Bull. London Math. Soc. 36 (2004) 119–129
DOI: 10.1112/S0024609303002443

OBITUARY

AUBREY WILLIAM INGLETON (1920–2000)

Aubrey Ingleton was born in Chester on 14 August 1920. His father William Ingleton was an accountant, whose father and paternal grandfather had been silversmiths and glass cutters.

Aubrey was educated at Tollington School in Muswell Hill, North London. After leaving school in 1937, he entered the Civil Service, an item in the local paper at that time noting,

> Congratulations to 16-year-old Aubrey William Ingleton, prefect at Tollington School Muswell Hill, who has been placed first out of 7,371 candidates in the recent Civil Service examination, general clerical class. Aubrey has had

> a brilliant school career and is exceptionally good at Mathematics. When he took the examination to enter Tollington from Tollington Preparatory School he gained 100 per cent. for his arithmetic paper.

In the Civil service, he worked initially in the Inland Revenue and then, from 1939, in the District Branch of the Ministry of Health. His career in the Civil Service was interrupted by the war, and in 1941 he was seconded to the Fleet Air Arm (Air Radio Branch), where he did research on radar. After the War he returned to the Ministry of Health, but by now he had decided that he wanted to pursue mathematics.

In 1946, Ingleton enrolled at the Northern Polytechnic (later to become the North London Polytechnic, and currently London Metropolitan University). Three years later he obtained a First Class external London BSc and received the Lubbock Prize, an award for 'the most meritorious candidate obtaining First Class Honours in Mathematics' in the University of London. He was also awarded the Sherbrooke Prize for his performance in the Final Examination. To have won these prestigious prizes as an external student was in itself quite exceptional.

In 1949, Ingleton began graduate research at King's College London, working in p-adic analysis under the supervision of Tony Ruston. George Temple had resumed as Head of the Mathematics Department after the war; other members of the Department with whom Ingleton interacted included Jack Semple, Richard Rado and Bernard Scott. Scott, who was a major source of exchange of ideas between colleagues, shared a room with Ruston. The King's Mathematics Department was known throughout the College and the University, not only for its academic excellence, but also for its informal and friendly atmosphere. There was one, short, departmental meeting a year, after the end of the summer term, when people would volunteer for the courses to be given the next year. The emphasis was very much on how to provide each member of staff with as much time as possible for research. There was also a strong tradition that anyone who had given a ten o'clock lecture (or who was to give one at eleven) would go down to the refectory to have coffee with the undergraduates.

Ingleton completed his doctorate in just two years, spending much of this period developing properties of normed linear spaces over non-Archimedean fields. Whereas it might be said of Ruston that he was totally committed to the machinery of analysis, the actual results playing a subsidiary role, for Ingleton – though everything had to be right – it was more important for there really to be something there to prove! He generally avoided results that could be taken over with little or no modification from the standard Banach space theory, and focused mainly on the differences. This work (published in [**1**]) was largely concerned with necessary and sufficient conditions for the Hahn–Banach theorem. To describe this, let K be a field with a non-trivial non-Archimedean valuation and let E be a normed linear space over K where the norm is a non-negative real-valued function with the usual properties. Ingleton observed that a necessary condition for the Hahn–Banach theorem to hold for linear functionals on subspaces of E is that E be non-Archimedean (that is, satisfy the strong triangle inequality: $\|x+y\| \leqslant \max(\|x\|, \|y\|)$ for all $x, y \in E$) and showed that in order for the Hahn–Banach theorem to hold in every non-Archimedean space over K, it is necessary and sufficient that the intersection of any totally ordered collection of spheres in K be non-empty. During his time at King's, Ingleton regularly attended the London Geometry Seminar that had recently been founded by Semple at King's and other London geometers such as Roth, Wren, Archbold and Scott. Although a retiring and quiet person, his contributions were numerous and invariably impressive.

