The first term is $O(\theta^{1+a}.\theta^{-1}) = O(\theta^a)$, and the second is

$$O\left\{\sum_{n>1/\theta} n^{-a-1}(u_n - u_{n-1})\right\} = O(\theta^{a+1}.\theta^{-1}) + O\left(\sum_{n>1/\theta} n.n^{-a-2}\right) = O(\theta^a).$$

This proves (7.1) and completes the proof of the theorem.

The result is "best possible" in the sense that we cannot take $a = 0$ or $\beta < 1$.

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SIR JOSEPH LARMOR.

E. CUNNINGHAM.

By the death of Sir Joseph Larmor on 19 May, 1942, the scientific world lost one who had made a large contribution to the transition from the classical mechanics to the new physics, and who had also served the interests of science and of the larger world as Secretary of the Royal Society and as Member of Parliament for the University of Cambridge. That his services had been widely recognized is shown by the list of distinctions that had been conferred upon him: Hon. F.R.S.Edin.; Hon. Mem. R. Irish Acad.; Hon. Mem. Asiatic Soc. of Bengal; Manchester Lit. and Phil. Soc.; Hon. Foreign Mem. U.S. National Acad. Sci.; American Acad. of Science and Arts, Boston; American Phil. Soc., Philadelphia; R. Accademia dei Lincei, Rome; Istituto di Bologna; Correspondant de l'Institut de France (Prix Poncelet, 1918); Lucasian Professor in the University of Cambridge; Fellow of St. John's College, Cambridge; D.Sc. London; Hon. D.Sc. Oxford and Dublin; LL.D. Glasgow, Aberdeen, Birmingham, St. Andrews; D.C.L. Durham.

Larmor was born at Magheragal, Co. Antrim, on 11 July, 1857. From the Royal Belfast Academical Institution he went to Queen's College, Belfast, and, after graduating there, to Cambridge, where he entered St. John's College as a Scholar. He took the Mathematical Tripos in 1880 and was Senior Wrangler, J. J. Thomson being Second Wrangler. He was also first Smith's Prizeman. Shortly afterwards he became a Fellow of his College.

In 1880 Larmor went back to Ireland as Professor of Natural Philsosphy in Queen's College, Galway, but returned to Cambridge five years later on his appointment as Mathematical Lecturer in St. John's College. This post he held until 1903, when he was elected to the Lucasian Professorship in the University of Cambridge in succession to Sir George Gabriel Stokes.

Elected a member of the London Mathematical Society at its meeting of 13 March, 1884, Larmor served on the Council of the Society from 1887 to 1912 and was a Vice-President in 1890 and 1891. He became Treasurer in 1892 and held the office for twenty years. In 1914 he was elected President and served for two years, continuing on the Council until 1919. In 1914 he was the recipient of the de Morgan Medal of the Society.

In 1901 the Royal Society made him one of its secretaries, an office which he held until 1912. Later he was honoured by the Royal Society by the award of a Royal Medal in 1915, and of the Copley Medal in 1921. During his service as Secretary he received, in 1909, the honour of knighthood.

From 1911–1922, Larmor represented the University in Parliament. He retired from the Lucasian Professorship in 1932, and shortly after, owing to ill-health, returned to Northern Ireland, where he lived until his death.

Most of Larmor's scientific work has been gathered together by himself in two volumes (Cambridge University Press, 1929, pp. 679 and 831). Of the contents of the first volume, he says in the preface "about half is of electrical character, the other half being mainly General Dynamics and Thermodynamics, including the dynamical history of the Earth, Formal Optics, and Geometry". We may trace the movement of the times in the gradual disappearance of interest in the geometrical optics, and the emergence of physical optics, in the second volume. But general dynamics, and particularly the principle of Least Action, is from beginning to end an abiding interest. In 1884 Larmor made his first contribution to the *Proceedings* of the London Mathematical Society. It consisted of a paper on "Least action as the fundamental formulation in dynamics and physics" [*Proc. London Math. Soc.*, 15 (1884), 158–184]. Thirty-six years later, in editing a new edition of Clerk Maxwell's *Matter and Motion* (1920), he dealt with the same theme in an appendix "On the Principle of Least Action". In the early paper the principle is associated with a second abiding interest, the principles of thermodynamics. He remarks: "The prominent place that this principle (least action) directly holds in general dynamical theory is illustrated by the attempts that have been made to place the Second Law of Thermodynamics on a dynamical basis. The investigations of Clausius, Boltzmann and Szily have shown that the Principle of Least Action in Dynamics is in close relationship with that law". He then takes up various problems of interest at that time, motions of particles, forms of catenaries and of systems of rays, and shows the analogies between them that flow immediately from the minimum principle.

