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OBITUARY

Nigel John Kalton, 1946–2010

Courtesy of Miami University (colour online)

1. *Family and early life*

Nigel Kalton was born in Bromley, Kent, on 20 June 1946. He was the third and last child of Gordon Edelbert Kalton (1903–1971) and Stella Vickery (1911–1981), 12 years younger than his sister Pam (who died of cancer at the early age of 38) and ten years younger than his brother Graham. His paternal grandfather was Gordon Edelbert Kaltenbach (1879–1955), a photographic dealer living in Birmingham. The family changed its name to Kalton at the time of the First World War, when anti-German feeling was extreme, but never did it legally, and hence Nigel's birth certificate bears both names. Both Stella and Gordon Kalton had only limited schooling, although it seems that Nigel's father was good at performing numerical calculations. But Gordon Kalton left school at a young age to help in the family business, which consisted of photographic shops in major cities in England. He met his wife Stella when he was running the London shop, and they married in June 1932.

The family business prospered before World War II; however, it was of course hard hit by shortages during and after the war, and the London shop was destroyed during the bombing of the capital. A few years after the war, Gordon Kalton gave up his stake in the business and relied thereafter on a modest income from stocks and shares. Nigel grew up in Bromley, on the outskirts of London, in a small, semi-detached house. His family did not have a telephone, a car, or a television until Nigel was a teenager.

Gordon Kalton was a recluse, being extremely hard of hearing. He was also very frugal. However, he did attach a great deal of importance to his children's education. As a result, Nigel, and Graham before him, commuted by train to Dulwich College rather than attend a local grammar school. Dulwich, an extremely well-endowed school (founded in 1619) with extensive playing fields, good science laboratories, and good teachers, had become a magnet school for bright students. Both Graham and Nigel went on to university at a time when only about five per cent of school-leavers did so. They were in fact the first members of their family to obtain higher education. Their sister Pam resisted attempts to persuade her to apply to university. Graham went to the London School of Economics to study mathematical statistics, before engaging in an academic career which led him to a full professorship in Great Britain and then to a distinguished career, continuing to this day, in the United States of America. Nigel's path took him to Cambridge to study mathematics, as described later.

When Nigel was very young, Pam and Graham were sometimes given the task of taking their baby brother in his pushchair for walks in the neighbourhood. On one such walk, when Nigel was about two years old, they taught him some lines based on Shakespeare's 'As You Like It', with a change made to reflect Gordon's taste for whisky. When they returned home Nigel recited to his father 'Oh good old man how well in thee appears the ancient vintage of the antique world'. But it was Nigel's unusual mathematical abilities that most distinguished him at an early age. An example of this occurred while Graham was a graduate student at LSE but still living at home for financial reasons. He was working through Kendall and Stuart's two-volume *The Advanced Theory of Statistics*, and would leave one or other volume at home when going to London. Nigel would read the volume left behind. One evening, Nigel, then around 13 years old, showed Graham a neat, simple derivation of the Poisson distribution as a limiting case of the binomial distribution that he had devised, resulting from his reading of Kendall and Stuart. Shortly after, and while still at Dulwich, Nigel completed his first refereed paper published in the *Cambridge Mathematical Gazette*, 'Quadratic forms that are perfect squares' [**1**].

As a teenager, Nigel did not have much interest in the usual sports of the public school tradition such as rugby, football, or cricket, but he certainly was an intense chess player, and a member of the Dulwich College chess club, which came second in the nation in the Sunday Times competition in 1964. But then came the time to leave Dulwich College for Cambridge, the right place for an outstanding boy who explained once that 'from a young age, I was very good at mental arithmetic, and somehow math was my subject. I never really thought to do anything else'.

2. *Cambridge*

Nigel Kalton entered Trinity College, Cambridge, in 1964. More precisely, he chose to miss out the first year (Mathematical Tripos Part I) and moved directly into the second year. This daring move was certainly appropriate, since Nigel quickly impressed his fellow students by his problem-solving abilities, even in the Cambridge environment, where such talents are not unknown. Peter Goddard, former director of the Institute for Advanced Studies at Princeton, had entered Trinity College in 1963 and thus took the same examinations as Nigel between 1964 and 1966. According to his testimony: 'Also marked was his ability to absorb new mathematics easily and quickly. He was cheerful and modest without being falsely so: he knew his abilities but he saw no need to base his whole persona on them; rather the reverse. His ability to absorb mathematics and solve difficult problems quickly suited him ideally to the formal Tripos examinations as they were then. Thus, unsurprisingly, he came first by a clear margin in the unofficial orders of merit for the Preliminary Examination (1965) and the Mathematical Tripos Part II (1966)'. It is plain that these mathematical and personal qualities remained Nigel's features throughout his life. Fortunately, his abilities were recognized and he was awarded the G. F. A. Osborn Prize, awarded to the most distinguished second-year mathematician at Trinity College, in 1965.

Besides Peter Goddard, contemporary students of Nigel Kalton included Garth Dales, Alexander Davie, Peter Dixon, and Ian Stewart. In Cambridge, Nigel had also met Jennifer Bursey. Jennifer's family tree has been rooted in England for a millennium, since her ancestry goes back to a companion of William the Conqueror named Sirloin de Burcy. The author of these lines had the chance to escort Jenny and Nigel to the church of Dives-sur-Mer (Normandy), where the names of the known companions of William are carved on the wall, and to check that Sirloin de Burcy's name was there. Jenny and Nigel were married in 1969, and they had two children, Neil (born in 1973) and Helen (born in 1976); later there were four grandchildren.

Even the most demanding studies allow for some amount of socializing. Most of Nigel's social life was devoted to chess, and a number of recorded games that he played at the time with Raymond Keene (who won the British Chess Championship in 1971) demonstrate that they were of comparable level. Nigel himself tied for the sixth place in the Eastbourne Open in 1966, he represented Cambridge University in the matches against Oxford University in 1967 and 1968, and he won the Major Open in Warwick in 1970. This qualified him for the British Championship in 1971 at Blackpool, won by Ray Keene, where Nigel scored 5/11 (one win, two losses, eight draws) and was ranked 26th among 36 participants, the very best players in the country. This result is impressive enough, given the fact that at that time he was a full-time mathematician. Nigel could have considered a professional career as a chess player, but mathematics remained his first passion. He retired from over-the-board play in 1976, although he played a number of games with the International Email Chess Group between 1993 and 1996.

The course 'Analysis 4' is one that was offered to second-year students at Cambridge. This course was taught by D. G. H. 'Ben' Garling, and was based on a book by Jean Dieudonné called *Treatise on Analysis*. It probably contributed to Nigel's choice of functional analysis, a subject that was at the time attracting a large number of graduate students at Cambridge, about fifteen, according to Garth Dales' testimony, with strong weekly seminars. Nigel became a student of Ben Garling in 1967, and wrote under his supervision the thesis entitled 'Schauder decompositions in locally convex spaces'; this was approved on 11 November 1970. The thesis earned Nigel the Raleigh Prize from Cambridge University.

Nigel's mathematical genealogy includes Ben Garling's advisor Frank Smithies, who had introduced functional analysis to Cambridge after studying the distribution theory of Laurent Schwartz. This new functional analysis was sometimes called 'soft analysis' as opposed to the 'hard analysis' of Hardy and Littlewood, although Frank Smithies was himself a student of Hardy. Á propos, it should be recalled that John Littlewood, the senior Wrangler at Cambridge in 1905, was still active during Nigel's student years, and they sometimes met, although to the best of my knowledge they never had a mathematical conversation together.

Ben Garling spent the academic year 1969–1970 at Lehigh University in Bethlehem, Pennsylvania, and Nigel Kalton accompanied him there as a visiting lecturer. This first contact with an American Department made a very positive impression on Nigel. In his own words: 'At Lehigh during a talk, people would chime in with no air about them. Questions were asked and discussed. No one tried to score off a speaker, and ultimately people had fun talking about mathematics'. This would have looked relaxing after the fiercely competitive atmosphere of Cambridge, and the remainder of Nigel's career confirms that this new-world attitude had a lasting influence on his choices. Also in 1970, Nigel's first papers appeared (if we rule out his promising early bird from the 1966 Mathematical Gazette); this launched a powerful flow of publications which only death broke off.

3. *From England to the United States*

After returning from the United States, Nigel Kalton spent the year 1970–1971 as a Science Research Council Fellow at Warwick University in Coventry. One of his good friends and collaborators there was Robert Elliott, whose wife Ann had taught Nigel's future wife Jenny at high school in 1962, and the two families became close friends. In 1971, Nigel was appointed as a lecturer at the University College of Swansea, part of the University of Wales. He was to stay there for eight years, until 1979. The Kaltons lived there with two young children on a modest lecturer's salary, and Nigel had no access to funds to allow him to travel to conferences. As a consequence of this relative isolation, he focused in part on somewhat unfashionable topics such as non-locally convex spaces. In this pre-internet era, his reasoning was that the risk was less that such topics would suddenly experience a dramatic overhaul that he would not know of, which would mean his working on a problem that someone had already solved. Anyhow Nigel's talent was so great that his research record quickly became impressive, but despite very strong support from the Department, the University of Swansea failed to promote him to a Readership.

Meanwhile, Nigel had been invited to the United States as a Visiting Associate Professor at the University of Illinois in Urbana-Champaign (by N. Tenney Peck, in 1977) and then at Michigan State University, East Lansing (by Joel H. Shapiro, in 1978). Several universities in the United States were interested in offering him a position, and the first to do so was the University of Missouri-Columbia through Dennis Sentilles; Nigel accepted this offer. When the Kaltons settled in Columbia in 1979, the place was still quite provincial, and during his first year there Nigel had the only NSF Grant of the Department. However, according to his testimony, he had 'jumped at the chance of a job at Missouri-Columbia, because the conditions were so much better and allowed me to pursue my research without impediment'. Nigel was to spend the rest of his life in Columbia, which thanks to his influence would become a major centre in functional analysis, not to mention other fields which also benefited from the Department's rise. He clearly preferred the quiet surroundings provided by a midwest college town to the buzzing and steaming of large cities, and Columbia was a place where he could work in peace and welcome collaborators, such as the author of these lines, who was privileged to share five academic years with him between 1985 and 1997. Hence Nigel Kalton fully became an American faculty member. It should, however, be mentioned that he kept his British citizenship to the end, without ever bothering to seek an American passport.

4. *Living and working in Missouri*

The University of Missouri at Columbia was prompt to realize what a catch Nigel Kalton was: several awards were bestowed upon him, such as the Chancellor's award for outstanding research in the physical and mathematical sciences in 1984 and the Weldon Springs presidential award for research and creativity in 1987. Nigel was named Houchins Professor of Mathematics in 1985 and became a Curator's Professor, the highest position that the University of Missouri can provide, in 1995. However, the Banach Medal awarded to him by the Polish Academy of Sciences in 2004 is surely his most prestigious award.

Public recognition is definitively of importance, but maybe not as important as the freedom to 'pursue research without impediment'. Nigel was left in peace by a wise Department, which valued his research as it deserved. He usually taught graduate courses in functional analysis, and his legendary finals were simply a gathering with the students over a beer in the nearby Heidelberg Pub. I attended some of these finals, where Nigel was sharing ideas and opinions with his students in his usual unassuming way. I suspect that some of these students remained unaware of the true stature of their professor. Fortunately, some of them understood who Nigel was. Adam Bowers, when a post-doctoral fellow in Columbia, attended the last course taught in 2009–2010 and the notes he took will be published shortly as a joint work by Nigel and himself [**271**].

Nigel Kalton worked extremely fast both to establish his results and to write them down. He typed his articles at high speed with no scrap paper around him, however complex the arguments were. He did not write much on paper, except for some explicit computations, and the whole work took place in his brain. I can actually testify to the quite amazing fact that he was able to solve highly non-trivial problems while talking about a completely different topic. And although he was entirely self-sufficient, he would listen to all those, students or colleagues, who approached him with sensible questions, and his attention would soon induce a drastic change in their mathematical landscape. Nigel seldom read articles or books, and would rather rebuild by himself what he needed in his work. It was indeed his way of *saving* time. This did not prevent him, however, from being very careful about references: he knew that work had been done by such and such, and he would quote the relevant articles. But the whole theory was in Nigel's mind anyhow. Let me call this a mystery for lack of a better word. Nigel also had a unique ability to use mathematical objects which are sometimes considered marginal, such as non-locally convex spaces or quasi-linear maps, not for their own sake, but as tools for showing spectacular results in main-stream analysis. His problem-solving power was famous and went much beyond answering open questions; he would build the proper framework in which the original problem was to be understood inside out with a collection of related results, and hence would prepare the ground for further work. He usually wrote down his theorems in the greatest generality, certainly not out of pedantry, but simply because his proofs reached this level and he trusted, maybe exceedingly, his reader's ability to find out what the applications were. As a rule, he submitted his articles to relatively modest journals and never attempted to publish in the most famous ones, although his work deserved the best. But it seems that his desire was to make things simple and be left in peace rather than to strive for fame. Vanity was foreign to him.