In 1951, Ingleton was appointed to a lectureship at Birkbeck College London. Cyril Offord had been appointed Professor and Head of the Birkbeck Mathematics Department in 1948. This department had always been small and somewhat ramshackle, and the war had done nothing to improve matters. The fact that Offord had a distinguished background (he had written five influential papers with Littlewood and became an FRS in 1952) raised the profile of the Department and helped him, during his 18-year tenancy, to secure some high-calibre members of staff. These included Hugh Dowker, David Cox and (just after Ingleton left) Roger Penrose, as well as a string of good analysts. Dowker (1950) was Offord's first major appointment. He was a Canadian in his late thirties, a brilliant Princeton topologist who wanted to leave McCarthy's America and accepted Offord's proposal that he should fill the vacant Readership in Applied Mathematics. His high reputation in America began, over the next few years, to give the Department its first experience of international standing – at least among topologists. Ingleton was Offord's second appointment. Assistant lecturers at small UK institutions with well-connected heads-of-department were generally recruited through the grapevine, and Offord will have heard about Ingleton, probably from Ruston. As an analyst, Offord will have particularly appreciated a geometer (which the Department lacked) whose first research topic had been a generalized Hahn–Banach theorem. The Department was friendly and very pleasant to work in; though (except for the analysts, who organized a seminar) it was too small and heterogeneous to generate much productive interaction. However, the proximity of King's and University College largely compensated for this. The geometry in the London Honours degree, which (at least initially) took up most of Ingleton's teaching time, was predominantly the projective theory of quadrics, bits about twisted cubics and some classical differential geometry.

In 1952, Aubrey married Joan Bremner, a family friend, and their daughter Annette was born in 1957.

Many of Ingleton's publications arose out of discussion with colleagues, from which he would quickly recognize the essential underlying features and be keen to set things in an appropriately generalized context. Characteristic of this was his note in *Nature* [**2**], in which he gave a substantial generalization of an existing derivation of the Lorentz transformation from its group of properties. The impetus for this work had been a conversation at King's with Clive Kilmister, who was writing, with Geoffrey Stevenson, a book ⟨**15**⟩ on special relativity, an area somewhat remote from Ingleton's main specialisms.

Ingleton's work on the rank of circulant matrices arose from conversations with Richard Cooke and Paul Vermes, colleagues at Birkbeck. Cooke's research interests could perhaps be summed up by the titles of his two substantial books, *Infinite matrices* and *Linear operators*, and Vermes was extending work by R. H. C. Newton on regular matrix methods of summability of divergent periodic sequences. In [**3**], Ingleton considers the problem of choosing $\{c_{ij}\}$, for given n and $c_{ij} = a_{i-j}$ with the sequence $\{a_k\}$ of period exactly n ((c_{ij}) then being a non-recurrent circulant matrix), such that the rank of (c_{ij}) is minimal. The paper is mainly concerned with matrices of 0's and 1's (as in Newton's work involving periodic sequences of 0's and 1's ⟨**10**⟩) where the minimum rank is determined for a certain class of values of n, and upper and lower bounds are given in the general case. For matrices with arbitrary rational coefficients, if $n = {p_1}^{\alpha_1} \dots {p_m}^{\alpha_m}$, the minimum rank is shown to be equal to $\sum_{\mu=1}^{m} {p_\mu}^{\alpha_\mu - 1}(p_\mu - 1) - \eta(n)$, where $\eta(n) = 1$ if $n \equiv 2 (\text{mod } 4)$, $n > 2$, and $\eta(n) = 0$ otherwise.

Ingleton's paper [**4**], published in 1959, was one of the very early papers on what is now the established area of matroid theory. It was based on an approach initiated by Richard Rado ⟨**12**⟩. This used the concept of I-functions, which were functions on *sequences* such that $I(x_1, \dots, x_n) = 0$ or 1 depending on whether or not the set $\{x_1, \dots, x_n\}$ was (abstractly) dependent or independent. Since an axiom of Rado was that $I(x, x) = 0$, these I-functions were really set functions and correspond exactly with Whitney's earlier theory of abstract dependence (or matroids) ⟨**16**⟩. In [**4**], Ingleton introduced a J-function which was an extension of the I-function and was indeed a genuine function on sequences. Thus, in a sense the J-function is a precursor of the now well-developed theory of greedoids introduced by Korte and Lovász in 1981 ⟨**7**⟩, though this does not appear to have been widely recognized.

