OBITUARY

BRIAN KUTTNER

With the death of Brian Kuttner on 2 January 1992, near Birmingham, England, summability theory has lost one of its leading exponents. During the period 1934–1991 he published over 100 research papers, including work on Fourier series, strong summability, Riesz means, Nörlund methods and Tauberian theory. Since his death, further papers with joint authorship have continued to appear; his influence persists.

Brian Kuttner was born on 11 April 1908 in London. The eldest child of George Henry Kuttner and Lilian Frances Kuttner (née Usherwood), he was educated at University College School, Hampstead, London, together with his brothers Conrad and Roland. Brian also had a sister, Elsa, who was the twin of Conrad. The Kuttners were a happy family, performing musical quartets at home, with Brian playing the piano.

Brian won a scholarship to Christ's College in the University of Cambridge, and following a distinguished undergraduate career he graduated BA in 1929, obtained his MA in 1932, and his PhD in 1934. His research supervisor seems to have been S. W. P. Steen, who is mentioned in Brian's first publication [**1**]. For a short period Brian also studied with the eminent German analyst Edmund Landau at the University of Göttingen.

In 1932 Brian was appointed Assistant Lecturer in Mathematics in the University of Birmingham, where he remained for the rest of his academic life. He became Lecturer in 1936, Senior Lecturer in 1952, Reader in 1955, and Professor of Mathematical Analysis in 1969. In 1932 the Mason Professor of Mathematics in Birmingham was the famous G. N. Watson, FRS, who had been Senior Wrangler at the University of Cambridge in 1907. Watson was, of course, the author of *The theory of Bessel functions*, and joint author, with E. T. Whittaker, of *A course of modern analysis*. The latter work can no longer be regarded as modern, but both books are of such quality that they are even now in regular use by many mathematicians who require analysis of the classical variety.

With G. N. Watson and Brian Kuttner, the University of Birmingham was twice blessed with formidable analytical power for over thirty years.

On retirement in 1975, Brian Kuttner became Emeritus Professor, fruitfully continuing his research activities until the end of his life. Analysts involved in collaboration with him include G. Das, I. J. Maddox, B. P. Mishra, R. N. Mohapatra, S. Nanda, M. R. Parameswaran, C. T. Rajagopal, M. S. Rangachari, B. E. Rhoades, B. N. Sahney and B. Thorpe.

Although Brian Kuttner and I, Ivor J. Maddox, researched together on relatively few occasions, I always looked forward to our meetings, and our joint publications gave me great satisfaction. It is difficult to describe our method of working, except to say that we were essentially complementary; Brian Kuttner concentrated on one aspect of the material and I on another.

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Brian enjoyed travelling, and attended the International Congress of Mathematicians at numerous venues, including the Moscow Congress in 1966, at which I too was present. Also, he regularly contributed to the seminars in Analysis at the British Mathematical Colloquium, meetings of which were held annually in various parts of the United Kingdom, usually giving the first talk of the series. It was his habit to arrive early for a Colloquium so that he could explore the surrounding countryside on foot. The quality of his map-reading of unknown territory was legendary. I recall that when the Colloquium was held in the University of Manchester, Brian took the train from Birmingham to Macclesfield, and then walked over hilly terrain to Manchester, a distance of over twenty miles, able to do so even though he was well into his sixties.

Conrad Kuttner has informed me that Brian began to take long walks, lasting all day, whilst still at school in Hampstead, years before hiking had become popular in England, and I well remember that whenever I met Brian I always had great difficulty in keeping pace with him as he strode powerfully towards whatever happened to be his current goal. I take this opportunity of thanking Conrad Kuttner for supplying me with some details of Brian's early years.

Throughout his life, Brian Kuttner maintained his enthusiasm for his subject and was always willing to give help to other researchers. In particular, he spent much time acting as a referee for research papers which had been submitted by numerous authors to a wide variety of learned mathematical journals, continuing this important and onerous work until the end, even though his sight was failing.

Dr Brian Thorpe, of the University of Birmingham, a former PhD student of Brian Kuttner, recently sent me a notebook in which Kuttner recorded refereeing and reviewing work. It indicates that in the year 1968 alone, Kuttner refereed 33 papers and reviewed a further 16. Also, many inexperienced researchers would send Brian Kuttner pre-publication manuscripts on which he would generously comment, often making corrections and suggesting further ideas for research.