Of course, Nigel Kalton accepted a number of invitations to various universities, including Paris, although that big city was not among his favourite places, and turned down a few, sometimes half-jokingly claiming that unfortunately he had to stay at home since his cats needed him. He attended scores of conferences in North America, Europe, and Australia. However, after being hired by the University of Missouri, he never stayed away from Columbia for more than a semester at a time. Nigel lived a quite well-regulated life there; he was not a morning person, and his routine was rather to work at home in the evening, and frequently well into the night. He would usually come to the Department around lunch time, happily explaining, for example to me, that all of yesterday's questions were solved, and much more. My contribution was to sit down and listen, but Nigel's generosity was such that this invariably resulted in a joint paper.

Nigel's working power was impressive, but he also knew how to relax, if not to rest. Racquetball is a popular sport in Columbia and Nigel played it on a regular basis, in his usual rather competitive way. And although his mind was constantly in gear, he was also excellent company, a man of taste who knew how to enjoy good food and fine wine, and besides mathematics a man of culture with a definite interest in historical matters. Hence sharing time with such a friend was both pleasant and instructive. Above all, he was a family man, Jenny's husband for 41 years and a proud father and grandfather.

Working under the supervision of a generous first-class mathematician is a PhD student's dream. A steady flow of students found their way to Columbia to turn this dream into reality, namely David Trautman (defense in 1983), Carolyn Eoff (1988), Camino Leranoz (1990), Beata Randrianantoanina (1993), Sik-Chung Tam (1994), Dan Cazacu (1997), Roman Vershynin (1999), Roman Shvidkoy (2002), Mark Hoffman (2003), Jakub Duda (2004), Pierre Portal (2004, joint supervision with Gilles Lancien from Besançon, France), Tamara Kucherenko (2005), Mikhail Ganichev (2006), Simon Cowell (2009). Daniel Fresen, who was Nigel's student in 2010, continued his PhD (defended in 2012) under the supervision of Alexander Koldobsky and Mark Rudelson. In my opinion, it would be fair to augment this list by the crowd of colleagues who benefited from Nigel Kalton's mathematical power, insight, and vision. Some among them gathered to celebrate Nigel's sixtieth birthday at the meeting organized in his honour by Beata and Narcisse Randrianantoanina in Oxford (Ohio).

Nigel Kalton suffered a devastating stroke on Sunday, 29 August 2010. He passed away peacefully in his sleep two days later in University Hospital, Columbia, in the presence of his wife Jennifer and his children Neil and Helen. A gathering in his honour was organized on 1 October 2010 in Columbia, to which his family, friends, and colleagues were invited to honour his memory, and following Jenny's wish to celebrate his life. The *Notices of the American Mathematical Society* devoted an obituary article ⟨**11**⟩ to Nigel Kalton with Peter Casazza as Coordinating Editor. Fritz Gesztesy has set up a website to honour Nigel's memory and achievements; this contains, in particular (with the publishers' permission), his publications (http://kaltonmemorial.missouri.edu). A selection of his articles, with for each one extensive comments by an expert of the field, edited by Fritz Gesztesy, Loukas Grafakos, Igor Verbitzky, and myself, is presently under completion and will be published by Birkhäuser under the title 'Kalton Selecta' ⟨**24**⟩.

None of those who were privileged to know Nigel Kalton will ever forget him. But Nigel was an achiever who always tried and never gave up. He left us his spirited example and his inspiring mathematics, and his will clearly was that research should go on, no matter what. Commenting now on some of his works is a modest attempt to fulfill this wish. Before doing so, I should make it clear that to present every item of Nigel's formidable list of publications is way beyond my abilities, and I apologize to any reader who is unhappy about the lack of comments on his/her favourite among Nigel's theorems. I will simply choose some fields with which I am relatively familiar, and in which his influence is especially important. These selected items should, I hope, give some idea of the width, depth, and scope of Nigel Kalton's contribution to mathematics.

5. *The Kalton zone*: $0 \leqslant p < 1$

Hahn–Banach theorems are cornerstones of functional analysis. But it turns out that non-locally convex spaces show up very naturally in many cases when there is no reason to 'stop at $p = 1$' and actually what happens when $p < 1$ provides precious information on the somewhat more classical locally convex setting. This is an invitation to visit what I suggest we call the *Kalton zone*: $0 \leqslant p < 1$. This terminology is amply justified by the fact that Nigel Kalton is the undisputed leader on non-locally convex analysis and its uses.

We recall that metrizable complete topological vector spaces (over $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$) are called *F-spaces*. An F-space X is locally bounded if and only if its topology can be generated by a *quasi-norm* $\|\cdot\|$, namely a map $\|\cdot\| : X \to \mathbb{R}^+$ such that:

(i) $\|x\| > 0$ if $x \neq 0$;
(ii) $\|\alpha x\| = |\alpha|\|x\|$ for all $x \in X$and $\alpha \in \mathbb{K}$;
(iii) $\|x + y\| \leqslant C(\|x\| + \|y\|)$ for all $(x, y) \in X^2$.

Here $C \geqslant 1$ is the *modulus of concavity* of the quasi-norm. A locally bounded F-space is called a *quasi-Banach space*. We refer to [**97**] for an authoritative book on F-spaces. It is clear that the Hahn–Banach theorem is sensitive to local convexity assumptions, to the point where it leads to a characterization [**31**]: a quasi-Banach space X is locally convex (that is, it is a Banach space) if and only if every continuous linear functional defined on a closed subspace of X has an extension to a continuous linear functional on X. In other words, a quasi-Banach space is a Banach space if and only if the weak and quasi-norm topologies have the same closed subspaces.

The proof of this theorem relies on the construction of Markushevich basic sequences (see [**203**, Propositon 3.4]), obtained by refining Mazur's classical argument. To be able to do this, one needs, however, a weaker topology, even if it is not 'weak' in the classical sense. A quasi-Banach space is *minimal* if it does not have any weaker Hausdorff vector topology. As suggested by Mazur's technique, a separable quasi-Banach space is minimal exactly when it contains no basic sequence. On the other hand, it is shown in [**39**] that an F-space satisfies the restricted Hahn–Banach extension property (that is, if L is an infinite-dimensional, closed subspace and $0 \neq x \in L$, then there exists an infinite-dimensional, closed subspace M of L such that $x \notin M$) if and only if every infinite-dimensional, closed subspace contains a basic sequence. It turns out that quite general assumptions, such as the existence of an equivalent pluri-subharmonic quasi-norm, force the existence of basic sequences. This applies to subspaces of L^p for $0 < p < 1$ and more generally to all *natural* quasi-Banach spaces, where 'natural' means 'subspace of a lattice-convex quasi-Banach lattice'. Along these lines, we refer to the articles [**200**, **216**] devoted to quasi-Banach sequence spaces such as $l_p(l_1)$ and $l_1(l_p)$ (with $0 < p < 1$), which have a unique unconditional basis up to permutation.

However, minimal quasi-Banach spaces do exist, and the following is shown in [**151**]: there is a quasi-Banach space M which contains a one-dimensional subspace E such that every infinite-dimensional, closed subspace Y of M contains E. In particular, M contains no basic sequence. Indeed, a basic sequence would provide a decreasing sequence of infinite-dimensional, closed subspaces with intersection equal to $\{0\}$, and this cannot be the case if they all contain E. Thus M is minimal. The reader may find it amusing to think of that space as a book: it has many pages but they all meet on the one-dimensional binding.

The proof of this theorem is the culmination of several works, which we now outline. Suppose that X and Y are quasi-Banach spaces. Then Y is an *extension of* X *by* E if $Y/E \simeq X$; when E is one-dimensional this extension is said to be *Minimal*. The reader should be warned that the word 'minimal' is used here with two different meanings, and to prevent confusion we shall use an initial capital letter to denote extensions by a one-dimensional space E. A Minimal extension is usually *not* a minimal quasi-Banach space, but the above theorem asserts that this may happen. A Minimal extension is said to be *trivial* if it splits, that is, if $Y \simeq X \oplus E$. The case where X is actually a Banach space is important, and indeed Kalton [**55**], Ribe ⟨**51**⟩, and Roberts ⟨**53**⟩ independently constructed non-trivial Minimal extensions of $X = \ell_1$, thus solving negatively the three-space problem for local convex spaces.

All Minimal extensions can be obtained in the following way [**53**]. Let X be a quasi-Banach space over the field $\mathbb{K}$, and let X_0 be a dense linear subspace of X. A map $F : X_0 \to \mathbb{K}$ is *quasi-linear* if: (i) $F(\alpha x) = \alpha F(x)$ for $x \in X_0$ and $\alpha \in \mathbb{K}$; (ii) there is $C > 0$ such that $|F(x+y) - F(x) - F(y)| \leqslant C(\|x\| + \|y\|)$ for all $(x, y) \in X^2$. Then one can define a quasi-norm on $\mathbb{K} \oplus X_0$ by

$$\|(\alpha, x)\|_F = |\alpha - F(x)| + \|x\| \quad (\alpha \in \mathbb{K},\ x \in X_0),$$

and the completion of $\mathbb{K} \oplus X_0$ for this quasi-norm is a Minimal extension of X, denoted by $\mathbb{K} \oplus_F X$. Conversely, any Minimal extension of X is obtained in this way, and $\mathbb{K} \oplus_F X$ splits if and only if there is a linear map $G : X_0 \to \mathbb{K}$ such that

$$|F(x) - G(x)| \leqslant C'\|x\| \quad (x \in X_0). \tag{5.1}$$

This approximation is related to Hyers–Ulam functional stability, for which we refer to ⟨**6**, Chapter 15⟩. The *Ribe space* is then obtained by considering the quasi-linear functional

$$F(x) = \sum_{k=1}^{\infty} x_k \log|x_k| - \left(\sum_{k=1}^{\infty} x_k\right) \log\left|\sum_{k=1}^{\infty} x_k\right|$$

on the dense subspace c_{00} of finitely-supported sequences in ℓ_1. Ribe's function F is closely related to Shannon's entropy function from information theory. This suggests the following terminology. Let X be a Banach sequence space. Then the quasi-linear map Φ_X defined on c_{00}^+ by

$$\Phi_X(x) = \sup_{\|t\|_X \leqslant 1} \sum_{k=1}^{\infty} x_k \log|t_k|$$

and extended to c_{00} by setting $\Phi_X(x) = \Phi_X(x^+) - \Phi_X(x^-)$ is called the *entropy function* of X ⟨**43**⟩. We shall see later that this entropy function can be understood as the logarithm of the sequence space X. For instance,

$$\Phi_{\ell_1}(x) = \sum_{k=1}^{\infty} x_k \log \frac{|x_k|}{\|x\|_1} \quad \text{and} \quad \Phi_{\ell_p} = \frac{1}{p}\Phi_{\ell_1}.$$

In order to construct a Minimal extension M of ℓ_1 with no basic sequence and such that $M/E \simeq \ell_1$, it suffices that every infinite-dimensional, closed subspace of M contains E. This is reminiscent of the Gowers–Maurey construction ⟨**30**⟩ of a Banach space X_{GM} without any unconditional basic sequence, which is such that, for any infinite-dimensional subspaces U and V of X_{GM}, we have

$$\inf\{\|u - v\| : \ u \in U, \ v \in V, \ \|u\| = \|v\| = 1\} = 0.$$

Indeed, this condition means that any two such subspaces U and V almost meet. The point is now to push the construction to a stage which is impossible to reach in normed spaces, namely that any pair of infinite-dimensional, closed subspaces actually meet (on the same line E). And it turns out that Gowers' modification ⟨**28**⟩ of the original construction, namely a space X with an unconditional basis and not isomorphic to its hyperplanes, gives an entropy function Φ_X which provides a Minimal extension $\mathbb{K} \oplus_{\Phi_X} \ell_1 = M$ with this intersection property [**151**]. Thus M is a minimal quasi-Banach space. Note that the function $\Phi_X = F$ fails to satisfy (5.1) when restricted to any infinite-dimensional subspace J of c_{00} or, equivalently,

$$\sup\{|F(x)| : x \in J, \ \|x\| \leqslant 1\} = \infty,$$

and hence $\Phi_X = F$ is distorted in the sense of ⟨**42**⟩.

