ADOLF HURWITZ.

JUST a week after the signing of the Peace, there passed away in the person of Adolf Hurwitz one of the most notable representatives of contemporary German mathematical science. Although a Jew by parentage, and for no less than twenty-seven years Professor at the Swiss Technische Hochschule, he retained his German nationality to the end. A product of the German academic system at its best, he can never have felt the temptation, to which so many of his countrymen have yielded, to change, even nominally, his nationality.

Born at Hildesheim in the year in which Riemann became Professor at Göttingen, Hurwitz entered the Andreanum little more than eighteen months after Riemann's untimely death, and was, before he had quite reached his eighteenth year, already at Munich attending the lectures of Klein, the most genial exponent of Riemann's ideas. A year later, he was at Berlin, in the mathematical Seminar, and gaining at first hand, from Weierstrass and Kronecker, a knowledge of the methods in which they were passed masters. But Hurwitz was to be above all a pupil of Klein, and, after three semesters spent at Berlin, we find him once more at Munich, and in October 1880 following Klein thence to Leipzig.

Untrammelled by examinations, Hurwitz was able, even when at Berlin, to collaborate with Klein and to afford him help in one of his most notable papers on elliptic modular functions,* a paper destined, with others of Klein's, almost equally remarkable, to be for many years the pivot on which Hurwitz's mathematical interests were to turn. Hurwitz was peculiarly fitted to carry out Klein's ideas.† He had gained from Schubert,‡ of Abzählende Geometrie fame, his master at the Andreanum, an interest

* *Math. Ann.*, Vol. 17, pp. 69 and 70. Some idea of the advance due to Klein may be gained by comparing the papers just referred to with H. J. S. Stephen's *B. A. Report*, 1865.

† Klein's strength, it may be remembered, was sometimes regarded as consisting still more in the fertility and the genial character of his ideas than in the power of developing them. Cf. Lie, *Transf. Gruppen.*

‡ We are told that Schubert gave up part of every Sunday to working at Geometry with the schoolboy Hurwitz, and the first of the latter's papers, written when he was still at the Andreanum, was a joint paper. It was also Schubert who persuaded Hurwitz's father to allow him to go to the University and who sent him with warm recommendations to Klein at Munich.

in Geometry and a familiarity with geometrical methods which were bound to serve him in good stead with Klein, and he had already entered on the field of original research. On the other hand, we have Hilbert's authority for the statement that the acquisition of Riemannian ideas, which intercourse with Klein rendered possible, of itself constituted at that time a transfer, so to speak, to a higher class among mathematicians. It is not surprising, then, that we find Hurwitz, a Göttingen *Privat-docent* of barely two years' standing and not yet 25, called in 1884 to Königsberg as *Extraordinarius*, with a record of important published work behind him.

At Königsberg he made the acquaintance of Hilbert, first the student and then the *Privatdocent*, and of Minkowski, whose family lived there, and who, when at home from Bonn for the holidays, joined them in their almost daily walks. During these walks, continued over the whole of the eight-year period of Hurwitz's residence at Königsberg, wellnigh every corner of the then known mathematical world was explored.*

We get some glimpse, incidentically, in studying Hurwitz's career, as to the way in which a professional mathematician may be formed.

From Königsberg, Hurwitz went to Zürich, where he remained until his death.

About a hundred papers were published by Hurwitz. In almost all of them, the influence of Klein,† direct or indirect, is perceptible, and many of them may be characterized as solutions, usually complete, of problems, of a fundamental nature and of no small difficulty, proposed by

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* Cf. Hilbert, *G. N.*, 1920. Hilbert adds: "Hurwitz mit seinen ebenso ausgedehnten und vielseitigen wie festbegründeten und wohlgeordneten Kenntnissen war uns dabei immer der Führer.