For myself, I am thankful for the chance circumstance which led to my acquaintance with Brian Kuttner in 1960. I was studying at the University College of North Staffordshire (now called the University of Keele) and my research for a Birmingham PhD was supervised externally by Brian Kuttner. Over time, our research interests tended to diverge, but I recall with great pleasure the all too few but extremely happy mathematical collaborations that we shared over the years in the University of Birmingham.

Brian's wife, Hilda, predeceased him, and he leaves a son, also named Brian, to whom condolence is extended.

Brian Kuttner was a kind, helpful and gentle man, revered by his research students, admired by the numerous analysts who collaborated with him in research, and greatly respected by all who were fortunate to have known him.

There are many who consider that the quality and extent of Brian Kuttner's work and the influence that it exerted did not receive the full recognition that it merited, but it continues to be appreciated by those with a true knowledge of its worth.

We shall not expect to look upon his like again.

Brian Kuttner's mathematical work

In the early years of the twentieth century, much analytical endeavour was devoted to the general development of the summability of divergent series, stimulated greatly by problems in the important theory of Fourier series, where it was known that continuous functions existed which had their Fourier series divergent at the points of an everywhere dense set; the first example of such a function being due to Du Bois Reymond in 1876 $\langle \mathbf{2} \rangle$.

Many of the summability methods in common use take the form of infinite matrix transformations of the type

$$t_n := \sum_{k=0}^{\infty} a_{nk} s_k, \tag{1}$$

where $A = (a_{nk})$ for $n, k = 0, 1, 2, \dots$ is an infinite matrix of real or complex numbers, and (s_k) is a given real or complex sequence. We define (s_k) to be summable A to a number s, written $s_k \to s(A)$, if and only if the series in (1) converge for all $n \geqslant 0$ and $t_n \to s$ $(n \to \infty)$. Most summability methods A are such that some divergent sequences (s_k) are summable A, thus enabling us to associate a generalized limit with a divergent sequence.

Moreover, with the aid of so-called Tauberian theorems, it is possible, with certain restrictions on the terms of a sequence, sometimes to deduce convergence from summability. The general importance of Tauberian theorems is due to the fact that in many problems of analysis the convergence, when it exists, that is desired cannot be established directly, though it is often possible to prove summability by some method A. The first result of this type was given by Tauber in 1897 $\langle \mathbf{11} \rangle$ for the Abel method of summability, which is defined by taking

$$a_{nk} := (n/(n+1))^k (n+1)^{-1}$$

in (1) above. With this choice of A it was proved by Tauber that if $s_k \to s(A)$ and (s_k) satisfies the Tauberian condition that $k(s_k - s_{k-1}) \to 0$ as $k \to \infty$, then (s_k) must be convergent to s.

Similar ideas apply to series $\sum a_k = a_0 + a_1 + \dots$, as well as sequences, and with parameters tending to limits other than infinity. For example, in his famous work on trigonometric series, Riemann $\langle \mathbf{10} \rangle$ effectively introduced $(R, 2)$ summability of a series $\sum a_k$ by defining $\sum a_k = s(R, 2)$ to mean that there exists

$$\lim_{h \to 0} \sum_{k=0}^{\infty} a_k \left(\frac{\sin kh}{kh} \right)^2 = s.$$

A method A is called a Toeplitz method, or regular, if whenever (s_k) converges to s, then $s_k \to s(A)$, and similarly for series. Necessary and sufficient conditions for regularity of A were first given by Toeplitz in 1911 $\langle \mathbf{13} \rangle$; see, for example, Hardy $\langle \mathbf{3} \rangle$.

By taking $a_{nk} := 1/(n+1)$ for $0 \leqslant k \leqslant n$ and $a_{nk} := 0$ for $k > n$ in (1), we obtain one of the simplest and most useful Toeplitz methods, that of the arithmetic means $(C, 1)$, also called Cesàro means of order 1. An early application to Fourier series, due essentially to Fejér, but in general form given by Lebesgue (see, for example, Zygmund $\langle \mathbf{15} \rangle$), tells us that the Fourier series of any 2π periodic integrable function f is summable $(C, 1)$ to $f(x)$ for almost all x.

The Riemann method $(R, 2)$ is regular, in that the convergence of $\sum a_k$ to s implies that $\sum a_k = s(R, 2)$. Also, Riemann proved that the Fourier series of any 2π periodic integrable function f is summable $(R, 2)$ to $f(x)$ at each point x at which f is continuous.