A minimal quasi-Banach space M is a rather strange object, since every one-to-one continuous linear map from M into a Hausdorff topological vector space is actually an isomorphism onto its range. However, existing examples are 'non-isotropic', in the sense that they contain a distinguished line, namely the orthogonal complement of the dual space. It in not known whether an even stranger example exists which would exhibit this behaviour everywhere, that is, is there a quasi-Banach space which contains no infinite-dimensional, proper closed subspace? Note that an algebraic complement of E in Kalton's space M is a quasi-normed space with the Hahn–Banach extension property that is not locally convex, and hence the characterization from [**31**] requires completeness.

The Ribe space, for instance, is a non-trivial Minimal extension of ℓ_1. However, there exist infinite-dimensional quasi-Banach spaces X which are such that every Minimal extension of X is trivial. Such spaces are called $\mathcal{K}$-*spaces* in [**59**], and it is shown in [**55**] that, for $0 < p < 1$, the spaces ℓ_p and L_p are $\mathcal{K}$-spaces, from which it follows, in particular, that L_p/E is not isomorphic to L_p [**59**], whenever E is a one-dimensional subspace of L_p.

Some Banach spaces are $\mathcal{K}$-spaces: it is shown in [**84**] that every quotient space of an $\mathcal{L}_\infty$-space is a $\mathcal{K}$-space, and in [**55**] that a Banach space with non-trivial type is a $\mathcal{K}$-space. In fact, Kalton conjectured that a Banach space is a $\mathcal{K}$-space exactly when its dual space has non-trivial cotype.

Minimal extensions of $\mathcal{L}_\infty$-spaces are trivial, in other words, all quasi-linear maps on 'cubes' are close to linear ones. This creates a link between this field and the Maharam problem, explored by Nigel Kalton and Jim Roberts, who showed, in particular, that the existence of a control measure is equivalent to the uniform exhaustivity of the given sub-measure. The

Maharam problem has been solved negatively in ⟨**59**⟩; this shows, in particular, that the Kalton–Roberts theorem is optimal. For sake of brevity, we simply state the following from [**84**].

THEOREM 5.1. *There is a universal constant K such that, if Σ is an algebra of subsets of some set Ω and $\varphi : \Sigma \to \mathbb{R}$ is a set function such that*

$$|\varphi(A \cup B) - \varphi(A) - \varphi(B)| \leqslant 1 \quad \text{whenever } A, B \in \Sigma \quad \text{and} \quad A \cap B = \emptyset,$$

then there is an additive set function λ on Σ such that $|\varphi(A) - \lambda(A)| \leqslant K$ for all $A \in \Sigma$.

The best value of K belongs to the interval $[3/2, 45]$, but its precise value seems to be unknown.

Let us conclude this section with Nigel's investigations on the fundamental theorem of calculus for functions which take values in F-spaces. He shows in [**73**] that, if X is an F-space with trivial dual and $x \in X$, then there exists an X-valued dyadic martingale (u_n) with $u_0 = x$ which converges uniformly to 0. It follows that, under these assumptions on X, there exists a non-constant Lipschitz function from $[0, 1]$ to X whose derivative vanishes identically [**73**]. This result is used in [**157**], where Nigel, answering a question of Popov ⟨**47**⟩, shows that, if X is a quasi-Banach space with trivial dual, every continuous function from $[0, 1]$ to X has a primitive. This result contrasts with ⟨**2**⟩, where it is proved that, if X is a non-locally convex quasi-Banach space with separating dual, then there is an X-valued continuous function which fails to have a primitive.

6. *Non-linear geometry of Banach spaces*

The subject 'non-linear geometry of Banach spaces' consists of consideration of a Banach space as a metric space, and checking how much of its linear structure is determined by the metric data. In other words, linear isomorphisms are replaced by weaker notions (bi-Lipschitz or merely uniform isomorphisms), and the question is to determine which properties are stable under such isomorphisms. Nigel Kalton's contribution to this subject began in 1998, with the proof [**176**] that the class of subspaces of c_0 is stable under Lipschitz isomorphisms, from which it follows that a space which is Lipschitz-isomorphic to c_0 is already linearly isomorphic to it. Shortly thereafter came [**182**], where the results of the first article are significantly deepened, since it is shown, for instance, that the class of spaces whose Szlenk index is the smallest possible (namely ω_0) is stable under uniform homeomorphisms, although it was known since Ribe's work ⟨**52**⟩ that the property of being an Asplund space is not uniformly stable. This second article leads to a simple and useful idea: asymptotic notions, such as the Szlenk index or moduli of asymptotic convexity or smoothness, provide natural invariants for non-linear isomorphisms. In this direction, the best results have again been shown by Nigel. Let us call a *coarse embedding* a map f between two metric spaces (M, d) and (N, δ) such that

$$\rho_1(d(x, y)) \leqslant \delta(f(x), f(y)) \leqslant \rho_2(d(x, y)) \quad ((x, y) \in M^2),$$

where ρ_1 and ρ_2 are two real-valued functions such that $\lim_{t\to\infty} \rho_1(t) = \infty$. The map f is *coarse-Lipschitz* if there is $\theta \geqslant 0$ such that $\rho_1(t) \geqslant At$ and $\rho_2(t) \leqslant Bt$ for $t \geqslant \theta$. In other words, a coarse-Lipschitz embedding is a map which is bi-Lipschitz for large distances; such maps describe the shape of a metric space at a large scale. It is shown in [**267**] that, if a Banach space X coarse-Lipschitz embeds into a Banach space Y, then the norms of the spreading models of X are controlled from below by the modulus of asymptotic convexity of Y, and from above by the modulus of asymptotic smoothness (provided that Y is reflexive). It follows for instance that, if $1 < p < \infty$, a Banach space which coarse-Lipschitz embeds into l_p is linearly isomorphic to a subspace of l_p. However, Nigel also constructed two subspaces of l_p (for $1 < p \neq 2 < \infty$) which are uniformly (in particular, coarse-Lipschitz) isomorphic, but not linearly isomorphic [**268**], in sharp contrast to the Lipschitz case. We refer to ⟨**27**⟩ for a recent survey on non-linear geometry of Banach spaces; it focuses on Nigel's works.

An important feature of Nigel's contribution to non-linear geometry concerns embeddings of special metric graphs into Banach spaces and 'concentration results' when the target space satisfies certain properties. Such investigations were motivated, in particular, by attempts to attack the Novikov conjecture by relating the geometry of groups with coarse embeddings into Hilbert spaces or super-reflexive spaces ⟨**67**, **34**⟩. Such embedding results are found in [**240**], where the Kalton–Randrianarivony graphs $\mathbf{G}_k(\mathbf{M})$ (increasing sequences of integers of length k equipped with a weighted Hamming distance) are used to show that the space $l_{p_1} \oplus l_{p_2} \oplus \cdots \oplus l_{p_n}$ is determined by its nets (in particular, by its uniform structure) provided that $1 < p_i < \infty$ for all i. Nigel also used these graphs (equipped this time with the graph distance, where two sequences are adjacent if they interlace) to show that, if c_0 coarsely embeds into a Banach space X, then one of the iterated duals is non-separable, and, in particular, X is not reflexive [**229**], although c_0 embeds uniformly and coarsely into a Banach space with the Schur property [**213**]. On the other hand, any stable metric space (where 'stable' means that the order of limits can be permuted in $\lim_k \lim_n d(x_k, y_n)$ whenever both limits exist) can be coarsely embedded into a reflexive space [**229**]. The line of thought that was opened in [**229**] leads to the property denoted $\mathcal{Q}$ by Kalton, a necessary condition for coarse embeddability into a reflexive space that could possibly be sufficient as well. The interlacing distance was also used in [**262**], being defined this time on increasing sequences of length k of countable ordinals, in order to obtain non-separable results. And indeed, Kalton shows in [**262**] that l_∞ / c_0 cannot be uniformly embedded into l_∞, and uses this result to show the existence of a (non-separable) Banach space which is not a uniform retract (in particular, not a Lipschitz retract) of its bidual; it is still not known whether such a separable example exists. Along different lines, it is shown in [**256**] that, if X is reflexive, then the Szlenk indices of both X and X^* are equal to ω_0 if and only if the tree of finite sequences of integers equipped with the hyperbolic distance does not Lipschitz embed into X. It follows that this class of spaces, which strictly contains the super-reflexive spaces, is stable under coarse-Lipschitz embeddings.

We now recall an important open question: let X and Y be two Lipschitz-isomorphic *separable* Banach spaces. Are X and Y linearly isomorphic? This problem is open even if $X = \ell_1(\mathbb{N})$, or if X and Y are assumed to be super-reflexive. Counter-examples are available in the non-separable case ⟨**1**⟩, and the relevance of separability is displayed in [**209**]. Let $\mathrm{Lip}_0(X)$ be the space of real-valued Lipschitz functions on a Banach space X which vanish at 0, and let $\mathfrak{F}(X)$ be the subspace of $\mathrm{Lip}_0(X)^*$ generated by the evaluation functionals at points of X. The Dirac map $\delta : X \to \mathfrak{F}(X)$, defined by $\langle g, \delta(x)\rangle = g(x)$, has a linear left inverse, namely the barycentre map $\beta : \mathfrak{F}(X) \to X$, such that $x^*(\beta(\mu)) = \langle \mu, x^* \rangle$ for all $x^* \in X^*$ and $\mu \in \mathfrak{F}(X)$. This setting provides canonical examples of Lipschitz-isomorphic spaces. Indeed, if we let $Z_X = \operatorname{Ker} \beta$, then it follows easily from the fact that $\beta\delta = Id_X$ that $Z_X \oplus X = \mathfrak{G}(X)$ is Lipschitz-isomorphic to $\mathfrak{F}(X)$.

Following [**209**], let us say that a Banach space X has the *lifting property* if, whenever Y and Z are Banach spaces and $S : Z \to Y$ and $T : X \to Y$ are continuous linear maps, the existence of a Lipschitz map $\mathcal{L}$ such that $T = S\mathcal{L}$ implies the existence of a continuous linear operator L such that $T = SL$. A diagram-chasing argument shows that $\mathfrak{G}(X)$ is linearly isomorphic to $\mathfrak{F}(X)$ if and only if X has the lifting property.

It turns out that non-separable reflexive spaces, and also the spaces $\ell_\infty(\mathbb{N})$ and $c_0(\Gamma)$ when Γ is uncountable, fail the lifting property, and this provides canonical examples of pairs of Lipschitz-isomorphic, but not linearly isomorphic, spaces. In the already quoted article [**262**], Nigel, inspired by ⟨**9**⟩, uses the pull-back operation applied to the exact sequence which defines the Johnson–Lindenstrauss space JL_∞ to produce a Banach space which is Lipschitz-isomorphic to $l_\infty \oplus c_0$ without being linearly isomorphic to it. It is still an open question whether l_∞ is determined by its metric structure. Another use of this technique allowed Nigel to show that, if X is a non-separable WLD space which contains a subspace isomorphic to c_0, then X fails to have unique Lipschitz structure [**262**].

On the other hand, every *separable* space X has the lifting property: to prove this, one can pick a Gaussian probability measure γ whose support is dense in X and use the fact that $\mathcal{L} * \gamma$ is Gateaux-differentiable. Then $L = (\mathcal{L} * \gamma)'(0)$ satisfies the equation $T = SL$.

The lifting property for separable spaces forbids the existence of canonical pairs of Lipschitz-isomorphic, but not linearly isomorphic, separable spaces. However, on the other hand it leads to positive results: for instance, if X is a separable Banach space and there exists an isometric embedding from X into a Banach space Y, then Y contains a linear subspace which is isometric to X [**209**].