During his time at Birkbeck, Ingleton continued to attend the London Geometry Seminar and various other London seminars, on algebra, geometry and topology. The middle 1950s was the era of the pioneering work of Borel, Hirzebruch, Chern, Serre, Grothendieck *et al.*, which, with the introduction of fibre bundles, sheaves, sheaf cohomology and homological algebra, was transforming the landscape of algebraic geometry. To study the new ideas and techniques, Ingleton, Bernard Scott and Scott's research student John Reeve got together to read Hirzebruch's book ⟨**5**⟩, and it was this that inspired Ingleton and Scott's interest in tangent and flag bundles. As was well known, on an algebraic variety V of dimension d, there is in general associated with a set of linear systems of hypersurfaces of total dimension d, a Jacobian variety (of dimension $d-1$), the locus of points at which the hypersurfaces of the linear systems have a common tangent line. In [**5**], Ingleton and Scott showed that this generalizes to a set of linear systems of total dimension $d + r$ $(0 \leqslant r < d)$, the generalized Jacobian then being of dimension $d - r - 1$, and obtained a general formula for the homology class of this Jacobian variety, considered as a cycle on V. Much of the interest of this paper lies in the cohomological and bundle-theoretic techniques established and applied to the tangent direction bundle of V. Subsequently, in [**7**] and [**8**], Ingleton announced a comprehensive definition of Jacobian subvarieties of an algebraic variety V, involving a number of nests of linear systems of primals on V and contact conditions expressed in terms of tangent flags to V. This definition included the classical Jacobian in its most general form ⟨**9, 14**⟩ and the 'generalized Jacobian' of [**5**] as very special cases. He showed that the cohomology class of such a Jacobian could be computed using the structure of the cohomology ring of the tangent flag bundle V^Δ of V, and gave an explicit formula for the cohomology class in a comparatively simple case that was still very much wider than the classical. Full details of this work, with proofs, were not published until the appearance of a series of papers with his student Samuel Ilori [**21–23**] some ten years later. Here V is a non-singular irreducible algebraic variety of dimension d defined over an algebraically closed field k. The flag construction can be applied to the tangent bundle TV of V to obtain an algebraic fibre bundle $\rho : V^\Delta \longrightarrow V$. This bundle has a fibre over $v \in V$,

$$F(d) = \{F = (F_0, F_1, \dots, F_d) \mid \{v\} = F_0 \subset F_1 \subset \dots \subset F_d = T_v V, \\ \text{where } F_i \text{ is a } k\text{-subspace and } \dim F_i = i\}.$$

The bundle V^{Δ} is called the tangent flag bundle of V and the Chow ring of V^{Δ} may be given by $A(V^{\Delta}) = \rho^* A(V)[\delta_1, \delta_2, \ldots, \delta_d]$ subject to the relation