† How much Hurwitz owed to Dedekind also is evident from his papers and from his own acknowledgments. But it would seem that they had never met, at any rate not before 1895. There is an interesting indication of this in Dedekind's answer to a question as to what he thought of the paper "Über die Theorie der Ideale" (*G. N.*, 1894), the first of Hurwitz's attempts in this direction. Dedekind explains that the mode of treatment of the fundamental theorem in the theory of Ideals there exposed, and based indeed on an algebraical lemma of his own, had been familiar to him for many years, and he gives in detail his reasons for not having adopted it. He had since found and published in Dirichlet's *Zahlentheorie* what he regarded as a much more natural and simple way of building up the subject. He quotes Gauss's "Ausspruch eines grossen Wissenschaftlichen Gedanken, 'die Entscheidung für das Innerliche im Gegensatz zu dem Aeusserlichen'," and then continues: "Hiernach wird man es auch erklärlich finden, dass ich meiner Definition des Ideals durch eine charakteristische innerliche Eigenschaft den Vorzug gebe vor derjenigen durch eine äusserliche Darstellungsform, von welcher Herr Hurwitz in seiner Abhandlung ausgeht. Aus denselben Gründen konnte der .... Beweis des Satzes .... mich noch nicht völlig befriedigen, weil durch die Einmischung der Functionen von Variabelen die Reinheit der Theorie nach meiner Ansicht getrübt wird." (*Göttinger Nachrichten*, 1895, p. 111.)

Klein. In some cases, the results obtained or the methods employed have an importance far beyond what we know or may presume to have been the occasion for writing them. In particular, the paper "Über algebraische Correspondenzen,"* may be referred to in this connection. Brill had succeeded in proving a theorem of the truth of which Cayley had persuaded himself by inductive processes, without being able however to devise anything of the nature of a demonstration save in a very special case. Hurwitz's work goes far beyond Brill's in generality,† besides being above all remarkable as an application, promised nine years before, of Abelian integrals to Geometry, and as the point of departure of Castelnuovo in his investigations on analogous matters in the theory of surfaces. And it may be said to generalise Abel's Theorem itself.

Other pairs of papers that have become classical are those entitled "Über algebraische Gebilde mit eindeutige Transformationen in sich,"‡ and "Über Riemannsche Flächen mit gegebenen Verzweigungspunkte"§; the second pair are also interesting because they show Hurwitz first failing to obtain the complete solution of the problem, taking up the thread ten years later in the light of a happy suggestion from Lasker, the international Chess Champion, met in the previous summer, and finally completing the solution by the use of a method‖ in the theory of abstract groups discovered in the meanwhile by Frobenius, Hurwitz's predecessor at Zürich.

Of the papers not more or less directly inspired by Klein, among the most original are those on the roots of algebraic and transcendental equations. Hurwitz was a recognised expert in the treatment of problems of this nature. His paper on the zeros of Bessel's functions,¶ which already marked a strikingly new departure, both as regards the methods employed and the character of the results obtained, was followed rapidly by several others on the roots of transcendental equations.** And when, soon after he had gone to Zürich, one of his Swiss colleagues turned to him for help

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* *Math. Ann.*, Vol. 28.

† Hurwitz was thus able to repay with interest a debt of Klein to Cayley (*Math. Ann.*, Vol. 17, p. 66). It was in studying the theory of modular correspondences that Hurwitz was led to consider the necessity of investigating correspondences defined by more than one equation on entities of genus p.

‡ *Math. Ann.*, Vols. 32, 41.

§ *Math. Ann.*, Vols. 39, 55.

‖ This method of Frobenius is also interesting to English readers as being closely connected with some of the most important of Burnside's work.

¶ *Math. Ann.*, Vol. 33.

** No fewer than seven papers of Hurwitz's deal with roots of equations in a technical problem involving the conditions under which an algebraic equation has the real part of its roots all negative, the skill shown by Hurwitz in furnishing the complete solution was noteworthy. Simple as are the conditions arrived at,* namely that certain determinants formed out of the coefficients of the equation have to be positive, the resources Hurwitz disposes of are seen in this, as in so many others of his papers, to be of the most varied description. He avails himself with equal freedom of the ideas and results of Sturm, of Hermite, of Frobenius, of Kronecker, and of Cauchy.