Free spaces constitute the proper framework for showing the gap which separates Hölder maps from Lipschitz ones. This is done in [**213**]. Suppose that $(X, \|\cdot\|)$ is a Banach space and that $\omega : [0,\infty) \to [0,\infty)$ is a sub-additive function such that $\lim_{t\to 0} \omega(t) = \omega(0) = 0$ and $\omega(t) = t$ for $t \geqslant 1$. Then the space $\mathrm{Lip}_\omega(X)$ of $(\omega \circ d)$-Lipschitz functions on X which vanish at 0 has a natural predual, denoted by $\mathcal{F}_\omega(X)$, and the barycentric map $\beta_\omega : \mathcal{F}_\omega(X) \to X$ (whose adjoint is the canonical embedding from X^* to $\mathcal{F}_\omega(X)$) is still a linear quotient map such that $\beta_\omega \delta = Id_X$. However, the Dirac map $\delta : X \to \mathcal{F}_\omega(X)$ is now uniformly continuous with modulus ω, for example, α-Hölder when $\omega(t) = \max(t^\alpha, t)$ with $0 < \alpha < 1$. Uniformly continuous functions fail the differentiability properties that Lipschitz functions enjoy, and thus one can expect that this part of the theory is more 'distant' from the linear theory than the Lipschitz one. It is indeed so, and Kalton showed [**213**, Theorem 4.6] that, if ω satisfies $\lim_{t\to 0} \omega(t)/t = \infty$, then $\mathcal{F}_\omega(X)$ is a Schur space, that is, weakly convergent sequences in $\mathcal{F}_\omega(X)$ are norm-convergent. It follows from this theorem that the uniform analogue of the lifting property fails unless X has the (quite restrictive) Schur property. Moreover, $\mathcal{F}_\omega(X)$ is (3ω)-uniformly homeomorphic to $X \oplus \operatorname{Ker} \beta_\omega$, and, as soon as $\lim_{t\to 0} \omega(t)/t = \infty$ and X fails the Schur property, we obtain canonical pairs of uniformly (even Hölder) homeomorphic separable Banach spaces which are not linearly isomorphic. We refer to ⟨**52**, **33**⟩ for other examples of such pairs.

Along with Hölder maps between Banach spaces, one may also consider Lipschitz maps between quasi-Banach spaces, and this is done in [**252**], where similar methods provide examples of separable quasi-Banach spaces which are Lipschitz, but not linearly, isomorphic.

7. *Interpolation, twisted sums, and the Kalton calculus*

The Banach–Mazur functional d_{BM} is a classical tool for estimating the 'distance' between two isomorphic Banach spaces, and similar functionals such as the Lipschitz distance d_L can be defined when more general notions of isomorphisms are taken into consideration. But in [**172**] 'distances' are defined between spaces which are not isomorphic, but are somewhat similar, such as ℓ_p and ℓ_q when p and q are close to each other. Indeed, if X and Y are two subspaces of a Banach space Z, let $\Lambda(X,Y)$ denote the Hausdorff distance between the closed unit balls B_X and B_Y, that is

$$\Lambda(X,Y) = \max\left\{ \sup_{x\in B_X} \inf_{y\in B_Y} \|x-y\|,\ \sup_{y\in B_Y} \inf_{x\in B_X} \|y-x\| \right\}.$$

The *Kadets distance* $d_K(X,Y)$ is the infimum of $\Lambda(\tilde{X},\tilde{Y})$ over all Banach spaces Z containing isometric copies $\tilde{X}$ and $\tilde{Y}$ of X and Y, respectively. The Kadets distance is a pseudo-metric which is controlled from above by d_{BM}, but there are non-isomorphic Banach spaces X and

Y such that $d_K(X,Y)=0$. The *Gromov–Hausdorff distance* d_{GH} is the non-linear analogue of the Kadets distance, defined along the same lines, except that the infimum is taken over all metric spaces containing isometric copies of X and Y. Of course, $d_{GH} \leqslant d_K$ and, for instance, $d_{GH}(\ell_p,\ell_1) \to 0$ as $p \to 1$, while $d_K(\ell_p,\ell_1)=1$ for all $p>1$. However, if X is a $\mathcal{K}$-space, then $d_{GH}(X_n,X)\to 0$ implies that $d_K(X_n,X)\to 0$. This can again be understood as an 'approximation by linear maps' on $\mathcal{K}$-spaces.

Interpolation theory provides families of Banach spaces which are not isomorphic, but tightly related, and the Kadets distance will make this remark precise, and usable. Moreover, interpolation leads to a 'differential calculus' on the 'manifold' of Banach spaces. We shall outline how Nigel Kalton's vision created links between this calculus, twisted sums, and quasi-linear maps.

Complex interpolation studies analytic families of Banach spaces. Let us restrict our discussion to a very important special case. Let W be some complex Banach space, and let X_0 and X_1 be two closed subspaces of W. We set $S=\{z\in\mathbb{C}: 0<\Re z<1\}$ and write $\mathfrak{F}$ for the space of analytic functions $F: S\to W$ which extend continuously to $\bar{S}$ and are such that $\{F(\mathrm{i}t): t\in\mathbb{R}\}$ is a bounded subset of X_0 and $\{F(1+\mathrm{i}t): t\in\mathbb{R}\}$ is a bounded subset of X_1. The space $\mathfrak{F}$ is normed by

$$\|F\|_{\mathfrak{F}} = \max_{j=0,1}\sup\{\|F(j+\mathrm{i}t)\|_{X_j} : t\in\mathbb{R}\}.$$

For $\theta\in(0,1)$ and $x\in W$, we define $\|x\|_\theta=\inf\{\|F\|_{\mathfrak{F}} : F(\theta)=x\}$ and

$$X_\theta=\{x\in W : \|x\|_\theta<\infty\}.$$

Set $W_0=\operatorname{span}\{X_\theta:\theta\in(0,1)\}$. A linear map $T: W_0\to W_0$ is *interpolating* if $F\longmapsto T\circ F$ is defined and bounded on $\mathfrak{F}$. If T is interpolating, then $T(X_\theta)\subseteq X_\theta$ for all $\theta\in(0,1)$.

The above space $X_\theta=[X_0,X_1]_\theta$ is said to be obtained from X_0 and X_1 by the *complex interpolation method*. It turns out that the Kadets distance is the right tool for discussing continuity properties of interpolation. Indeed, the following is shown in [**172**]: for $0<\theta<\varphi<1$, we have

$$d_K(X_\theta,X_\varphi)\leqslant 2\frac{\sin(\pi(\varphi-\theta)/2)}{\sin(\pi(\varphi+\theta)/2)}.$$

This continuity of the interpolation method with respect to the Kadets distance permits us to apply connectedness arguments. Indeed, let us call a property (P) *stable* if there exists $\alpha>0$ so that Y has (P), whenever X has (P) and $d_K(X,Y)<\alpha$. For instance, each of the following properties (P) is stable: separability, reflexivity, $X\supseteq\ell_1$, super-reflexivity, and $\operatorname{type}(X)>1$. Connectedness thus shows that, if $0<\theta<1$ and $X_\theta=[X_0,X_1]_\theta$ has (P), then X_φ has (P) for every $\varphi\in(0,1)$.

This line of thought opens an exciting field of research. It can be shown that the connected component of any separable Banach space X contains all isomorphic copies of X. It follows from ⟨**45**⟩ that the connected component of ℓ_2 contains all super-reflexive Banach lattices, but it is not known whether it contains all super-reflexive spaces. It is conjectured that the component of c_0 consists of all spaces isomorphic to a subspace of c_0. These concepts are also relevant to non-linear isomorphisms: it follows from instance from Sobczyk's theorem that, if $d_{GH}(X_n,c_0)\to 0$, then we have not only that $d_K(X_n,c_0)\to 0$ (since c_0 is a K-space), but actually that $d_{BM}(X_n,c_0)\to 0$ [**172**]. This implies for instance that, if the uniform distance between X and c_0 is small, then X is linearly isomorphic to c_0 [**182**, Theorem 5.7]. It is not known whether a space which is uniformly homeomorphic to c_0 is linearly isomorphic to c_0.

The Kadets and Gromov–Hausdorff distances are clearly topological notions, but interpolation points to some kind of differential structure, which we shall now briefly describe. Minimal extensions have been discussed in § 5. Let us say more generally that, if X, Y, and Z are quasi-Banach spaces, then Z is an *extension of X by Y* if $Z/Y\simeq X$. An extension Z is also called a *twisted sum* of X and Y (a *non-trivial twisted sum* if Y is not complemented in Z). We refer to ⟨**12**⟩ for a comprehensive survey of this matter.

Quasi-linear maps $\Omega : X \to Y$ were defined in § 5. The *extension* $X \oplus_\Omega Y$ of X by Y is the space $X \oplus Y$ equipped with the quasi-norm given by $\|(x, y)\| = \|x\| + \|y - \Omega x\|$. Even when X and Y are Banach spaces, $X \oplus_\Omega Y$ is not necessarily a Banach space: it is a Banach space if there exists $C > 0$ such that

$$\left\| \sum_{k=1}^{n} \Omega(x_k) - \Omega\left(\sum_{k=1}^{n} x_k\right) \right\| \leqslant C \sum_{k=1}^{n} \|x_k\| \quad (n \geqslant 1)$$

for all sequences all (x_k) in X. The latter condition always holds when X is a $\mathcal{K}$-space [**55**]. Indeed, every extension can actually be obtained with such an Ω: if $q : Z \to X$ is the quotient map, take $\Omega = S - R$, where $qS = qR = Id_X$, the map S is homogenous (but not necessarily linear) such that $\|Sx\| \leqslant 2\|x\|$, and R is linear (but not necessarily continuous). As in the case of Minimal extensions, the existence of a bounded linear projection from $X \oplus_\Omega Y$ onto Y is equivalent to the existence of $C > 0$ and a linear map $L : X \to Y$ such that $\|\Omega x - Lx\| \leqslant C\|x\|$ for all $x \in X$.

When $X = Y$, the space $X \oplus_\Omega X$ is called a *self-extension* of X; this space is denoted by $X \oplus_\Omega X = d_\Omega X$. When $X = \ell_2$, a non-trivial self-extension of ℓ_2 is called a *twisted Hilbert space*. It was shown in ⟨**20**⟩ that such spaces exist. An alternative example, the *Kalton–Peck* space Z_2, is constructed in [**60**] with the help of the Ribe functional (see § 5). Indeed, let $\Omega : \ell_2 \to \mathbb{R}^{\mathbb{N}}$ be defined by

$$\Omega((\xi_n)) = \left(\xi_n \log\left(\frac{|\xi_n|}{\|\xi\|_2}\right)\right)_{n \geqslant 1}.$$

The space $Z_2 = d_\Omega \ell_2$ is therefore the space of pairs $(\xi, \eta) = ((\xi_n), (\eta_n))$ of sequences such that

$$\|(\xi, \eta)\| = \left(\sum_{n=1}^{\infty} |\xi_n|^2\right)^{1/2} + \left(\sum_{n=1}^{\infty} \left|\eta_n - \xi_n \log\left(\frac{|\xi_n|}{\|\xi\|_2}\right)\right|^2\right)^{1/2} < \infty.$$

Of course, Z_2 is a Banach space since ℓ_2 is a $\mathcal{K}$-space. This Kalton–Peck space Z_2 exhibits remarkable features, which are not yet fully understood. It is plain that Z_2 has an unconditional finite dimensional decomposition (F.D.D.) consisting of two-dimensional spaces; however, it has no unconditional basis and no local unconditional structure ⟨**32**⟩. Actually, an unconditional F.D.D. with spaces of bounded dimension provides an unconditional basis which can be chosen from the subspaces *if* the space has local unconditional structure [**154**]. It is unknown, however, whether a twisted Hilbert space can have local unconditional structure; the best result so far is that it has no unconditional basis in full generality [**202**]. The space Z_2 is also an example of a symplectic Banach space which is not the direct sum of two isotropic subspaces [**80**]. In fact, intuition suggests that the space Z_2 is 'even-dimensional', and thus that it should not be isomorphic to its hyperplanes: this 40-year-old conjecture is still open, although examples of infinite-dimensional Banach spaces which are not isomorphic to their hyperplanes are now known ⟨**28**⟩. In fact, there are (non-separable) $C(K)$ spaces which are indecomposable and not isomorphic to their hyperplanes ⟨**37**, **46**⟩, and even with a minimality property ⟨**4**⟩. We note along these lines that spaces with two-dimensional unconditional F.D.D., but no unconditional bases, show up in the classification results shown in ⟨**35**⟩, which play a crucial role in Gowers' homogeneous space theorem ⟨**29**⟩.