$$\prod_{h=1}^{d}(1-\delta_h) = \rho^* c(V),$$

where $c(V) = \sum_{i=0}^{d} c_i(V)$ is the total Chern class of the tangent bundle of V; see ⟨**1**⟩. In [**21**], by using nests of linear systems of primals on V and indices $\mathbf{k} = (k_0, k_1, \ldots, k_d)$, Ingleton and Ilori define 'Ehresmann' subvarieties of V^{Δ} and compute the cycle classes (in the Chow ring $A(V^{\Delta})$) of Ehresmann subvarieties that have codimension one in V^{Δ}. In [**22**], they prove an intersection formula that calculates the intersection of any of the Ehresmann subvarieties of V^{Δ} with one of the Ehresmann subvarieties of codimension one, generalizing Monk's formula ⟨**9**⟩ from the special case when $V = \boldsymbol{P}_n(\mathbf{C})$. Using this intersection formula, together with the knowledge of Ehresmann classes of codimension one, obtained in [**21**], they prove, in [**23**], an invariance principle that states that the cycle classes of Ehresmann subvarieties of V^{Δ} in the Chow ring of V^{Δ} can be determined using a knowledge of the easier corresponding calculus on $F(n+1)$ (equal to the tangent flag bundle of $\boldsymbol{P}_n$). They then apply this to calculate the cycle classes of Jacobian subvarieties of V. Other work in this area was the joint paper [**16**] with Ilori and Lascu, in which an elegant and quite simple proof is given of Scott's formula ⟨**13**⟩, where they generalize this to the situation in which the varieties are over an algebraically closed field, homology is replaced by rational equivalence, and tangent direction bundles by arbitrary vector bundles. They also deduce an extension of the formula to bundles of flags.

Ingleton's paper [**6**], published in 1966, deals with the problem of whether there is a unique solution to the system of equations $\mathbf{y} = A\mathbf{x} + \mathbf{c}$ with all of the components of both $\mathbf{x}$ and $\mathbf{y}$ non-negative and $\mathbf{x}^T\mathbf{y} = 0$, where $A = NBN^T$ for some matrix N and (symmetric) positive definite matrix B, and $\mathbf{c}$ is in the column space of A. The way he came to look at this problem is again typical of much of his mathematical activity. It was when John Reeve and Clive Kilmister were working on their book on mechanics ⟨**6**⟩ and were considering the problem of the impulsive initial motion of a uniform heavy cube struck by a blow whilst at rest on a smooth horizontal plane. To simplify the problem, they supposed the normal reactional impulses of the plane to be idealized at the four lower corners of the cube. Reeve had noticed that one could solve this problem by adjoining to the usual equations for the impulsive initial motion of a three-dimensional body, the following set of equations and inequalities:

$$R_i \geqslant 0, \quad V_i \geqslant 0, \quad R_iV_i = 0, \qquad i = 1, 2, 3, 4,$$

where R_i are the upward normal reactions of the plane at the four lower corners of the cube and V_i are the upward vertical components of initial velocity of the corresponding corners. (The point of the condition $\mathbf{R}^T\mathbf{V} = 0$ was that if a corner of the cube leaves the table, it can hardly remain there to receive an impulse; contrarywise, if it receives an impulse it must remain in contact with the table to get it.) Kilmister pointed out that it was remarkable that the above family of equations and inequalities should have a solution, and indeed a unique solution; and he suggested that it must be a consequence of the positivity of the energy form. At this point they handed it over to Ingleton who, in a very short time, produced the existence and uniqueness proofs contained in [**6**]. The way in which he dealt with the problem is also typical of much of his work. Although the symmetry of the matrix A is inevitable in the dynamical application, as well as in a related geometrical problem considered by Du Val ⟨**4**⟩, it is not relevant to the purely algebraic problem and Ingleton was interested in precisely those constraints on A that were relevant. Thus in addition to showing that the system always has a unique solution for $\mathbf{y}$ (with $\mathbf{x}$ being unique only if A is non-singular), the paper gives the result for a more general class of matrices A, termed 'adequate' (which includes positive definite matrices but has non-empty intersections with both the set of positive semidefinite matrices and its complement) and proves that a necessary and sufficient condition for a unique solution for all $\mathbf{c} \in \mathbb{R}^n$ is that all the principal minors of A are positive. It later became apparent that the problem of solving the system $\mathbf{y} = A\mathbf{x} + \mathbf{c}$ with $\mathbf{x} \geqslant \mathbf{0}$, $\mathbf{y} \geqslant \mathbf{0}$ and $\mathbf{x}^T\mathbf{y} = 0$, was of some importance in programming theory and was known as the *linear complementary problem*. Indeed, the answer to the original question had been available from the literature on quadratic programming ⟨**2**⟩. Whereas the treatment in [**6**] is elementary and a significant extension of previous work on positive definite matrices, R. W. Cottle ⟨**3**⟩ later applied standard techniques of programming theory to obtain Ingleton's generalization to adequate matrices. Ingleton returned to this problem in [**9**], where he gave necessary and sufficient conditions on A for there to be a $\mathbf{y}$-unique solution for all $\mathbf{c} \in \mathbb{R}^n$. These require: (i) that the principal minors of A be non-negative, (ii) that if a principal minor of A vanishes then the columns passing through it are dependent, and (iii) a more technical constraint on the signs of the entries in a vanishing principal minor.

