Papers quoted.

1. L. P. Eisenhart and M. S. Knebelman, *Proc. Nat. Acad. Sci.*, 13 (1927), 38–42.
2. M. S. Knebelman, *American J. of Math.*, 51 (1929), 527–564.
3. W. Ślebodziński, *Prace. Mat. Fiz.*, Warsaw, 39 (1932), 55–62.
4. J. Douglas, *Math. Annalen*, 105 (1931), 707–733.
5. J. A. Schouten and D. J. Struik, *Einführung in die neueren Methoden der Differentialgeometrie* (Groningen, Noordhoff, 1 (1935), 2 (1938).
6. R. Weitzenböck, *Invariantentheorie* (Groningen, Noordhoff, 1923).
7. W. V. D. Hodge, *Harmonic integrals* (Cambridge, 1940).

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TULLIO LEVI-CIVITA

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Tullio Levi-Civita was born in Padua on 29 March 1873, the son of Giaccomo Levi-Civita and his wife Bice Lattis. His family was a wealthy one, well known for its strong Liberal traditions. Giaccomo Levi-Civita was a barrister, jurist and politician, and was for many years mayor of Padua, and a Senator of the Kingdom of Italy. As a young man he had served as a volunteer and fought with Garibaldi in the campaign of 1866, and he had played an important part in the Risorgimento.

Giaccomo Levi-Civita was anxious that his son should follow in his footsteps as a barrister, but Tullio's interest in the physical and mathematical sciences was apparent even in early childhood, and when he expressed a wish to follow his own inclinations his father never opposed him; and in later years the son's eminence in the scientific world was a source of great pride to the father. Consequently, when he completed his classical studies at the Ginnasio-Liceo Tito Livio in his native city at the age of seventeen, Tullio Levi-Civita entered the faculty of science at the university of Padua as a student of mathematics, and four years later he took his degree.

Amongst his teachers at the university of Padua were D'Arcais, Padova, Veronese, and Ricci-Curbastro. The two last-named were the most distinguished, and both had considerable influence on the future career of their brilliant pupil. The influence of Ricci is the more obvious, since it developed into active collaboration, but probably Veronese's influence was quite as important, since it is largely to him that Levi-Civita owed the remarkable spacial intuition and familiarity with multidimensional space which characterises the younger man's contribution to the Ricci–Levi-Civita partnership in the absolute differential calculus.

Levi-Civita's undergraduate days were not over before he began to write mathematical papers, and his ability was quickly recognized. Indeed, in the year after he took his degree, his application for the chair of mechanics at Messina was strongly favoured by Volterra and Morera, two of the electors; but the other two electors and Cremona, the chairman, supported Marcolongo, who was considerably Levi-Civita's senior both in years and in experience, and he was consequently elected. Three years later Padova, one of his former teachers at Padua, died, and Levi-Civita was elected in his place to the chair of mechanics, at the age of twenty-five. For twenty years he held this post, and these were among the most productive of his life. In 1918 he was called to the chair of mechanics at Rome, a post which he held for another twenty years, until racial discrimination, introduced into Italy in 1938, brought about his removal from office. Until then his life was uneventful, spent in the happy pursuit of his mathematical interests.

Levi-Civita's researches covered a vast field of mathematics, and it is not possible to say he was this or that kind of mathematician; one can only say he was a mathematician, and a great one. Viewing his work as a whole, however, the dominating impression one receives is of an astounding command of the technicalities of pure mathematics, aided by an acute geometrical intuition, applied mainly to problems of applied mathematics. This is, of course, only a general impression, for there is plenty of Levi-Civita's work which is first-class pure mathematics, and plenty that is genuinely applied. But one sometimes feels that parts of his work are insufficiently appreciated simply because on the one hand so much of it deals with special problems outside the range of interest of the pure mathematician, and on the other hand it is regarded as too theoretical by the applied mathematician. This fate has befallen other mathematicians who can be described as pure mathematicians whose main interest is in applying their knowledge to physical problems, but in Levi-Civita's case some of his contributions in this no-man's land between pure and applied mathematics have been of such importance that they could not be hidden; and in particular his contributions to the absolute differential calculus have placed both pure and applied mathematicians under a debt of gratitude which they have been glad to acknowledge.