As so often in Nigel Kalton's work, the conceptual framework in which the construction is completed provides flexibility, and leads to more results. Suppose that $F : \mathbb{R} \to \mathbb{C}$ is any Lipschitz map and that E is a Banach sequence space. Set

$$\Omega_F(\xi) = \left(\xi_n F\left(\log \frac{|\xi_n|}{\|\xi\|_E}\right)\right)_{n \geqslant 1} \quad \text{and} \quad d_{\Omega_F} E = E \oplus_{\Omega_F} E.$$

Taking $E = \ell_2$ and $F(t) = t^{1+i\alpha}$ $(\alpha \neq 0)$ provides a complex Banach space $Z(\alpha)$ which is not complex-isomorphic to its conjugate space $\overline{Z(\alpha)} = Z(-\alpha)$ [**147**]. The existence of such spaces had been shown in ⟨**8**, **58**⟩ by probabilistic methods. We refer to ⟨**21**⟩ for work on this topic and the existence of Banach spaces with exactly n complex structures for any given integer n, and also to ⟨**22**⟩.

The notation $d_\Omega X$ is reminiscent of differential calculus, and this is not by chance. With the above notation of the complex interpolation method, and following ⟨**55**⟩, we define a subspace dX_θ (called the *derived space*) of $W \times W$ by $dX_\theta = \{(x_1, x_2) : \ \|(x_1, x_2)\|_{dX_\theta} < \infty\}$, where

$$\|(x_1, x_2)\|_{dX_\theta} = \inf\{\|F\|_{\mathfrak{F}} : \ F(\theta) = x_1, F'(\theta) = x_2\}.$$

The space $Y = \{(x_1, x_2) \in dX_\theta : \ x_1 = 0\}$ is isometric to X_θ, and so is dX_θ / Y. Hence dX_θ is a self-extension of X_θ. By the above, one has $dX_\theta = d_\Omega X_\theta$ for some quasi-linear map $\Omega : X_\theta \to W$; actually, $\Omega(x) = F'(\theta)$, where $F \in \mathfrak{F}$ is such that $\|F\|_{\mathfrak{F}} \leqslant C\|x\|_\theta$ and $F(\theta) = x$, does the work. Now, if T is an interpolating operator, then $(x_1, x_2) \mapsto (Tx_1, Tx_2)$ is bounded on dX_θ, and this yields the Rochberg–Weiss commutator estimate:

$$\|T(\Omega x) - \Omega(Tx)\|_\theta \leqslant C\|x\|_\theta \quad (x \in X_\theta).$$

The sequence spaces ℓ_p, where $1 \leqslant p \leqslant \infty$, provide first examples of interpolation lines, and the above calculations applied to $X_0 = \ell_1$ and $X_1 = \ell_\infty$ provide the Kalton–Peck space $Z_2 = dX_{1/2}$, and more generally the spaces Z_p (which are twisted self-extensions of l_p) are the derived spaces to this interpolation line. Similar calculations are possible for the function spaces $L_p(\mathbb{T})$. For this interpolation scale, the Hilbert transform H is a very important example of an interpolating operator, and in this case the commutator estimate becomes

$$\|H(f \log|f|) - H(f)\log|H(f)|\|_p \leqslant C_p\|f\|_p \quad (f \in L_p(\mathbb{T}))$$

for $1 < p < \infty$ and some $C_p < \infty$.

Following [**111**, **130**], we now relate this differential calculus with the entropy functions defined in § 5. If X_0 and X_1 are separable sequence spaces, then the interpolation spaces X_θ are given by the Calderón formula $X_\theta = X_0^{1-\theta} X_1^\theta$, that is,

$$\|x\|_\theta = \inf\{\|x_0\|_0^{1-\theta}\|x_1\|_1^\theta : \ |x| = |x_0|^{1-\theta}|x_1|^\theta\}.$$

The entropy function Φ_X can conveniently be described as the *logarithm* of the sequence space X. Indeed one has $\Phi_{X_\theta} = (1-\theta)\Phi_{X_0} + \theta\Phi_{X_1}$, and, by the Lozanovskii factorization theorem, $\Phi_X + \Phi_{X^*} = \Phi_{\ell_1}$, where Φ_{ℓ_1} is the Ribe functional, whilst one also has $\Phi_{\ell_p} = (1/p)\Phi_{\ell_1}$, as in § 5, and $\Phi_{\ell_\infty} = 0$. It now becomes natural to see the Hilbert space as the geometric mean between any sequence space X and its dual X^*. The map '$X \mapsto \Phi_X$' is logarithmic-like, but in order to complete the picture we need conversely an *exponential* function which maps a quasi-linear map Φ to a Banach space. Suppose that $\Phi : c_{00}^+ \to \mathbb{R}$ is any functional. Then there exists a Banach sequence space X such that $\Phi_X = \Phi$ if and only if Φ and $\Phi_{\ell_1} - \Phi$ are both convex functions and Φ is positively homogeneous [**130**], and this space X has closed unit ball

$$B_X = \left\{(x_k) : \ \sum_{k=1}^{\infty} u_k \log|x_k| \leqslant \Phi(u) \text{ for all } u \geqslant 0\right\}.$$

This exponential map leads to what I suggest be called the *Kalton calculus*. It bears an uncanny resemblance to the exponentiation map from a Lie algebra to its Lie group, and creates 'lines' from infinitesimals; in other words, it yields *extrapolation*. For instance, if X is p-convex for some $p \in (1, 2)$ and also p^*-concave, then $X = Y^{1/p}$ for some sequence space Y, and so $(1/p)\Phi_{\ell_1} - \Phi_X$ is convex. Similarly, p^*-concavity means that $(1/p)\Phi_{\ell_1} - \Phi_{X^*}$ is convex. Now the equation

$$\Phi_X = (1-\theta)\Phi + \theta\Phi_{\ell_2} = (1-\theta)\Phi + \frac{\theta}{2}\Phi_{\ell_1}$$

provides a convex function Φ such that $\Phi_{\ell_1} - \Phi$ is also convex, and thus $\Phi = \Phi_Z$ for some Z. Exponentiating, we find that $X = Z^{1-\theta}\ell_2^{\theta}$, a result from ⟨**45**⟩.

To close the circle of ideas relating the entropy functions with derived spaces, we note that, if $X_\theta = X_0^{1-\theta}X_1^{\theta}$, then $dX_\theta = d_\Omega X_\theta = X_\theta \oplus_\Omega X_\theta$, where the quasi-linear map Ω satisfies

$$|\langle x^*, \Omega x\rangle - \Phi(xx^*)| \leqslant C\|x\|_{X_\theta}\|x^*\|_{X_\theta^*}.$$

Here $\Phi = \Phi_{X_1} - \Phi_{X_0}$ and xx^* denotes the pointwise product of the sequences x and x^*.

Special properties of the derived space $d_\Omega X_\theta$ can 'spread out' by exponentiation to a segment $\{X_\varphi : \ |\varphi - \theta| < \varepsilon\}$. Indeed, if X_0 and X_1 are acceptable function spaces on $\mathbb{T}$ and if R is the vector-valued Riesz transform, then R is bounded on X_θ for $|\theta - \theta_0| < \delta$ if and only if $\|R\Omega - \Omega R\|_{X_{\theta_0}} < \infty$. It follows that there exist twisted Hilbert spaces which are not UMD [**130**], although Z_2 is UMD [**104**].

We note at this point that higher-order derivatives can be considered, and this has been done, for example, in ⟨**10**, **54**⟩.

As seen before, differentiating interpolation lines yields quasi-linear maps Ω with $dX_\theta = d_\Omega X_\theta$. If for instance X_1 is obtained from X_0 through a change of weight, then the map Ω enjoys a commutation property, namely

$$\|\Omega(ax) - a\Omega(x)\|_{X_\theta} \leqslant C\|a\|_\infty\|x\|_{X_\theta}.$$

These special maps are called *centralizers* in [**111**], and the corresponding space $d_\Omega X_\theta$ is a *lattice twisted sum*. Note that the Kalton calculus, which we displayed here (following [**204**]) for sequences, is designed for function spaces, and this is what is done in [**111**, **130**]. Centralizers yield to extrapolation results: if, for instance, X is a super-reflexive sequence space and Ω is a real centralizer on X, then there exist super-reflexive Banach sequence spaces X_0 and X_1 such that $X = X_0^{1/2}X_1^{1/2} = X_{1/2}$ and moreover $dX_{1/2} \simeq d_\Omega X$. The Rochberg–Weiss commutator estimates now state that, if $X_\theta = X_0^{1-\theta}X_1^{\theta}$ and $dX_\theta = d_\Omega X_\theta$, then

$$\|T(\Omega x) - \Omega(Tx)\|_{X_\theta} \leqslant C\|x\|_{X_\theta}$$

for interpolating operators T. When, for instance, Ω is a centralizer, this estimate says that Ω nearly commutes not only with multiplication operators, but with all interpolating operators.

At this point, the extrapolation technique allows a change of perspective: starting from an operator T on X, Kalton considers all pairs (X_0, X_1) such that $X = X_0^{1-\theta}X_1^{\theta}$ and T is interpolating between X_0 and X_1, and he obtains a whole family of estimates on T. Doing this for $X = \ell_p$ yields the family of quasi-linear maps

$$\Phi_G(u) = \sum_{n=1}^{\infty} u_n G(\log|u_n|),$$

where G runs through the family $\mathcal{G}$ of 1-Lipschitz maps with compactly supported derivative from $\mathbb{R}$ to $\mathbb{R}$. This leads to consideration of the quasi-Banach space h_1^{sym} consisting of sequences ξ such that

$$\|\xi\|_{h_1^{\mathrm{sym}}} = \sum_{k=1}^{\infty}|\xi_k| + \sup_{G\in\mathcal{G}} \Phi_G(\xi) < \infty.$$

This 'tangent space' h_1^{sym} is conveniently described as the space of all sequence (ξ_k) in ℓ_1 such that

$$\sum_{n=1}^{\infty} \frac{1}{n}|\xi_1 + \xi_2 + \cdots + \xi_n| < \infty.$$

The same construction applied to function spaces leads to the symmetric Hardy function space $H^1_{\mathrm{sym}}(\mu)$ of all functions $f \in L^1(\mu)$ such that

$$\|f\|_{H^1_{\mathrm{sym}}} = \int |f| \, \mathrm{d}\mu + \sup_{G \in \mathcal{G}} \int |f| G(\log |f|) \, \mathrm{d}\mu < \infty.$$

Commutator estimates on interpolating operators are then used in [**111**] to show that, if $1 < p_0 < p < p_1 < \infty$ and $p^{-1} + q^{-1} = 1$ and if $T : L_{p_j} \to L_{p_j}$ is a bounded linear operator for $j = 0, 1$, then the bilinear form

$$B_T : (f, g) \longmapsto f \cdot T^* g - g \cdot T f, \quad L_p \times L_q \longrightarrow H^1_{\mathrm{sym}},$$

is bounded (where the above dot denotes the pointwise product of functions).

The above theorem can be applied to a variety of interpolating operators. When applied to the Riesz projection onto $L^2(\mathbb{T})$, it gives the inequality

$$\|fg\|_{H^1_{\mathrm{sym}}} \leqslant C \|f\|_2 \|g\|_2 \quad (f, g \in H^1(\mathbb{T}) \text{ with } g(0) = 0).$$

Since $H^1_0 = H^2 \cdot H^2_0$, one obtains the inequality $\|h\|_{H^1_{\mathrm{sym}}} \leqslant C\|h\|_1$ for every function $h \in H^1$ with $h(0) = 0$; this was first shown in ⟨**13**, **14**⟩.

The ideas developed above have non-commutative analogues, and the bridge which brings us to the non-commutative world is the concept of trace. If X is a symmetric Banach sequence space, then we denote by $\mathcal{C}_X$ the space of all operators T on ℓ_2 whose sequence $(s_n(T))_{n \geqslant 1}$ of singular numbers belongs to X. When X_0 and X_1 are reflexive, we have

$$[\mathcal{C}_{X_0}, \mathcal{C}_{X_1}]_\theta = \mathcal{C}_{X_0^{1-\theta} X_1^\theta} = \mathcal{C}_{X_\theta},$$

and interpolation tools apply to the spaces $\mathcal{C}_X$.