In 1961, Ingleton was appointed as the Mathematics Tutor at New College Oxford. Influential in this move was E. C. Titchmarsh, who was then Savilian Professor of Geometry at Oxford (a chair associated with New College), and who had known Ingleton from his lectures at University College London and thought very highly of him. Ingleton's stay at New College was relatively brief, but long enough to inspire several cohorts of undergraduates who will testify to his excellence as a mathematics tutor, always being so quick to get to the heart of their difficulties and resolving them with a clarity hard to match. While at New College, he held the demanding post of Domestic Bursar for a year and was also the first Secretary of the new Mathematics Faculty at Oxford.

In 1966, Ingleton was elected to a Chair in Pure Mathematics at Cardiff. The spring of that academic year was the one in which the British Mathematical Colloquium was held at Swansea, and Ingleton gave the opening talk, entitled 'Flags', outlining the on-going work on tangent flag bundles and generalized Jacobian varieties. Of incidental interest, some also recall this as being the first lecture they had attended in which an overhead projector was used!

In 1967, Balliol College Oxford needed a new Pure Mathematics Tutor to fill the gap caused by the untimely death of Kenneth Gravett. Knowing that Ingleton was not entirely happy in Cardiff, where the department had proved resistant to some of the changes he tried to introduce, Balliol seized the opportunity to appoint an experienced and well-respected tutor, and offered him the Fellowship. He was happy at Balliol and remained there for the rest of his academic life. He was known to be a man of sound judgement, who was totally open, one who did not court disputes but who was not afraid to speak his mind, though never in such a way as to give offence. Within the College he served as Keeper of the Minutes to the Governing Body and was Estates Bursar between 1978 and 1980 (where he carried out a major review and reform of the College's catering arrangements). Outside Balliol, he served for many years on the Oxford Faculty Board of Mathematics and became very much an elder statesman in the Oxford Mathematical Institute, who was trusted, and whose advice was respected, by all. For the last few years before his retirement in 1985, he was a member of the General Board, which oversaw the University's academic activities across all subjects. He chaired the committee that undertook one of the major reforms of the undergraduate syllabus and was a member of the Dover Commission, which completely reformed the Oxford Entrance system and did away with Entrance Scholarships. He also served as Editor of the *Proceedings of the LMS* from 1968 to 1974 and, for four years sometime around 1980, as an external examiner at Glasgow University. With regard to the latter position, Dan Martin writes,

> Aubrey was an excellent external examiner. Not only was his judgement sound, but he was able to vet the questions on *all* the optional subjects. Few examiners can do that!

The quality that served Ingleton so well, as a mathematician, as a tutor and in his administrative posts, was his complete clarity of mind. He could see his way through intricate problems quickly and explain them with a rare lucidity. Indeed, his two final-year undergraduate courses on Lebesgue integration were regarded by many as the clearest they had attended. They were unusual at the time in covering the integration first and then deriving the measure theory from it, rather than the other way round. It was unfortunate that the appearance of Weir's textbooks on the subject discouraged Ingleton from completing his own. (His Mathematical Institute lecture notes [**14**] on the first part of the course were still in use by Oxford undergraduates until very recently, but they were to have been extended by the material from his second, more advanced, course.)