But however much Levi-Civita's interest may have moved towards applied mathematics, his very earliest paper was quite unambiguously on pure mathematics. While he was a student at Padua his teacher, Veronese, had started a discussion on non-Archimedean geometry, and in 1893, before taking his degree, he contributed a paper to the discussion which was published in the *Atti Lincei*. After this one paper Levi-Civita does not seem to have taken any further part in the discussion, though Veronese became involved in a heated argument, mainly with Peano, until five years later when he published a couple of notes in the same journal which effectively resolved the difficulties and finally answered the question under discussion.

Some of Levi-Civita's other early papers suggest that his first attraction was towards pure mathematics. Thus in 1895 he published an improvement on Riemann's formula for expressing the number of prime numbers lying in a given interval as a contour integral. To about this same time also belongs a paper devoted to the solution of the integral equation

$$u(x) = \int_{a(x)}^{b(x)} f(x, y)\, v(y)\, dy$$

for the function $v(y)$, and this was followed by a paper applying the result to a problem in electrostatic induction. This is apparently the first of the many occasions on which he used his skill as a pure mathematician to solve a problem of physical importance. Other such occasions now followed in rapid succession.

To this same period, the closing years of last century, belong Levi-Civita's first researches in differential geometry and the absolute differential calculus, and broadly speaking we may say that by the time Levi-Civita was twenty-five he had established himself as a mathematician and had settled down to a continuous programme of work of the type he found most congenial to him. From then on papers appeared continuously in which he displayed great analytical skill in the solutions of all sorts of problems in applied mathematics. One can best review his work by abandoning the chronological order and considering his work as a whole in the various fields to which he contributed.

The work by which Levi-Civita is best known is certainly that on the absolute differential calculus, with its applications to relativity theory. The study of the particular class of invariants known as tensors goes back to the work of Riemann and Christoffel on quadratic differential forms (though the name tensor was only introduced by Voigt in 1898). In 1887 Ricci published his famous paper in which he developed the calculus of tensors, including the important operation of covariant differentiation. For a considerable number of years following the publication of this paper he was engaged in working out his "absolute differential calculus", aided by a number of able pupils, foremost among them being Levi-Civita. The results of the work of Ricci, Levi-Civita and others were finally published in a joint memoir by Ricci and Levi-Civita which appeared in 1900 under the title, "Méthodes de calcul différential absolu et leurs applications", and which presented the theory of tensors essentially in the form used by Einstein and others fifteen years later.

In 1917 Levi-Civita made an advance in the absolute differential calculus of fundamental importance, with the introduction of the concept of parallel displacement. Few mathematical ideas have found such diverse applications so quickly. It is the basis of the unified representation of gravitational and electromagnetic fields in relativity theory, and there are still more far-reaching consequences which are not yet fully recognized in physics. The idea is no less important in pure mathematics. About the same time as Levi-Civita published his great paper on parallel displacement, Hessenberg published a paper in which it was shown that the notion of covariant differentiation did not depend in any essential way on a Riemannian metric, but was capable of considerable generalization. The ideas of the two papers were quickly taken up and developed, first by Weyl who used them for his unified theory of gravitation and electromagnetism, then by many others, and from them has developed the whole of the modern differential theory of generalized spaces.

Levi-Civita's direct contributions to relativity theory are substantial, but they are of a less conspicuous nature. From 1917 to 1919, in a series of papers, he and his students treated very elegantly the problems arising in the special case of a static gravitational field, including systems in a state of steady rotation. In 1937 he announced a result which, although it has proved to be erroneous, drew attention to a difficult and interesting question and greatly stimulated the development of relativistic mechanics. Since no exact solution of the problem of two bodies in relativity theory has been found, much attention has been paid to methods of approximation, which often involve very laborious methods of calculation as well as subtle points of theory. In pursuing this problem, Levi-Civita reached the conclusion that (relatively to distant objects) the centre of gravity of a double star has a secular acceleration in the direction of the major axis of the orbit, towards the periastron of the larger mass. This result could not be said to be inconsistent with recognized principles, but it was sufficiently surprising to awake keen suspicion. Direct criticism was scarcely possible, the calculation being too extensive for detailed publication. The obscurity deepened when it was found that de Sitter's earlier formulae led to a similar result but disagreed as to the magnitude. The problem was re-investigated by Einstein, Infield and Robertson, and by Eddington and Clark: these two investigations agreed in contradicting the supposed acceleration. Ultimately Levi-Civita found an algebraic mistake in his own calculation and came into agreement. Though in one sense a failure, his intervention in the two-body problem greatly benefited relativity mechanics, which had been languishing for want of a definite aim.