His papers on continued fractions and on the approximate representation of irrational numbers are also very original, as well as curious. And all his algebraical work is marked by a rare insight into underlying principles. One of Hurwitz's greatest triumphs was his complete solution of a question concerned with the reducibility of quadratic forms of any number of variables, a part of which had baffled the united efforts of a Cayley and a Roberts, equipped though they might be with all the resources of an empirical science and of a power of calculation that shunned no labour.†

But perhaps Hurwitz's main interests really lay in the theory of numbers as a whole, including that part of it which attaches itself naturally to the theory of modular functions, such as the relations connecting numbers of classes of quadratic forms. On this latter subject he wrote seven papers, and one of the earliest of his papers written independently was devoted to the proof that a theorem of Stieltjes, giving the number of modes of expressing a prime as the sum of five squares, holds in a generalised form for every integer. The interest of four others lies in their connection with the theory of ideals. And of his last sixteen papers,‡ almost all those whose interest is not chiefly pedagogic, were devoted to the solution of Diophantine equations and analogous problems,

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* *Math. Ann.*, Vol. 46.

† Hurwitz's account of the matter is worth quoting:—"Roberts und Cayley haben sich im 16ten und 17ten Bande des *Quarterly Journal* mit den Nachweis beschäftigt, dass ein Product von Zwei Summen von je 16 Quadraten nicht als Summe von 16 Quadraten darstellbar sei. Ihre äusserst mühsamen auf Probiren beruhenden Betrachtungen besitzen indessen keine Beweiskraft, weil ihnen bezüglich der bilinearen Formen $z_1, z_2, \ldots$ specielle Annahmen zu Grunde liegen, die durch nichts gerechtfertigt sind." (*Gött. Nach.*, 1898, p. 310, Note 1.)

‡ It is noteworthy that in one of these later papers he concerns himself with an equation first employed by Klein in investigations (*Math. Ann.*, Vols. 14, 15) with which Hurwitz's dissertation was connected.

while his only publication in book form is a reprint of one of his papers on "Quaternion Theory of Numbers."*

Brilliant as Hurwitz's researches were known to be, he was honoured at Zürich most as a teacher, and the tradition of his success there is likely to be long preserved.

But he loved to employ the deductive method of exposition, alike in his writings and in his lectures. His papers even weary by their completeness, although this is almost always atoned for by a finished elegance of form. And if but few are merely didactic, a relatively large proportion are concerned with new proofs of known theorems;† while of his pupils, only those in close personal contact with him can have been able to form a just idea of the processes by which he was led to his results. Perhaps had he been less successful as a teacher, he might have been better able to found a great school of mathematics.

Hurwitz always remained a nineteenth century mathematician. One of the first, if not the very first, to utilise Cantor's theorem on the non-countability of the continuum,‡ and possessed, as he several times showed, of an acquaintance with, and the ability to apply, the elements of the theory of sets of points, he had to content himself with appreciating the nature, without fully grasping the magnitude of the revolution brought about in mathematical analysis by the extension of our knowledge of the Real Variable, so characteristic of the century in which we live. The very thoroughness of his early preparation may indeed have rendered him inapt or unwilling to do more than skirmish on ground§ relatively unfamiliar to him, and it is noteworthy that he did not follow Poincaré in the exploitation of the generalisation of the elliptic modular function constituted by the automorphic function. Indeed, though in an improved version‖ of a portion of his dissertation, published in later years, he remarks that the methods he employs are obviously applicable to automorphic functions, his sole

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* *Gött. Nach.*, 1896.

† The interest of these proofs is undeniable, and some have become classical. In one the motive is the desire to give a purely algebraic proof of an algebraic theorem previously established with extreme facility by the use of the Calculus.

‡ *Crelle*, Vol. 95.

§ His interest in new work is shown in more than one of his later papers—for example, in his use of Fejér-Cesàro methods in dealing with Fourier series, in his proof of a theorem of Fatou, and in the application of his old modular elliptic function equipment to a new proof of Landau's extension of Picard's theorem.

‖ *Math. Ann.*, Vol. 58.

contribution to that subject is a paper* in which he shows how the fundamental region may be determined for automorphic functions of any number of variables.