Let $\mathcal{C}_{\ell_1} = \mathcal{C}_1$ be the ideal of trace-class (or nuclear) operators on ℓ_2. A *trace* on $\mathcal{C}_1$ is a linear map τ such that $\tau(AB) = \tau(BA)$ for all $A \in \mathcal{C}_1$ and all bounded operators B. We write $\mathrm{Comm}(\mathcal{C}_1)$ for the linear span of all commutators $[A, B] = AB - BA$ with $A \in \mathcal{C}_1$ and B bounded. Clearly, if $S \in \mathcal{C}_1$, then $S \in \mathrm{Comm}(\mathcal{C}_1)$ if and only if $\tau(S) = 0$ for every trace τ. It was shown in ⟨**64**⟩ that $\mathrm{Comm}(\mathcal{C}_1)$ is strictly contained in $\{T \in \mathcal{C}_1 : \tau(T) = 0\}$, or, equivalently, that there exist *discontinuous* traces on $\mathcal{C}_1$. The precise description of $\mathrm{Comm}(\mathcal{C}_1)$ was obtained in [**117**] by interpolation arguments, and reads as follows:

THEOREM 7.1. *Let $T \in \mathcal{C}_1$ be a trace-class operator. Then $T \in \mathrm{Comm}(\mathcal{C}_1)$ if and only if its eigenvalue sequence $(\lambda_n(T))_{n \geqslant 1}$ belongs to h^1_{sym}.*

It was shown in [**117**] that every $T \in \mathrm{Comm}(\mathcal{C}_1)$ is the sum of 6 commutators, but this number has now been reduced to 3, and the case of general ideals of operators is also treated in ⟨**17**⟩ and [**165**, **166**]. We refer to [**120**] for characteristic determinants of trace-class operators and their use in this context. Alain Connes' trace theorem is extended in [**269**], and it is shown there that pseudo-differential operators of order $(-d)$ on $\mathbb{R}^d$ do not have a unique trace.

8. *Multipliers, and some of their uses*

Bases are valuable tools for computing in linear spaces and for representing (or defining) linear operators. Nigel Kalton's experience of all the subtle properties of bases allowed him to attack successfully a number of problems through original constructions of 'diagonal' operators.

We shall begin with Hilbertian theory. A linear operator T on a complex Hilbert space H is *power bounded* if $\sup_{n \in \mathbb{N}} \|T^n\| < \infty$. A classical theorem of Rota ⟨**56**⟩ shows that power-bounded operators are similar to operators of norm close to 1. A stronger requirement would be to show that a power-bounded operator T is similar to an operator such that $\sup_{n\in\mathbb{N}}\|T^n\|$ is close to 1; whether this is always possible is a question that was asked by Peller ⟨**44**⟩. Basis theory joined forces with harmonic analysis to provide a negative answer to Peller's question in [**195**].

A *weight* w on $\mathbb{T}$ is a non-zero function from $L^1(\mathbb{T})$ with $w \geqslant 0$. We denote by $L^2(w)$ the corresponding weighted L^2-space, and set

$$H^2(w) = \overline{\operatorname{span}}\{e^{in\theta} : n \geqslant 0\} \subseteq L^2(w).$$

The Riesz projection is formally defined on $L^2(w)$ by

$$R : \sum_{n\in\mathbb{Z}} \hat{f}(n)e^{in\theta} \longmapsto \sum_{n\geqslant 0} \hat{f}(n)e^{in\theta}, \quad L^2(w) \longrightarrow H^2(w),$$

and w is called an A_2-*weight* if R is a bounded projection, with norm $\|R\|_w$. Take $\varphi \in [0, \pi/2)$. Then [**195**, Proposition 2.2] proves the following Helson–Szegö estimate: $\|R\|_w \leqslant (\cos\varphi)^{-1}$ if and only if there exists $h \in H^1(\mathbb{T})$ with $|w - h| \leqslant w \sin\varphi$. In the case where $\alpha \in (0, 1)$ and $w(e^{i\theta}) = |\theta|^\alpha$ for $\theta \in (-\pi, \pi]$, this estimate leads to the formula

$$\inf\{\|R\|_v : \ v \sim w\} = \frac{1}{\cos(\pi\alpha/2)},$$

where $v \sim w$ means that w/v and v/w both belong to $L^\infty(\mathbb{T})$.

We consider a basis $(e_n)_{n\geqslant 0}$ of a separable Hilbert space H, and call $T : H \to H$ a *fast monotone multiplier* (with respect to (e_n)) if

$$T\left(\sum_{k=0}^{\infty} a_k e_k\right) = \sum_{k=0}^{\infty} \lambda_k a_k e_k$$

for an increasing sequence (λ_k) in $(0, 1)$ such that

$$\lim_{k\to\infty} \frac{1-\lambda_k}{1-\lambda_{k-1}} = 0.$$

Easy computations then show that $\sup_{n\geqslant 0}\|T^n\|$ is at most equal to the basis constant b of (e_n). If we now consider the (usually conditional!) basis $(e_k)_{k\geqslant 0}$ of $H^2(w)$, where w is an A_2-*weight*, and we set $e_k(\theta) = e^{ik\theta}$ for $k \geqslant 0$, then this basis constant is $b = \|R\|_w$.

The main result of [**195**] implies, in particular, the following theorem.

THEOREM 8.1. *Let $\alpha \in (0, 1)$, and set $w_\alpha(e^{i\theta}) = |\theta|^\alpha$ for $\theta \in (-\pi, \pi]$. Let T be a fast monotone multiplier with respect to the basis $(e_n)_{n\geqslant 0}$ of $H^2(w_\alpha)$. Then T is power bounded, and*

$$\inf_A \left\{\sup_{n\in\mathbb{N}} \|(A^{-1}TA)^n\|\right\} = \frac{1}{\cos(\pi\alpha/2)},$$

where the infimum is taken over all invertible operators A.

Therefore, a negative answer to Peller's question is obtained with fast monotone multipliers with respect to Babenko's conditional bases of H ⟨**5**⟩. More general weights are also considered in [**195**], which show that the infimum is usually *not* attained in Theorem 8.1.

Multipliers can also be unbounded, and such objects provide a negative answer to another important open question. We consider the following Cauchy problem:

$$u'(t) + B(u(t)) = f(t),$$

with the initial condition $u(0) = 0$. Here $t \in [0, T)$, $-B$ is the closed, densely-defined infinitesimal generator of a bounded analytic semi-group on a complex Banach space X, and u and f

are X-valued functions on $[0,T)$. One says that B satisfies *maximal regularity* if $u' \in L^2(X)$ as soon as $f \in L^2(X)$. Hence maximal regularity refers to a property of linear partial differential equations of the form $\partial_t u = Lu + f$, and states that the time derivative of the solution u belongs to the same space as the forcing term f. Such a regularity is useful for solving related non-linear problems. When X is a Hilbert space, every such B has maximal regularity ⟨**57**⟩, and the question of the converse occurs, in particular when $X = L^p$ with $1 < p < \infty$. A quite general answer is obtained in [**177**].

THEOREM 8.2. *Let X be a Banach space with an unconditional basis. Then every closed, densely-defined operator B such that $-B$ generates a bounded analytic semi-group on X has maximal regularity if and only if X is isomorphic to a Hilbert space.*

The proof goes as follows. Suppose that B satisfies maximal regularity. Then solving the Cauchy problem for well-chosen functions $f \in L^2(X)$ shows that, for every X-valued trigonometric polynomial $g(t) = \sum \hat{g}(n)e^{in\theta}$, one has

$$\left\| \sum_{n\in\mathbb{Z}} in(\mathrm{i}n + B)^{-1}\hat{g}(n)e^{in\theta} \right\|_{L^2(X)} \leqslant C\|g\|_{L^2(X)}.$$

This is applied to closed, densely-defined operators B of the form

$$B : \sum_{n=1}^{\infty} a_n e_n \longmapsto \sum_{n=1}^{\infty} a_n b_n e_n,$$

where (e_n) is an unconditional basis of X and (b_n) is an increasing sequence of positive real numbers. Any such multiplier $B = M((b_n))$ is *sectorial* of type ω for each $\omega \in (0, \pi)$ and, in particular, $-B$ generates a bounded analytic semigroup. Note that when the sequence $(\hat{g}(n))$ is unconditional, we have

$$\|g\|_{L^2(X)} \simeq \left\| \sum \hat{g}(n) \right\|_X .$$

Then the maximal regularity inequality applied with a proper choice of the scalar sequence (b_n) shows after some work that, for any block basis (u_j) of any permutation $(e_{\pi(n)})$ of (e_n), the space $\overline{\mathrm{span}}\{u_j\}$ is complemented in X. It then follows from ⟨**41**⟩ that (e_j) is equivalent to the canonical basis of c_0 or ℓ_p for some $p \in [1, \infty)$. Since $\ell_p \simeq (\sum_{n\geqslant 1} \oplus \ell_2^n)_p$ has non-equivalent unconditional bases whenever $1 < p < \infty$ and $p \neq 2$, it follows that X is c_0, ℓ_1, or ℓ_2. Finally, c_0 and ℓ_1 can be discarded with the help of a multiplier relative to the (conditional) summing basis of c_0.

Since the Haar system is an unconditional basis of L^p, Theorem 8.2 answers negatively the maximal regularity problem for L^p when $1 < p < \infty$ and $p \neq 2$. The case of L^1 had been covered in ⟨**39**⟩. The unconditionality assumption in Theorem 8.2 can be weakened, but it is not known whether the conclusion holds for every space X with a basis (see [**193**]). The discrete-time analogue of the maximal regularity problem, which presents some specific difficulties, has been investigated in ⟨**7**, **48**, **49**, **249**⟩.

9. *Differential equations*

The maximal regularity problem can be phrased more generally as follows: When can we 'solve' an equation

$$Au + Bu = f$$

for commuting sectorial (unbounded) operators A and B on a Banach space X by the 'formal' solution' $u = (A+B)^{-1}f$? For this, we must be sure that $u \in D(A) \cap D(B)$ and, moreover, that

$$\|Au\| + \|Bu\| \lesssim \|Au + Bu\|. \tag{9.1}$$

We recall that an operator A is *sectorial* of angle ω if the spectrum $\sigma(A)$ is contained in a sector $\Sigma_\omega = \{\lambda \in \mathbb{C} : |\arg\lambda| < \omega\} \cup \{0\}$ and further satisfies the resolvent estimate that $\|(\lambda - A)^{-1}\| \lesssim |\lambda|^{-1}$ for $\lambda \notin \Sigma_\omega$. Let $\omega(A)$ be the infimum over all such ω. The special case where $-A$ generates an analytic semigroup e^{-tA} on a Banach space Y and B is the time derivative ∂_t on $X = L^p([0,T], Y)$ is precisely the 'maximal regularity property' of A.

The maximal regularity property for operators in UMD spaces X was characterized in ⟨**63**⟩ in terms of R-sectoriality. A set τ of bounded operators on X is *R-bounded* if, for all $T_1, \ldots, T_n \in \tau$ and $x_1, \ldots, x_n \in X$, we have

$$\mathbb{E}\left\|\sum_{j=1}^{n} \varepsilon_j T_j x_j\right\|_X \lesssim \mathbb{E}\left\|\sum_{j=1}^{n} \varepsilon_j x_j\right\|_X,$$

where (ε_j) is a sequence of Rademacher functions or, equivalently, an independent sequence of Bernoulli random variables. A sectorial operator A is *R-sectorial* if, for some $\nu > \omega(A)$, the set $\{\lambda R(\lambda, A) : \lambda \notin \Sigma_\nu\}$ is not just bounded, but an R-bounded set. Again $\omega_R(A)$ is the infimum over all such ν. Lutz Weis showed in ⟨**63**⟩ that, if A is the generator of a bounded analytic semigroup on a UMD space X, then maximal regularity of A is equivalent to its R-sectoriality.

This characterization of maximal regularity hinted at the following theorem for operator sums [**187**]: if A and B are resolvent commuting operators on a Banach space X, if B has a bounded H^∞-calculus on Σ_ω and A is R-sectorial with $\omega_R(A) + \omega < \pi$, then $A + B$ is closed on $D(A) \cap D(B)$ and (9.1) holds. The strength of this theorem lies in the asymmetry of the assumptions on A and B; in applications, B is usually a 'standard' operator such as a partial derivative or the Laplace operator for which the boundedness of the H^∞-calculus is well known, whereas A may be a differential operator with rough coefficients.