As time goes on and his interest in the deeper problems of the structure of matter extend, we find him turning always to this principle of least action as standing firm among all changing concepts of structure. In the preface to the second volume of papers (September, 1928), he writes: "It must be premature to conclude that the classical foundations of electrical and optical science are to be regarded as undermined. It has never, of course, been suggested that any mechanical model is more than an aid to insight. . . . The oft-repeated demur, that if there could be one model there would be an infinite number, can only be a different mode of expression of the familiar common ground that, after the groups of relations relevant to the model have been reduced by its aid to coherent and condensed statement, the scheme of equations that survives may be interpreted as the manifestation of a unique abstract dynamical action of local structure, independent of models altogether".

In the prime of his activity Larmor is concerned with a picture of the physical world as a unity. Of his book *Aether and Matter* (Cambridge University Press, 1900), Sir Horace Lamb, at the British Association in 1904, humorously said that it would be more correctly entitled "Aether and no matter". Here is Larmor's summary of the conception of matter that he is considering: "We shall now consider the material system as consisting of free aether pervaded by a system of electrons which are to be treated individually, some of them free or isolated, but the great majority of them grouped into material molecules. . . . The medium in which the activity occurs is, for our present purpose, the free aether whose dynamical equations have been definitely ascertained . . . so that there will be nothing hypothetical in our analysis on that score. An electron will occur in the analysis as a singular point in the aether, on approaching which the elastic strain constituting the aethereal displacement (f, g, h) increases indefinitely according to the type

$$-\frac{e}{4\pi}\cdot\left(\frac{d}{dx}, \frac{d}{dy}, \frac{d}{dz}\right) r^{-1};$$

it is in fact analogous to what is called a simple pole in the two dimensional representation that is employed in the theory of a complex variable. It is assumed that this singularity represents a definite structure, forming a nucleus of strain in the aether, which is capable of transference across that medium independently of motion by the aether itself. . . . The aether is stagnant on this theory, while the molecules constituting the earth and all other material bodies flit through it without producing any finite flow in it". Elsewhere he says (*Aether and Matter*, p. 86): "An electric charge is a nucleus of intrinsic strain in the aether. It is not at present necessary to determine what kind of configuration of strain this can be, if only we are willing to admit that it can move or slip freely about through the medium in the way that a knot slips through a rope".

In forming his theory of the aether, Larmor acknowledged his debt to a fellow-countryman. MacCullagh had worked on the propagation of light waves through a continuous medium. He had shown that he could obtain an adequate basis for this by conceiving a medium in which the potential energy depended, not on the distortion, that is change of shape and size, of the small elements, but on the rotational displacement of these elements relative to an absolute background in space. After Maxwell had effected the identification of light with electromagnetic waves, Larmor carries over this thought to a medium in which the kinetic energy T and the potential energy W are

$$T = \tfrac{1}{2}A\int(\dot{\xi}^2+\dot{\eta}^2+\dot{\zeta}^2)\,dt, \qquad W = \tfrac{1}{2}B\int(f^2+g^2+h^2)\,dt,$$

$(\dot{\xi}, \dot{\eta}, \dot{\zeta})$ being the velocity of the medium and (f, g, h) being defined as regards its rate of change by

$$(\dot{f}, \dot{g}, \dot{h}) = \frac{1}{4\pi}\,\text{curl}\,(\dot{\xi}, \dot{\eta}, \dot{\zeta}). \qquad \text{(I)}$$

To obtain the dynamical equations of the medium he develops the variational equation

$$\delta\int(T-W)\,dt = 0,$$

and derives the equation

$$-A(\ddot{\xi}, \ddot{\eta}, \ddot{\zeta}) = \frac{B}{4\pi}\,\text{curl}\,(f, g, h). \qquad \text{(II)}$$

Equations (I) and (II) are of the form of Maxwell's field equations, with $(\dot{\xi}, \dot{\eta}, \dot{\zeta})$ representing the magnetic induction, and (f, g, h) the aethereal displacement.