Despite the administrative demands during his 18 years at Balliol, Ingleton supervised a succession of PhD students and continued to produce influential mathematics. His paper [**10**], which is based on the content of his talk on matroids at the first Combinatorial Conference to be held in Britain (Oxford 1969), is a beautiful and much quoted paper. It is part survey and part original work, and is a classic in the area of representability of matroids. It contains a wealth of geometrical examples and constructions, giving new necessary conditions for representability over some field or particular fields or even particular characteristic sets. There are also examples exhibiting matroids representable over the complex numbers but not over the reals, and similarly over the reals but not the rationals. Here also appears the first explicit discussion of whether or not a matroid is *algebraic*, in other words can be interpreted as the independence structure arising from the rank function induced by algebraic dependence on a subset of a field. He shows that any matroid that is algebraic over a field of characteristic zero must also be linearly representable over that field. However, this does not extend to general fields and he ends his paper with a rare conjecture, namely that not every matroid is algebraic.

As one would expect of a conjecture by Ingleton, this turned out to be correct, but it was not until 1975 that he and his student Roger Main [**17**] proved that the well-known and much-studied 8-element matroid V_8, shown to be not *linearly* representable over any field by Peter Vamos in a private communication in 1968, was also not algebraic over any field.

Throughout the period 1965–80, one of the main applications and interests in matroid theory was due to its impact on transversal theory; see for example the book by L. Mirsky ⟨**8**⟩. It had been discovered that for any finite family $\mathcal{A} = (A_i \mid i \in I)$ of subsets of a set S the partial transversals of $\mathcal{A}$ were the independent sets of a matroid $M[\mathcal{A}]$ on S and unsurprisingly any matroid which was isomorphic to one arising in such a way was called a *transversal matroid*. Characterizing these matroids was a natural, although seemingly difficult, problem. D. J. A. Welsh and M. J. Piff showed that they were linearly representable over any sufficiently large field ⟨**14**⟩; thus they could not be easily distinguished by any very natural algebraic route. In [**11**], Ingleton found a complete geometrical characterization in terms of 'compatible quasi-simplices'. These are not easy to describe; however, the paper shows Ingleton's fascination and skill with geometric configurations. With hindsight it is not surprising that these compatible quasi-simplices do not have easy descriptions as there are complexity-theoretic arguments for believing that unless there is a very surprising collapse in the complexity hierarchy, then there is no 'easy' classification of transversal matroids.

Ingleton's paper [**12**] is a short survey based on his invited lecture at a small and enjoyable Anglo-French workshop held in Brest in May 1970. Dominic Welsh, who made this trip with Ingleton, recalls his clockwork organization of the long train–boat trip from Oxford, and was therefore considerably surprised when Ingleton commented halfway through the trip that this was the first time that he had been outside the UK.

The paper [**13**] on supermatroids, jointly written with Dunstan and Welsh, was an attempt to generalize the concept of a matroid regarded as its collection of independent sets, and hence as a lower ideal of the Boolean algebra, to general partially ordered sets. This idea has never quite taken off, partly due to the fact that in order to get interesting theorems one has to restrict the class of posets to well-behaved lattices such as semi-modular. Another possible factor is that it appeared in the Proceedings of the Second British Combinatorial Conference, held in Oxford in 1972, and access to it is quite difficult. However, again these supermatroids are precursors of the greedoids of Korte and Lovász ⟨**7**⟩ mentioned earlier.

The main result of Ingleton's paper [**15**] with M. J. Piff is an extremely important structural theorem. It gives a beautiful, clear-cut characterization of the matroids that are duals of transversal matroids. Essentially, it can be seen as highlighting a general duality theory between matchings in bipartite graphs and linking sets of nodes by paths in general graphs. A particular example of this duality is the relationship between the classic theorems of König for bipartite graphs and Menger for directed graphs. Indeed, this paper contains some of the nicest structure theorems in combinatorics, though unfortunately it has been bedevilled by a gruesome collection of names, such as 'deltoid' and 'gammoid'.