Here, as well as in results on maximal regularity and Fourier multiplier theorems on Bochner spaces (see, for example, ⟨**15**, **38**⟩), R-sectorial operators 'behave' like Hilbert space operators. So, when extending Hilbert space results to a Banach space setting, one can often 'replace' the missing Hilbert space structure by assuming R-boundedness instead of norm-boundedness for the relevant set of operators. This phenomenon is explained by the following theorem, taken from [**272**].

THEOREM 9.1. *Let X be a Banach space of finite cotype, and suppose that $\tau \subset \mathcal{B}(X)$ is an R-bounded set. Denote by $\bar\tau$ the closure of* $\operatorname{absco}(\tau)$ *in the strong operator topology, and define $\|T\|_{\bar\tau} = \inf\{\lambda > 0 : \lambda^{-1}T \in \bar\tau\}$ on the linear span B_τ of $\bar\tau$ in $\mathcal{B}(X)$. Then there exists a subalgebra B of $\mathcal{B}(H)$ for some* (*abstract*) *Hilbert space H, a bounded linear map $\sigma : B_\tau \to B$, and a continuous algebra homomorphism $\rho : B \to \mathcal{B}(X)$ such that $(\rho \circ \sigma)(T) = T$ $(T \in B_\tau)$, $\|\rho\| \leqslant 2$, and $\|\sigma\| \leqslant 4R(\tau)$.*

So R-boundedness is a rather strong property for a set τ of operators because it implies that τ is closely related to a set of Hilbert space operators. At the same time large classes of operators central to spectral theory, evolution equations, and harmonic analysis are known to be R-bounded. We refer in particular, to [**201**, **226**, **231**, **235**, **272**, **273**] for the articles which followed [**187**].

It should be mentioned that much earlier in his career, Nigel Kalton had met differential equations in a quite different context. Optimal control considers a system of ordinary differential equations over a time interval $I = [0, 1]$:

$$\frac{dx}{dt} = f(t, x, y(t)). \tag{9.2}$$

Here $x \in \mathbb{R}^m$ and $f : I \times \mathbb{R}^m \times Y \to \mathbb{R}^m$ is a continuous function which is Lipschitz in x. At each time $t \in I$ a controller chooses measurably $y(t)$ from a compact metric space Y. Given $x_0 \in \mathbb{R}^m$, there is a trajectory $x(\cdot)$ satisfying (9.2) for any such 'control' $y(\cdot)$. We assume that there is an associated total 'reward':

$$C(y) = g(x(1)) + \int_0^1 h(t, x(t), y(t))\,\mathrm{d}t,$$

where the functions g and h are assumed to satisfy suitable continuity conditions. Control theory discusses how the control $y(\cdot)$ should be chosen at each time $t \in I$ to maximize $C(y)$.

A chess player such as Nigel was bound to get involved in game theory. And indeed, the articles [**11**, **13**] investigate problems which links control theory with non-discrete extensions of von Neumann's game theory and his saddle-point result. In a two-person, zero-sum deterministic differential game, there are two controllers, or players, J_1 and J_2. At each time $t \in I$, player J_1 chooses an element $y(t)$ from a compact metric space Y and J_2 chooses an element $z(t)$ from a similar space Z. The dynamics are now given by

$$\frac{dx}{dt} = f(t, x, y(t), z(t)). \tag{9.3}$$

The 'reward' for player J_1 is given by

$$P(y, z) = g(x(1)) + \int_0^1 h(t, x(t), y(t), z(t))\,\mathrm{d}t.$$

The game is zero-sum, and so what is gained by J_1 is lost by J_2, and vice versa. Therefore, J_1 will choose the control $y(\cdot)$ to maximize $P(y, z)$, whilst J_2 will choose $z(\cdot)$ to minimize $P(y, z)$. An initial problem is how to model dynamically the choice of control values by J_1 and J_2 as time evolves. The players do not know in advance the control values chosen by their opponent. However, each player should react to the other's choice as time evolves.

Nigel Kalton and Robert Elliott proceed as follows. Let M_1 (respectively, M_2) be the set of measurable functions $y : I \to Y$, (respectively, $z : I \to Z$). A function $\alpha : M_2 \to M_1$ is a *pseudo-strategy* for J_1. It produces a 'reply' in M_1 to any control function $z(\cdot)$ chosen by J_2 from M_2. A pseudo-strategy α has a *value*

$$u(\alpha) = \inf_{z \in M_2} P(\alpha z, z),$$

which is the worst outcome for J_1 if α is used. However, α does not know future values of $z(\cdot)$, and so $\alpha : M_2 \to M_1$ is a strategy if, whenever $0 < T \leqslant 1$ and $z_1(t) = z_2(t)$ for $0 \leqslant t \leqslant T$, then $(\alpha z_1)(t) = (\alpha z_2)(t)$. Similar definitions are given for strategies $\beta : M_1 \to M_2$ for J_2. Writing Γ (respectively, Δ) for the set of strategies for J_1 (respectively, for J_2), the value of the game to J_1 is $U = \sup_{\alpha \in \Gamma} u(\alpha)$. Similarly, the value of the game to J_2 is

$$V = \inf_{\beta \in \Delta} v(\beta) = \inf_{\beta \in \Delta} \sup_{y \in M_1} P(y, \beta y).$$

These strategies are now called *Elliott–Kalton* strategies in the literature (see ⟨**23**⟩).

If $U = V$, then the differential game is said to have a *value*. Rather than considering the dynamics (9.3) starting at time $t = 0$ and location $x_0 \in \mathbb{R}^m$, we can consider games with dynamics starting at $t \in [0, 1]$ at position $x \in \mathbb{R}^m$. This gives rise to similar quantities $U(t, x)$ and $V(t, x)$. It turns out (see ⟨**18**⟩) that $V(t, x)$ is a viscosity solution of the equation

$$\partial V/\partial t + \min_z \max_y ((\nabla V \cdot f) + h) = 0$$

with $V(1,x) = g(x)$ and, similarly, $U(t,x)$ is a viscosity solution of

$$\partial U/\partial t + \max_y \min_z((\nabla U \cdot f) + h) = 0$$

with $U(1,x) = g(x)$. The *Isaacs* (*saddle point*) *condition* holds if

$$\min_z \max_y((p \cdot f) + h) = \max_y \min_z((p \cdot f) + h) \quad (p \in \mathbb{R}^m).$$

It is shown in [**13**] that, if the Isaacs condition holds, then $U(t,x) = V(t,x)$ and the game has a value. More generally, the domains of f and h can be extended to the sets of probability measures on Y and Z, respectively, and [**13**] then shows that, if the players J_1 and J_2 use such relaxed controls, then the Isaacs condition always holds and the game has a value.

We refer to [**16**, **19**, **22**, **27**, **28**, **29**, **30**, **32**, **35**, **36**] for subsequent articles on differential games, most of these being joint works with Robert Elliott.

10. *Greedy bases*

When Banach spaces are used in applied mathematics, coordinate systems are usually needed. Schauder bases constitute the primary way for providing such coordinates, and this leads to many theoretical as well as practical problems. Nigel Kalton contributed very significantly to every subfield of the study and use of Schauder bases, as can be checked throughout this survey of some of his works. In this respect, one should single out the article [**192**] and its links to Casazza's fundamental contributions to frame theory. We focus in this section on a part of Nigel's work which is clearly directed towards numerical analysis and concrete applications.

Let (e_i) be a normalized Schauder basis for a Banach space X with dual basis (e_i^*). For $x \in X$, the *error* in the best n-term approximation to x is given by

$$\sigma_n(x) := \inf\left\{\left\|x - \sum_{i\in\Lambda} a_i e_i\right\| : a_i \in \mathbb{R},\ |\Lambda| \leqslant n\right\}.$$

The *Thresholding Greedy Algorithm* (TGA) was introduced by Temlyakov ⟨**60**⟩ for the trigonometric system and extended to the Banach space setting by Konyagin and Temlyakov ⟨**36**⟩. See ⟨**61**⟩ and the recent monograph ⟨**62**⟩ for a history of the problem and for background information on greedy approximation. The TGA is defined as follows. For $x \in X$ and $n \in \mathbb{N}$, let $\Lambda_n(x) \subset \mathbb{N}$ be the set of indices corresponding to a choice of the n largest basis coefficients of x in absolute value, that is, $\Lambda_n(x)$ satisfies

$$\min\{|e_i^*(x)| \colon i \in \Lambda_n(x)\} \geqslant \max\{|e_i^*(x)| \colon i \notin \Lambda_n(x)\}.$$

We call $G_n(x) := \sum_{i\in\Lambda_n(x)} e_i^*(x)e_i$ an *nth greedy approximant to* x, and say that the TGA *converges* if $G_n(x) \to x$. Note that these operators G_n are not linear, and that $G_n(x)$ is well-defined only if the above inequality is strict.

In ⟨**66**⟩, it was proved for the multi-variate Haar system, normalized in $L_p[0,1]^d$, with $d \geqslant 1$ and $1 < p < \infty$, that there is a constant $C > 0$ such that

$$\|x - G_n(x)\| \leqslant C\sigma_n(x) \quad (x \in L_p[0,1]^d).$$

The case where $d = 2$ is especially interesting for its applications to image compression. A basis is said to be *greedy* (or C-greedy) if it satisfies such an inequality. More generally, a basis (e_i) is *quasi-greedy* if there exists $K < \infty$ such that, for all $x \in X$ and $n \geqslant 1$, we have $\|G_n(x)\| \leqslant K\|x\|$. By a result of Wojtaszczyk ⟨**66**, Theorem 1⟩ a basis (e_i) is quasi-greedy if and only if the TGA converges for all target vectors $x \in X$. Greedy bases are precisely the bases for which the TGA essentially provides a best n-term approximation to any target vector x (up to a factor of the greedy constant C).

Konyagin and Temlyakov ⟨**36**⟩ showed that a basis is greedy if and only if it is unconditional and democratic, where (e_i) is *democratic* with constant Δ if, for all finite subsets A and B of $\mathbb{N}$ with $|A| = |B|$, we have

$$\left\|\sum_{i\in A} e_i\right\| \leqslant \Delta \left\|\sum_{i\in B} e_i\right\|.$$

It is clear that an unconditional basis is quasi-greedy, but the converse is false, even in a Hilbert space ⟨**66**⟩.

The article [**205**] introduces a property that is intermediate between quasi-greedy and greedy. A basis is *almost greedy* if there is a constant C such that $\|x - G_n(x)\| \leqslant C\tilde{\sigma}_n(x)$, where we let

$$\tilde{\sigma}_n(x) = \inf\left\{\left\|x - \sum_{k\in A} e_k^*(x)e_k\right\| : \ |A| \leqslant n\right\}.$$

A major theme of this work is the performance of the TGA in the case of an almost greedy basis. In this direction, the main result, from [**205**], is the following important characterization of almost greedy bases in the spirit of the characterization of greedy bases given by Konyagin and Temlyakov.

THEOREM 10.1. *Suppose that (e_i) is a basis of a Banach space X. Then the following are equivalent:*

(a) (e_i) *is almost greedy*;
(b) (e_i) *is quasi-greedy and democratic*;
(c) *for any* (*respectively*, *every*) $\lambda > 1$, *there is a constant* $C = C_\lambda$ *such that*

$$\|x - G_{[\lambda m]}(x)\| \leqslant C_\lambda \sigma_m(x) \quad (x \in X).$$

What matters now is that many classical Banach spaces which do not have an unconditional basis (and thus have no greedy basis) can be shown to admit an almost greedy basis. For instance, a space X with a basis such that X contains a complemented subspace with a symmetric basis and has finite cotype has an almost greedy basis [**208**], and this applies, in particular, to $L_1[0,1]$ and to the Schatten ideals S_p. However, the Haar basis is not quasi-greedy in $L_1[0,1]$, and it seems to be an open problem to find a 'natural' quasi-greedy basis of that space. Clause (c) of the above theorem is important because it implies that the TGA, while no longer optimal (up to the greedy constant) for m-term approximation, nevertheless performs very effectively. Setting for instance $\lambda = 2$, we see that the greedy approximant $G_{2m}(x)$ provides an approximation which is essentially as good as the best m-term approximation. The convergence of various greedy algorithms is also investigated in [**207**].