In addition to the large number of papers which Levi-Civita published on the absolute differential calculus and relativity he published two books, *Questioni di meccanica classica e relativistica* (1924) and *Lezioni di calcolo differentiale assoluto* (1925). Both of these have become standard works.

While he is most famous for his work in relativity theory, Levi-Civita has had an important influence on many other branches of mathematics. It is particularly necessary to mention his work in analytical dynamics, to advance which subject he did as much as anyone during the earlier years of the twentieth century. Most of this work is however too detailed to permit of any analysis in this notice, and it must be sufficient if we mention some of the more important. The largest individual group of these papers deals with the problem of three bodies, either in the general case or in the restricted case in which two of the bodies describe circular orbits while the mass of the third is so small that it does not affect the motion of the other two particles. In these papers Levi-Civita has done much to increase our knowledge of the types of analytic solution possible, and he has also found an analytic condition for a collision resulting from given initial conditions, in the restricted case.

There is also a group of papers dealing with various problems that arise in connection with the integrals of the equations of a general dynamical system. Perhaps the most important of these is one in which he establishes the existence of a system of integrals of a Hamiltonian system corresponding to given invariant relations. Other papers on dynamics deal with such a diversity of subjects as the analytic solution of Kepler's equation connecting the mean anomaly with the eccentric anomaly and eccentricity of a particle moving in an elliptic orbit, the effect of neglecting terms in the criteria for the stability of orbits, the Kowalevsky top, and Saturn's rings. And here we must also mention Levi-Civita's important contributions to the theory of adiabatic invariants. In 1923 Levi-Civita, in association with Amaldi, published a three-volume work on rational mechanics, *Lezioni di meccanica razionale*, which is now one of the accepted classics on the subject.

Hydrodynamics is another subject which attracted Levi-Civita's attention, and to which he made a considerable number of contributions. Most of these deal with the solution of problems in classical hydrodynamics which require considerable analytical skill, but it is perhaps true to say that his work is too theoretical to appeal to modern experts in hydrodynamics. Nevertheless he has performed an extremely useful function by supplying much needed rigour at several points of the theory. His work on hydrodynamics is to a certain extent bound up with his work on the general theory of systems of partial differential equations. His work on this subject forms an important addition to the well-known Cauchy-Kowalevsky theory. An excellent account of this is given in a booklet, *Caratteristiche dei sistemi differenziali e propagazione ondosa*, which he published in 1931 and which was later translated into French. The Cauchy-Kowalevsky theorem affirms the existence of regular solutions of a normal system of partial differential equations in the neighbourhood of given initial values. The equations of the small motion of a fluid or of an elastic solid, the equations of the electromagnetic field, and Schrödinger's equation, all come under this theory. Levi-Civita's work is largely concerned with solutions having given initial values in a neighbourhood in which the equations cease to be normal, and his booklet is concerned not only with the general theory but with special cases of physical interest; thus, for example, he is able to deduce the impossibility of the propagation of a discontinuity in a viscous medium. He was particularly interested in the relation between the classical theory of wave propagation and modern wave mechanics, a topic to which he devoted a considerable part of an address which he gave in Chicago to a joint meeting of the American Mathematical Society and the American Association for the Advancement of Science in 1933.

It was natural that the efforts made to find a common framework to contain both quantum mechanics and the general theory of relativity should prove of the greatest interest to Levi-Civita. In 1933 he published a paper in which he proposed to replace Dirac's first order equations by a set of second order equations which took into account the gravitational field. When the two sets of equations are compared in the case in which there is no gravitational field it is found that Levi-Civita's equations are reconcilable with Dirac's when the electromagnetic field is either purely electric or purely magnetic, but not in the general case.

Though reference has been made to some of Levi-Civita's more outstanding researches there are very many which must be passed over. It ought to be mentioned, however, that in addition to his work in the realm of pure science he was frequently consulted by technicians and engineering firms on problems of practical engineering. In this way he was brought to do work of considerable value to the outside world, notably in connection with the construction of submarine cables and the vibration of bridges.

Levi-Civita was a born teacher. He lectured on a very wide field of mathematics, and his lucid discourses attracted large audiences. His quick grip on a problem, and his ability to give a vivacious and precise account of abstract problems, served to make him a most attractive expositor, both in formal lectures and in more intimate discussions. He followed with keen interest all sorts of developments in mathematics, and his interests and knowledge extended far beyond the considerable ground covered by his researches. Particularly, he followed closely the work of his former pupils, many of whom were able to profit by his advice long after they had passed out of his classes.