Although many important and difficult problems were solved in convergence and summability theory in the first three decades of the present century, the 1930s were exciting times for analysis at large. Banach's seminal book ⟨**1**⟩ on complete normed linear spaces, now called Banach spaces, directed attention into uncharted areas of increased abstraction and generality, and Wiener's great paper ⟨**14**⟩ on Tauberian theorems may, in retrospect, be regarded as the apogee of a major aspect of classical summability theory.

Brian Kuttner commenced publication in 1934, and much of his early work was motivated by his interest in Fourier series and the summability methods that had been applied to them. However, his first paper [**1**] is a long and rather technical work on Fourier integral theorems related to a special class of so-called divisor functions, which were introduced by S. W. P. Steen in 1930. I believe that Steen had been Brian Kuttner's supervisor for the PhD at Cambridge.

In [**2**] the relation between the Riemann method $(R,2)$ and the general Cesàro method (C,k), of order k, is examined, and Kuttner proves that if a series $\sum a_k$ is summable $(R,2)$ to s, then $\sum a_k$ is summable (C,k) to s for every $k > 2$. It is this proof of Kuttner that Hardy gives in his celebrated treatise *Divergent series* ⟨**3**⟩.

Then, at the age of only 26, Kuttner [**3**] proved a basic theorem in the general theory of trigonometric series, a result delightful for both the deceptive simplicity of its statement and the elegance of its proof. In a recent letter, Professor A. Cyril Offord kindly informed me that Zygmund greatly admired this theorem of Kuttner, which now occupies an honoured place in Zygmund's monumental work on trigonometric series ⟨**15**⟩. Indeed, another proof of Kuttner's result was obtained by Marcinkiewicz and Zygmund in 1936 ⟨**9**⟩.

Kuttner's result [**3**] asserts that if a general trigonometric series

$$a_0/2 + \sum_{k=1}^{\infty} (a_k \cos kx + b_k \sin kx)$$

converges in a set H of positive measure, then the conjugate series

$$\sum_{k=1}^{\infty} (a_k \sin kx - b_k \cos kx)$$

is convergent at almost all those points of H at which the conjugate series is summable $(C,1)$. The Tauberian character of the theorem is clear, with the convergence on H of the original series acting as the Tauberian condition for the conjugate series. Being a result on general trigonometric series, it may, in particular, be applied to the Fourier series of any 2π periodic integrable function f. With the aid of the Fejér–Lebesgue theorem mentioned above, and a similar result on the conjugate Fourier series, due to Privalov and Plessner (see ⟨**15**⟩), Kuttner was able to deduce the very interesting fact that the set of points at which the Fourier series of f is convergent, and the set of points at which its conjugate series is convergent, differ only by a set of measure zero.

In [**15**], again through a link with Fourier series, he established a very interesting result on strong summability, now known as Kuttner's theorem. If $p > 0$ is given, then a complex sequence (s_k) is defined to be summable $[C,1]_p$ to a number s if and only if

$$(1/n) \sum_{k=1}^{n} |s_k - s|^p \to 0 \quad (n \to \infty).$$

The method $[C,1]_p$ is called strong Cesàro summability of order 1 and index p, and had been used by Hardy and Littlewood ⟨**4**⟩ in the theory of Fourier series.

Kuttner proved in [**15**] that in the case when $0 < p < 1$, given an arbitrary Toeplitz matrix A, there is always a sequence (s_k) which is summable $[C,1]_p$ but is not summable A. Extensions and generalizations of Kuttner's theorem, using functional analytic methods, have been made by Maddox $\langle$**6**, **7**, **8**$\rangle$ and Thorpe $\langle$**12**$\rangle$.

Kuttner also made several contributions to the theory of Riesz means, which had been introduced into analysis by Marcel Riesz in the first decade of the twentieth century. Many general properties of these means and their connection with Dirichlet's series may be found in the classic Cambridge Tract of Hardy and Riesz $\langle$**5**$\rangle$.

The Riesz mean of type λ and order $\alpha \geqslant 0$, of a given series $\sum a_k$ is defined to be

$$C^\alpha(x) := x^{-\alpha} \sum (x-\lambda_k)^\alpha a_k,$$

where the summation is over all $\lambda_k < x$. It is assumed that λ is any real sequence with $0 \leqslant \lambda_0 < \lambda_1 < \lambda_2 < \ldots < \lambda_n \to \infty$, and we define $\sum a_k = s(R,\lambda,\alpha)$ to mean that $C^\alpha(x) \to s$ as $x \to \infty$.