In the paper [**208**], Kalton and his collaborators also studied the *thresholding operators* defined by

$$\mathcal{G}_a(x) = \sum_{|e_i^*(x)|\geqslant a} e_i^*(x)e_i \quad (a > 0,\ x \in X).$$

The natural boundedness conditions imposed on these operators yield a corresponding class of *thresholding-bounded* bases. It is shown in [**208**] that this class of bases coincides with the class of *nearly unconditional* bases introduced by Elton ⟨**19**⟩, and that it strictly contains the class of quasi-greedy bases. We recall that a normalized basis (e_n) in a Banach space X is *nearly unconditional* if, for every $0 < a \leqslant 1$, there exists a constant $\varphi(a)$ such that, for every

$x = \sum_{n=1}^{\infty} e_n^*(x)e_n \in X$ and $A \subseteq \{n \in \mathbb{N}\colon |e_n^*(x)| \geqslant a\}$, we have

$$\left\| \sum_{n\in A} a_n e_n \right\| \leqslant \varphi(a)\|x\|.$$

With this notation, a basis (e_n) is unconditional if and only if $\sup_{a>0} \varphi(a) < \infty$.

Elton proved that every normalized, weakly null sequence in a Banach space admits a nearly unconditional subsequence. It is now known ⟨**30**⟩ that some Banach spaces contain no unconditional basic sequence, but the question whether every infinite-dimensional Banach space contains a quasi-greedy basic sequence is still open. In this direction, it was shown in [**208**] that, if (e_n) is a semi-normalized, democratic, weakly null basic sequence in a Banach space, then (e_n) has an almost greedy subsequence. In particular, if X is a Banach space which does not have c_0 as a spreading model (for example, if X has finite cotype), then every semi-normalized, weakly null sequence in X has an almost greedy subsequence. A further generalization of Elton's theorem was given by Dilworth, Odell, Schlumprecht, and Zsák ⟨**16**⟩. Their techniques could be also used to prove other partial unconditionality results. On the other hand, we refer to [**175**] for quantitative results on the approximation of smooth functions by polynomials of a given degree on bounded, convex domains of $\mathbb{R}^n$.

11. *Isometric theory*

It is sometimes useful to work with special norms on Banach spaces: they might be canonical or easy to compute, or they can be tightly related with the structure of operators of the space, or they can provide isomorphic information on the space. All this motivates us to investigate isometric theory, that is, the study of Banach spaces equipped with a given norm. We refer to ⟨**65**⟩ for a very useful survey on Nigel Kalton's work in isometric theory.

It should be pointed out that the real and complex isometric theories are quite different.

On a complex Banach space X, one can define the notion of an *Hermitian* linear operator T by: $\|e^{isT}\| = 1$ for all $s \in \mathbb{R}$. Let us say that $x \in X$ is *Hermitian* if there exists $x^* \in X^*$ such that $x^* \otimes x$ is an Hermitian operator. It is not difficult to check that a projection P is Hermitian if and only if $\|P + \lambda(I - P)\| = 1$ for all $\lambda \in \mathbb{C}$ with $|\lambda| = 1$. In other words, Hermitian projections are 'orthogonal'. It follows that a complex Banach space with a 1-unconditional basis is the closed linear span of its Hermitian elements. A remarkable result of Kalton and Wood [**43**] states the converse.

THEOREM 11.1. *A complex Banach space X which is the closed linear span of its Hermitian elements has a* 1*-unconditional basis.*

An important corollary of Theorem 11.1 is that, if X is a 1-complemented subspace of a complex Banach space Y with a 1-unconditional basis, then X has a 1-unconditional basis. It is still unknown, however, whether a contractively complemented subspace of a separable, complex, order-continuous Banach lattice is isometric to an order-continuous complex Banach lattice. Note that, in this same paper, Kalton and Wood show that the space $C([0, 1])$ of complex-valued, continuous functions on the unit interval has a maximal norm, that is, there is no equivalent norm with a strictly larger group of isometries. Such results are motivated by the (still open) Banach–Mazur problem: whether a separable Banach space whose invertible isometries act transitively on the sphere is isometrically Hilbertian.

It should be noted that in the *real* case, the existence of 1-unconditional bases does not pass to 1-complemented subspaces ⟨**40**⟩. We refer to ⟨**50**⟩ for important results on 1-complemented subspaces of spaces with 1-unconditional bases. The Kalton–Wood theorem is one of the few available positive results, whilst it is still not known whether a complemented subspace of a space Y with an unconditional basis has an unconditional basis. A negative answer looks plausible, since for instance it is observed in [**123**, § 4] that a space with the bounded approximation property, but no F.D.D., as constructed by C. J. Read (unpublished) is complemented in a space with an unconditional F.D.D.

The field of approximation properties in Banach spaces is filled with ingenious and deep counter-examples. However, [**123**] contains a major positive result.

THEOREM 11.2. *Let X be a separable Banach space having the metric approximation property* (M.A.P.). *Then X has the commuting metric approximation property* (C.M.A.P.).

In other words, if Id_X is the uniform limit on compact sets of a sequence of finite-rank linear contractions, then there is such an approximating sequence consisting of commuting operators. Along these lines, it is shown in [**159**] that the unconditional M.A.P. is equivalent to its commutative version for all separable spaces (with a simpler proof for complex spaces, again using Hermitian operators). It is, however, still open whether every separable Banach space with the bounded approximation property has the commuting bounded approximation property.

An isometric concept which turned out to be very useful was defined by Alfsen and Effros ⟨**3**⟩: a closed subspace X of a Banach space Y is an *M-ideal* in Y if there is a subspace V of Y^* such that $Y^* = V \oplus_1 X^\perp$, where $\oplus_1$ means that

$$\|y^* + z^*\| = \|y^*\| + \|z^*\| \quad (y^* \in V,\ z^* \in X^\perp).$$

We refer to ⟨**31**⟩ for the theory as it was in 1993, immediately after Kalton's breakthrough [**134**].

Although the notion of an M-ideal is independent of any algebraic structure, it turns out to be tightly related to the notion of 'ideal' from operator theory, and for instance ideals $\mathcal{K}(X)$ of compact operators in spaces $\mathcal{B}(X)$ of bounded operators provide a wealth of examples of M-ideals (see Chapters V and VI in ⟨**31**⟩). Following [**134**], let us say that a Banach space X has *Property* (M) if, for any sequence (x_n) in X with $w - \lim_{n\to\infty} x_n = 0$, one has

$$\overline{\lim}\|x + x_n\| = \overline{\lim}\|y + x_n\|$$

for all $(x, y) \in X^2$ with $\|x\| = \|y\| = 1$. In other words, the norm of X is 'asymptotically isotropic' and all vectors of the sphere 'look the same' when seen from infinity. Note that Property (M) is clearly hereditary. It is shown in [**134**] that this property forbids distortion: if X has (M), then there is some $p \in [1, \infty]$ such that, for every $\varepsilon > 0$, the space X contains a subspace that is $(1 + \varepsilon)$-isomorphic to ℓ_p when $p < \infty$ and to c_0 when $p = \infty$.

Note that it is obvious that ℓ_p spaces equipped with their natural norms have Property(M), whilst L_p spaces fail to have it. In fact, if X is a separable, order-continuous non-atomic Banach lattice and X has an equivalent norm with Property (M), then X is lattice-isomorphic to L_2 [**134**]; this is, in particular, the case when X is isomorphic to a subspace of an Orlicz sequence space h_F. More generally, Fenchel–Orlicz spaces can be renormed to have Property (M) [**163**], and this applies for instance to the Kalton–Peck spaces Z_p.

The isotropy condition (M) implies that some unconditionality is available. Indeed [**148**, Theorem 2.13] reads:

THEOREM 11.3. *Let X be a separable Banach space. Then $\mathcal{K}(X)$ is an M-deal in $\mathcal{B}(X)$ if and only if X does not contain a copy of ℓ_1, X has Property (M), and X has the metric compact approximation property.*

It is also shown in [**148**] that, for $1 < p < \infty$ with $p \neq 2$ and X an infinite-dimensional, closed subspace of L_p, the closed unit ball B_X is $\|\cdot\|_1$-compact if and only if, for each $\varepsilon > 0$, there is a subspace X_ε of ℓ_p such that $d_{BM}(X, X_\varepsilon) < 1 + \varepsilon$. This theorem has been pushed to the case where $p = 1$ in [**156**] to characterize subspaces of L_1 which ε-embed into ℓ_1; this requires a visit to the Kalton zone $0 \leqslant p < 1$. The main result of [**156**] states that, if X is a closed subspace of L^1 with the approximation property, then B_X is L_p-compact and locally convex for some (equivalently, for all) $p \in [0, 1)$ if and only if, for each $\varepsilon > 0$, there is a quotient space E_ε of c_0 such that $d_{BM}(X, E_\varepsilon^*) < 1 + \varepsilon$.

Following [**123**], we say that $X \subseteq Y$ is a *u-ideal* in Y if

$$Y^* = V \oplus X^\perp \text{ and } \|y^* + z^*\| = \|y^* + \lambda z^*\| \quad (y^* \in Y^*,\ z^* \in X^\perp,\ |\lambda| = 1).$$

The article [**136**] is devoted to this notion which, thanks again to Hermitian operators, is rather nicer in the complex case. Let us denote by $Ba(X)$ the subspace of X^{**} consisting of weak*-limits of weak*-convergent *sequences* of elements of X. With this notation, it follows from [**136**, Theorem 6.5] that, if X is a separable, complex Banach space which is a u-ideal in its bidual, then there exists an Hermitian projection from X^{**} onto $Ba(X)$. Moreover, X has Pełczyński's property (u). Hence, if one thinks of $Ba(X)$ as the 'band' generated by X in X^{**}, then one can say that the embedding of a u-ideal in its bidual looks very much like the embedding of an order-continuous Banach lattice. However, u-ideals (such as $\mathcal{K}(X)$ spaces, with X reflexive and with the unconditional compact approximation property) in general bear no usable order structure. We refer to [**92**] for (order) ideal properties of (algebraic) ideals of operators between Banach lattices.

Nigel Kalton returned one last time to Hermitian operators in the article [**266**]. He had completed this work a few days before passing away, and his draft manuscript was kindly edited later on by Garth Dales. Nigel showed for instance in this last article the unexpected theorem that, if E is a complex Banach lattice and $T \in \mathcal{B}(E)$ is an Hermitian operator, then T^2 is Hermitian; it was known that this property fails to hold in general for complex Banach spaces. His results supplement earlier work from [**43**] for the special case of spaces with 1-unconditional bases. Another important result from this last paper is that, if $E = F \oplus G$ and $\|x + y\| = \|(|x|^p + |y|^p)^{1/p}\|$ for some $p \geqslant 1$ with $p \neq 2$ and all $x \in F$ and $y \in G$, then this direct sum is a band decomposition. The case $p = 2$ in the above equation holds true when the decomposition is Hermitian.

12. *As you like it*

Around 1948, a very young Nigel recited to his father a few lines from Shakespeare, or almost so. Now that life is over, we should recall what these lines really are:

O good old man, how well in thee appears
The constant service of the antique world,
When service sweat for duty, not for meed!

Nigel has never been old. His life was, alas, too short for that, and moreover he kept to the very end the mind and the abilities of his youth. But he certainly was a good man, dedicated to constantly serving mathematics, and through mathematics all of us. And although he would not have used such big words, his service was motivated by duty before being a quest for recognition, fame, or wages. He gave us the outstanding example of a giant of mathematics who bends as much as it is necessary to be understood, but never lower. His sleepless soul has now come to rest. But his influence is stronger than ever.

I am thankful to the London Mathematical Society for bestowing upon me the daunting honour of writing Nigel Kalton's obituary. Special thanks are due to Professors Nick Bingham and Garth Dales for their careful editing of the present text. I am very grateful to Professor

Graham Kalton, who kindly shared with me his memories of Nigel's early years. Mrs Jennifer Kalton is a friend of thirty years, whose home has always been open to Nigel's collaborators and friends. She kindly edited this obituary: for this, and much more, I am truly thankful to her.

My deepest gratitude goes to Nigel Kalton, the generous genius to whom I owe more than I can express. On behalf of all his colleagues, students, and collaborators, and in my own name, let me simply say: thank you, Nigel. We all miss you.

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