Though he was educated in the Real Gymnasium† section of the Andreanum, and though, curiously enough, the friend of his father to whose benevolence he owed his University career, and to whom he dedicated his dissertation, had the English or at least British name of Edwards, Hurwitz was not sufficiently master of our language to be able to read English mathematical papers, except with very great difficulty. His knowledge of the work of English writers was indirect. But he had great familiarity with French and several of his papers are written in this language. The value of his work was appreciated outside Germany and Switzerland. Some of his papers written in German were translated into other languages, and he was elected honorary or corresponding member of several learned bodies. He became an honorary member of our Society in November 1913.

Hurwitz was very generally liked, not only as a teacher, but as a man, and this in spite of the fact of his life being one long struggle‡ with a wasting disease, which must have rendered him little disposed for social intercourse. In points of honour he was punctilious. It fell to his lot to be anticipated in several of his results, and he was never slow to publish his recognition of the priority of another. And it is related that nothing but his refusal to break his word pledged to the Swiss Schulratspresident, who had secured him for the Technische Hochschule by travelling all the way from Zürich to Königsberg for the purpose, prevented his going to Göttingen as *Ordinarius* in succession to H. A. Schwarz. How great a sacrifice this entailed will be realised if it be borne in mind that a Chair in a Swiss University was, for a German, never, before the War, regarded as more than a stepping stone to a Chair in Germany. That the call

* *Math. Ann.*, Vol. 60.

† It was owing to this circumstance that he did not become *Privatdocent* at the University of Leipzig, as he, a pupil of Klein's, then holding a Chair there, would naturally have done. Evidence of a knowledge of Greek was regarded by the Philosophical faculty of Leipzig as an indispensable requisite for the *venia legendi*. Göttingen was, as would now be said, more advanced in its ideas.

‡ That this struggle was waged with comparative success for so many years appears almost incredible, and can only be accounted for by the constant care and devotion of Hurwitz's wife, a daughter of Professor Samuel, a well known member of the Medical faculty at Königsberg.

was not repeated at a later date may be attributed to the state of Hurwitz's health, supposed to render such a call undesirable.*

W. H. Y.

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* Since the above notice was in print, I have received the following statement from Dr. Vermeil, Klein's assistant, written at Klein's request. It confirms in various points what is given above, and adds some details of interest: "Hurwitz hat als Schüler von Schubert schon als Secundaner Resultate im Gebiete der abzählenden Geometrie gefunden. Als er dann im Sommer 1877 nach München kam, stellte ihm Klein sofort die Aufgabe, die Resultate der abzählenden Geometrie auf zuverlässige Grundlagen zu stellen. Leider aber erlitt Hurwitz sehr bald einen Typhusanfall (der Typhus grassierte damals in München), und kehrte darum erst nach mehreren Semestern nach München zurück, wo er die beste Hilfe von Klein im Ausbau der Theorie der elliptischen Modulfunktionen wurde. Seine Leipziger Dissertation, die in den *Math. Ann.* erschienen ist, ist nicht nur durch die selbständige Entwicklung der Eisenstein'schen Methoden bemerkenswert, sondern insbesondere dadurch, dass er $\sqrt[12]{\Delta}(\omega_1, \omega_2)$ als eine Kongruenzform 12ter Stufe erkannte und dadurch die einfachste Grundlage für die neuen Multiplikatorgleichungen schuf. Inzwischen hatte Gierster aus den von Klein gefundenen Modulargleichungen höherer Stufe neue Zahlentheoretische Resultate, zum Teil auf induktivem Wege, abgeleitet, und es bleibt eine der grössten Leistungen von Hurwitz, durch die Theorie der zugehörigen überall endlichen Integrale, die Gierster'schen Resultate endgültig begründet zu haben und überhaupt eine allgemeine Theorie der algebraischen Korrespondenzen auf algebraischen Kurven begründet zu haben. Später machte die räumliche Trennung die Beziehung Zwischen Klein und Hurwitz seltener. Aber Klein wünscht die Förderung an zuerkennen, die Hurwitz der Theorie der endlichen Gruppen linearer Substitutionen von der Theorie der elliptischen Modulfunktionen her erteilt hat. Hurwitz war wesentlich ein zahlentheoretisches Talent und ergänzte dadurch die mehr intuitive Art von Klein auf glückliche Weise."