Levi-Civita was a man of small stature, and was handicapped throughout his life by defective eyesight. Nevertheless, he was very robust, and enjoyed excellent health. In his younger days he was an ardent mountaineer, but eventually he had to give this up on account of worsening eyesight. He was also a keen cyclist, and was able to go for long runs, usually among the mountains which he could no longer climb, until he was over sixty.

He had a passion for foreign travel, a form of relaxation which his private fortune and freedom from domestic worries enabled him to indulge to the full. He received invitations to attend and address scientific gatherings all over the world, and he took the greatest pleasure in these visits. On these journeys he was always accompanied by his wife—Libera Trevesani, a former pupil whom he married in 1914. She was a clever and affectionate companion to him, and the couple became widely known and liked in all parts of the world; few mathematicians have been more liked for their personal qualities than Levi-Civita. There were no children of the marriage.

Levi-Civita did not follow his father's example by taking any active part in politics. Nevertheless he followed political developments closely, and remained true to the liberal traditions of his family. The bond between him and his father was very close, and the father's portrait always hung, beside that of Garibaldi, in his study. He viewed with strong displeasure the advent of fascism in Italy, and he was a signatory of the "Manifesto Croce" issued by a large body of Italian scientists after the Matteoti affair in 1925. His scientific renown, however, protected him from persecution until the introduction of the anti-Jewish laws in Italy in 1938 resulted in his removal from his chair at Rome. This was a heavy blow to him, from which he never recovered. He received offers of asylum from many parts of the world, but severe heart trouble which had appeared rendered him unable to travel to accept any of these. At the end of 1941 he had a stroke, and he died on December 29.

Universities and scientific academies throughout the world honoured him. In this country he was a foreign member of the Royal Society, an honorary fellow of the Royal Society of Edinburgh, and an honorary member of the Edinburgh Mathematical Society. He was made an honorary member of the London Mathematical Society in 1924.

In preparing this notice I have received valuable assistance from Dr. Enrico Volterra, Prof. Beniamino Segre, Sir Arthur Eddington, Mr. L. A. Pars, and Prof. E. T. Whittaker.

ÉMILE PICARD

J. HADAMARD.

Avec la mort d'Emile Picard, le 12 Décembre 1941, une des plus grandes personnalités de la science contemporaine a disparu.

Charles Émile Picard* naquit à Paris le 24 Juillet 1856. Au Lycée Henri IV (en ce temps-là Lycée Napoléon) où il fit ses études secondaires, il excellait en version grecque, en vers latins, en histoire, mais détestait délibérément la géométrie, qu'il apprenait *par coeur*, pour éviter les punitions ! Au contraire, à l'âge de 15 ans, lorsque, dans la classe de seconde, il fit connaissance avec l'algébre, il fut aussitôt fortement séduit, tendance qu'il garda toute sa vie. Cependant, deux ans plus tard, dans la classe de mathématiques, il avait cessé d'être rebelle à la géométrie; ses maîtres se formèrent rapidement une haute opinion de lui: ils eurent à triompher des hésitations de sa mère pour lui faire poursuivre une carrière scientifique. Comme tout jeune français de notre époque possédant des dons scientifiques, il eut à choisir entre l'École Polytechnique, préparant en principe à des carrières d'ingénieurs, et l'École Normale consacrée à la science pure. Il se décida en faveur de cette dernière, où il fut reçu premier: on dit que cette décision fut prise après une émouvante visite à Pasteur, dans laquelle le père de la bactériologie parla de la science pure et désintéressée en termes si nobles que son jeune interlocuteur fut définitivement convaincu.

Émile Picard passa à l'École Normale Supérieure les trois années réglementaires, de 1874 à 1877, puis fut nommé "agrégé préparateur" pour l'année scolaire 1877–1878. Entre temps, il avait obtenu le titre de Docteur avec une thése sur les "Applications des complexes linéaires à l'étude des surfaces et des courbes gauches".

Immédiatement après sa quatrième année d'École Normale, il fut nommé à l'Université de Paris, à titre temporaire cependant: car l'année

* Cette notice est conforme au texte anglais qui a paru dans les *Proceedings of the Royal Society*. Toutefois, ce qui est relatif aux courbes et surfaces algébriques ainsi qu'aux transcendantes qui leur sont attachées est dû à M. Claude Chevalley, que je tiens à remercier ici de sa [illegible]cieuse collaboration.