A long-standing problem concerned the limitation theorem for Riesz means, which dealt with the behaviour of $C^\beta(x)$ for $0 < \beta < \alpha$, when we are given that $\sum a_k = s(R,\lambda,\alpha)$. In an enigmatic footnote of their tract, Hardy and Riesz stated: 'It is curious that the simpler result, which holds for the integer case, should not always hold, but it is possible to show by examples that this is so.'

Curiouser and curiouser, prior to 1983 it would appear that no such examples had ever been seen, except perhaps by Hardy and Riesz. Then Kuttner and Maddox [**99**], as a by-product of work on lacunary series, proved that the statement of Hardy and Riesz was true, by relating the $(R,\lambda,1)$ mean with the (R,λ,β) mean in the case when $0 < \beta < 1$ and $\lambda_{2n-1} := 2^n$ and $\lambda_{2n} := 1+2^n$.

Nörlund methods of summability were also of considerable interest to Kuttner, and in [**70**] he considered the class N_r of all sequences which are summable by some real and regular Nörlund method, and proved that N_r is a linear sequence space. This result is far from obvious if we view it as follows. Denote by (N,q) the space of all sequences $x = (x_k)$ such that the sequence

$$\left(Q_n^{-1} \sum_{k=0}^{n} q_{n-k}\, x_k\right)$$

converges, where (q_k) is a given real or complex sequence such that $Q_n := q_0+q_1+\ldots+q_n$ is non-zero for all $n \geqslant 0$. Suppose also that q is such that the (N,q) transformation is regular. By an ingenious argument, Kuttner proved, in effect, that the union

$$N_r := \bigcup \{(N,q)\colon q \text{ is real and regular}\}$$

is a linear sequence space. In general, of course, the union of linear spaces is not always linear.

Kuttner also posed the problem, still apparently unsolved, as to whether or not

$$N_c := \bigcup \{(N,q)\colon q \text{ is complex and regular}\}$$

is also a linear sequence space.

Once, in conversation with me, L. S. Bosanquet remarked that Kuttner had a 'genius for counterexample', and although I cannot now recall the exact context in which this comment arose, it is on this point that I shall conclude my tribute.

Counterexamples of considerable ingenuity are to be found in many of Kuttner's papers, though I consider that one of the most interesting appears in [**78**], since it demonstrates his ability to exploit just those properties of the special functions of analysis which he needs for the problem in hand, whilst leaving one with a sense of wonder as to how he came to discover the example.

First, in [**78**], Kuttner proves the unusual result that the condition

$$\sum_{r=0}^{n} |a_r| = O(1) \left| \sum_{r=0}^{n} a_r \right| \tag{2}$$

acts as a Tauberian condition for (C, k) summability of any positive order k, but does not act as a Tauberian condition for Abel summability. This result is in sharp contrast with the usual situation in which there is some Tauberian condition for the Abel method, which is then necessarily a Tauberian condition for (C, k) summability.

For the counterexample, essentially Kuttner uses the fact that Jacobi's infinite product for the theta function $\theta(x) := \theta_4(\mathrm{id}/2, x)$ is such that

$$|\theta(x)| = \left| \prod_{n=1}^{\infty} (1-x^{2n})(1-e^{d}x^{2n-1})(1-e^{-d}x^{2n-1}) \right|$$

$$\leqslant \prod_{n=1}^{\infty} (1-x^{2n}) \leqslant 1-x^2,$$

for $0 < x < 1$, where d is chosen such that $d > 0$ and $e^d < 2$. Hence we have $\theta(x) \to 0$ $(x \to 1-)$. However, it is known that $\theta(x)$ may also be expressed in the form

$$\theta(x) = \sum_{r=-\infty}^{\infty} (-1)^r e^{dr} x^{r^2},$$

whence

$$(1+e^{-d})\,\theta(x) = (1+e^{-d}) \sum_{r=0}^{\infty} (-1)^r e^{dr} x^{r^2} + \sum_{n=1}^{\infty} b_n x^n,$$

say, where $\sum |b_n| < \infty$, which implies that $\sum b_n$ is Abel summable. Consequently, the series $\sum a_n$ is Abel summable, where $a_n := 0$ for $n = 0$,

$$a_n := (-1)^r (1+e^{-d})\, e^{dr} \quad (n = r^2,\ r = 1, 2, \ldots)$$

and $a_n := 0$ otherwise. We find that

$$\sum_{k=0}^{n} a_k = (-1)^r e^{dr} \quad (r^2 \leqslant n < (r+1)^2),$$

and so $\sum a_n$ is Abel summable, but divergent, and such that (2) holds.

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