, II, III ", *Proc. Camb. Phil. Soc.*, **41** (1945), 111–117 (pt I), 117–126 (pt II) 42 (1946), 11–14 (pt III).
25. " Note on the conditions for a *p*-cycle of an algebraic manifold to be of rank *k* ", *Proc. Camb. Phil. Soc.*, 43 (1947), 577–580.
26. " Harmonic integrals on algebraic varieties ", *Proc. Camb. Phil. Soc.*, 44 (1948), 37–42.
27. " The finite algebraic form of the theory of harmonic integrals ", *Proc. Symp. Brussels* (1948), 43–54.
28. Federigo Enriques (obituary notice). *Yb. R. Soc. Edinb.* (1946–48).
29. " On the topology of three-folds whose hyperplane sections have geometric genus zero ", *Annali Mat. pura appl.* (4), 29 (1949), 115–119.
30. " Some recent developments in the theory of algebraic varieties,", *J. Lond. Math. Soc.*, 25 (1950), 143–157.
31. " The topological invariants of algebraic varieties ", *Proc. Int. Congr. Math.*, (1950), 182–192.
32. " A special type of Kähler manifold ", *Proc. Lond. Math. Soc.*, (3) 1 (1951), 104–117.
33. "The characteristic classes on algebraic varieties ", *Proc. Lond. Math. Soc.*, (3) 1 (1951), 138–151.
34. " Differential forms on a Kähler manifold ", *Proc. Camb. Phil. Soc.*, 47 (1951), 504–517.
35. " Tangent sphere-bundles and canonical models of algebraic varieties ", *J. Lond. Math. Soc.*, 27 (1952), 152–159.
36. " Structure problems for complex manifolds ", *Rc. Mat. Applic.* (5) 11 (1952), 1–10.
37. Elie Cartan (obituary notice). *J. Lond. Math. Soc.*, 28 (1953), 115–119.
38. Guido Castelnuovo (obituary notice). *J. Lond. Math. Soc.*, 28 (1953), 120–125. (Tribute on the centenary of his birth in 1965, *Simposio int. di Geometria algebrica, Roma*, (1965), 54–55.)
39. " Differential geometry and the theory of algebraic varieties ", *Conv. int. Geom. diff., Roma* (1953), 1–8.
40. Professor Lefschetz's contributions to algebraic geometry: an appreciation, *Algebraic Geometry and Topology* (1954), 3–23.

41. (With M. F. Atiyah) " Formes de seconde espèce sur une variété algébrique ", *C. r. hebd. Séanc. Acad. Sci., Paris* 239 (1954), 1333–1335.
42. (With M. F. Atiyah) " Integrals of the second kind on an algebraic variety ", *Ann. Math.*, 62 (1955), 56–91.
43. " A note on the Riemann-Roch theorem ", *J. Lond. Math. Soc.*, 30 (1955), 291–296.
44. " Changing views of geometry ", *Math. Gaz.*, 39 (1955), 177–183.
45. Henry Frederick Baker (obituary memoir), *Biog. Mem. Fellows R. Soc. Lond.*, 2 (1956), 49–68. (Reprinted with modifications in *J. Lond. Math. Soc.*, 32 (1957), 112–128. Shorter notice in *Nature, Lond.* 177 (1956). Biography in *DNB.*)
46. Peter Fraser (obituary notice), *J. Lond. Math. Soc.*, 34 (1959), 111–112.
47. Oswald Veblen (obituary notice), *J. Lond. Math. Soc.*, 36 (1961), 507–512.
48. " Differential forms in algebraic geometry ", *Rc. Mat.*, 20 (1961), 172–234.
49. " The Riemann-Roch theorem ", Seminario dell'Istituto Nazionale di Alta Matematica (1962–63), 17–28.
50. Joint Mathematical Council for the United Kingdom. *Math. Gaz.*, 47 (1963), 130–131.
51. " The topology of varieties in complex projective space ", *Rc. Mat.*, 25 (1966), 1–16.
52. Francis Puryer White (obituary notice). *Bull. Lond. Math. Soc.*, 3 (1971), 82–84.
53. Solomon Lefschetz (obituary memoir). *Biog. Mem. Fellows R. Soc. Lond.*, 19 (1973), 433–453. Reprinted *Bull. Lond. Math. Soc.*, 6 (1974), 198–217.

Books

54. *The theory and applications of harmonic integrals* (Cambridge University Press. 1st ed. 1941; 2nd ed. 1952, reprinted 1959.)
55. (With D. Pedoe) *Methods of algebraic geometry.* (Cambridge University Press. Vol. 1 1947, reprinted 1959, 1968; vol. 2 1952, reprinted 1968; vol. 3 1954.)