A matroid M is *base orderable* if there is a bijection φ between any two bases B and B' such that for each $x \in B$, $(B \backslash x) \cup \varphi(x)$ and $(B' \backslash \varphi(x)) \cup x$ are bases. The concept is natural, is closed under minors, and is possessed by both transversal matroids and their duals. The very elegant main result of [**18**]: (a) exhibits a complete family of excluded minors for the property of being base orderable and (b) shows that any such family must be infinite. In other words, in more modern terminology, the set of base orderable matroids is not well quasi-ordered under the minor ordering.

Although [**19**], written with J. A. Bondy, appears to be a paper about graphs, it is basically a result about matroids. Given a connected matroid M with at least three elements, its basis graph is formed by taking the bases of M as vertices and joining two bases by an edge if their symmetric difference has cardinal 2. The main result of [**19**] is that every edge is in a circuit of length k for all k in the range $(3, n)$ where n is the number of bases. This greatly strengthens the previously known property that every basis graph is Hamiltonian.

Ingleton's last paper [**20**] on matroid theory, and indeed on any aspect of combinatorics, is his invited lecture at the NATO Advanced Study Institute in Berlin in 1977. This is a valuable mix of survey and new material, concentrated principally on the geometric description of transversal matroids and related structures. In it he also introduced the interesting concept of a *complete class* of matroids; from reading the paper thoroughly again, one gets the feeling that there is a lot more good material to be found in this area. It is also a paper that is not typical of Ingleton's style in that it has quite a few conjectures, open problems and even opinions, such as that the search for a forbidden set of excluded minors for the class of ternary gammoids was probably futile.

It was only a few months after he retired in 1985 that illness struck, and Ingleton effectively gave up mathematical research. He continued to pursue other interests, however, and, as a member of the British Chess Problem Society, regularly solved and compiled chess problems. The illness was successfully treated, but recurred some fifteen years later. Aubrey died on 28 June 2000. In his address at Aubrey's funeral, Sir Anthony Kenny, Master of Balliol from 1978 to 1989, said,

> Aubrey ... spent few words, but those words he spent were words well spent. You always knew that what he told you was true, and that what he promised you would get done. He had no great taste for college feasts or festivals, and ceased to attend them once he was no longer obliged by duty to do so. However, even as an emeritus fellow, he liked to come regularly into the common room to read the newspapers amid the companionship of the fellows. ... Most of us knew little of his hobbies: of his gift for composing chess problems, for instance, or his learned interest in the local history of Headington, or his encyclopaedic knowledge of vintage railway engines. But his passion for the Hebrides, the invariable scene of his annual holidays, was an open secret. ... I recall Aubrey as a paradigm of the type of devoted tutor and conscientious college officer that has been, throughout the century just ended, the backbone of the Oxford collegiate system.

In concluding the account of Ingleton's work on matroids and abstract independence theories, prepared for this obituary, Dominic Welsh writes,

> Finally, on a personal note, my memory of Aubrey is as a totally reliable, wonderful colleague. We met every week at the Tuesday combinatorics seminar. He was usually quiet, not one to venture wild conjectures or the like, but what he said was almost always correct. Indeed, I lost count of the times that shortly after the seminar, either that evening or the next day, I would get a phone call or precisely written note with a counterexample to some conjecture or problems that had been put forward at the tea conversation after the seminar.

The above sentiments are entirely consistent with my own regard for Ingleton as a wise, extraordinarily clever and very nice man.

Acknowledgements. I should like to thank Joan and Annette Ingleton for providing many of the biographical details, Professor C. W. Kilmister and Dr. J. E. Reeve for background to some of Ingleton's early work and Dr. E. Kronheimer for sharing with me reminiscences of the Birkbeck Mathematics Department of the late 1950s. Professor I. G. Macdonald kindly commented on a draft of the summary of Ingleton's work on tangent flag bundles and Jacobian varieties. I am also grateful to Dr. K. C. Hannabuss for allowing me to draw on the obituary he wrote for the Balliol College Record, and to Sir Anthony Kenny for allowing me to quote from his oration at Ingleton's funeral. Particular thanks are due to Professor D. J. A. Welsh for writing all the material covering Ingleton's work on matroids and abstract independence theories.

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