There follows, however, a point of difficulty. Assuming that there is nothing involved in the energy except a strain form, no inertia or energy foreign to the aether, Larmor infers that the above equations will determine the state of the field at any instant from its state, supposed completely known, at the previous instant. In other words, he supposes that the unrolling of the field and the motion of the electrons will be given by the solution of the differential equations from the initial conditions. This may be so if the strain form is geometrically completely specified, after the manner of Lorentz, who develops the mechanics of the electron from a special configuration of the electron (the contracting sphere) together with the hypothesis of no non-electromagnetic inertia. Larmor alternates between a picture of an electron as a singularity of unspecified configuration with an associated energy containing a part $\frac{1}{2}LV^2$ (L being a quantity not determined by his analysis and V being the velocity), and a picture of the electron as a point singularity, in which case the differential equations do not determine its motion at all, since it is possible to obtain an explicit solution of the equations with any arbitrarily assigned motion of such singularities.

Thus Larmor's main contribution at this point is to maintain the relevancy of a minimal principal as summarizing the equations of the field, but not contributing greatly to the conception of the nature of the electron.

He goes on, however, to give an analysis of the equations of a material medium, in terms of aggregates of electrons moving in the aether, which is effectively identical with that given by Lorentz, and has become so much a permanent basis of our thinking that it is difficult to realize that fifty years ago it had not been touched upon.

But Larmor contributed more than he realized in thus analyzing matter into aethereal actions. Although he himself never adopted the principle of relativity enthusiastically, he did much to prepare the way for it. Again parallel to, but independently of, Lorentz he examined the condition of a material body moving uniformly through the aether. On the hypothesis that the Maxwell equations of the free aether were sufficient to determine the whole motion of an electronic system, and using the fact, which he demonstrated, of the invariance of the equations under a certain transformation of dependent and independent variables, he gave a rational basis for Fitzgerald's suggestion of a contraction of a moving body in the ratio $\sqrt{(1-v^2/c^2)} : 1$ in the direction of its motion through the aether. The argument was not complete. It was only given as an approximation up to and including the order v^2/c^2. It also failed at the point mentioned above, that, if the electrons are considered as points, the equations do not determine their motions at all; while, if they are considered as having a definite finite structure. that structure must itself be assumed to be subject to the same invariance under the transformation as it is desired to demonstrate of the whole system (*cf.* Lorentz's assumption of a contracting electron to explain a contracting body). Nevertheless

Larmor anticipates Einstein in making known the transformation later to become so famous generally under the term "Lorentz-Einstein" transformation. That the invariance of the equations is exact became known to Larmor at a later date before its publication by Einstein.

It is by this work, largely paralleled by that of Lorentz, that Larmor is chiefly known. As summarized and augmented in his book, *Aether and Matter*, it gained for him the Adams Prize in the University of Cambridge. In preparing the way for Einstein, in removing the material continuum from a place more fundamental than an electromagnetic medium to a derived place, and so putting electromagnetic concepts in the first place, he was preparing the way for a radical revision of Newtonian concepts. Mass became a coefficient in the "action" of an electromagnetic medium. Absolute time and absolute velocity received their first serious blow, and so began the emancipation from that rigidity of the Newtonian system which had marked the two hundred years from its first statement.

After the publication of this work Larmor's mind seems to have been occupied more with the difficulties which had arisen in connexion with the distribution of energy in the radiation from a black body and in general thermodynamical matters. His Bakerian Lecture to the Royal Society (1909) was entitled "The statistical and thermodynamical relations of radiant energy". In this he shows that he was loath to adopt Planck's suggestion that energy is divisible only into specific quanta associated with the frequency of a vibrator. At the same time, however, he was convinced that the statistical method, in Boltzmann's form, in some way held the key to the problem of radiation. He gives a statistical discussion, leading to Planck's formula, in which there is no implication that energy is constituted on an atomic basis. In its place he adopts the idea of cells of equal opportunity. The test of equality of opportunity is that when an element of disturbance of one type is transformed physically so as to be of another type—as, for example, when a train of waves has its period and energy changed by reflexion at a moving mirror—the regions occupied before and after are, in the sense of geometry of many dimensions, equivalent. The implication of quanta only remains in the form that the ratio of the energy element to the extent of the cell is an absolute physical quantity.

The new problems in the theory of radiation caused Larmor to think afresh of the part played by the representation of fields by means of Fourier series and integrals. Is the representation of radiation by means of a frequency spectrum one which is intrinsic to the disturbance or one which is imposed upon it by periodicity in the analyzing apparatus, the grating or the prism? This led Larmor to the main theme of his presidential address to the London Mathematical Society in 1916. On this occasion he presented to the Society a memoir on "The Fourier harmonic analysis and its scope in physical science". The address contains an appeal, which he often made, for a closer linking of activity of abstract mathematicians with physicists. It contains, too, one of the rare passages in which he reveals his thought about men: "Nature is never irrational, but our main intellectual aim is the redemption of our views of her operations from that reproach". In this memoir Larmor surveys the relation of Fourier analysis to the recorded statistics of tides, of meteorology, of sunspots and of radiation. What evidence is there for a unique permanent period in sunspots? What is the range of record necessary to secure purity in the spectrum? How comes it that a structureless temperature radiation may be represented by a continuous spectrum? Over such questions the address ranges. To him the interest in the analysis lies in its correspondence with physical data, and he is somewhat out of sympathy with the analysts who pursue the behaviour of the Fourier series for its own sake. "Progress might be assisted, with benefits not all on one side, by continuous and sympathetic scrutiny of the nature of the operations, and the scope and the degree of validity of the conclusions, and by interest in their improvement, on the part of modern analysts, to whom minute discussion of the nature and limitations of functional discontinuity in other aspects has become highly congenial. No apology is required for reviewing the practical side of the subject, in an elementary way, before the Mathematical Society, especially as a variation from the purely critical and negative attitude on such matters that is not unfamiliar to us".

On 31 October, 1896, Zeeman presented to the Amsterdam Academy his famous paper on the broadening of a spectral line when the emitting substance was subjected to a magnetic field. In 1897 Larmor gave the explanation of this phenomenon in a paper in the *Philosophical Magazine*, and in the same paper gives the now well-known expression $\frac{2}{3}e^2f^2/c$ for the rate of radiation of energy from an electron which has acceleration f.

To his great services in the realm of electromagnetic theory Larmor added another in the stimulus which he gave to the study of thermodynamics; witness his tribute to Willard Gibbs in the *Proceedings of the Royal Society* (1904), and his Bakerian Lecture mentioned already. Those who attended his lectures on thermodynamics in Cambridge as early as 1904 remember him as one whose mind was escaping from the toils of special solutions of the differential equations of mechanical media, sometimes at the price, as it seemed at the time, of discursive generality.

In 1924 Larmor gave considerable stimulus to the theory of radio-propagation by attacking the problem of the propagation of electric waves in an ionized atmosphere. His suggestions were quickly taken up and developed by Appleton and led to great advances.

One other very substantial service to science remains to be noted. In his later years Larmor gave much time and energy to the editing of the works of his great predecessors. For the collected papers of Henry Cavendish he edited a description of unpublished mathematical and dynamical manuscripts, and he was editor of the collected works of James Thomson (1912), of the fourth and fifth volumes of the works of Sir George Stokes, and of the fourth, fifth and sixth volumes of those of Lord Kelvin.

Between the old and the new physics Larmor stands, always conscious of his debt to the past, always labouring to free science from the shackles of the past, building the foundations of the new physics, but critical of its enthusiasm for the new and of its impatience with the past. A deeply honest thinker, with wide interest in the world at large, never seeking publicity, but winning respect always for his judgment and his directness, he was one of the band of great men who adorn the title "Professor of Natural Philosophy".