OBITUARY

EDWARD HUBERT LINFOOT

Edward Hubert Linfoot, formerly John Couch Adams Astronomer at the University of Cambridge, died on 14 October 1982 at the age of 77. Although the bulk of his scientific work was in the field of mathematical optics—an area to which he made many important contributions—during the initial stages of his career he was a pure mathematician of distinction. He was a member of the Society from 1926 to 1971.

Edward Hubert Linfoot was born in Sheffield on 8 June 1905, the eldest child and only son of George Edward Linfoot, musician, and his wife Laura (nėe Clayton). He attended the King Edward VII school in Sheffield and won a mathematical scholarship to Balliol College, Oxford, at the early age of 16. He matriculated in 1923 and in 1926 took his B.A. in mathematics with first class honours. During this period he first came into contact with G. H. Hardy, who inevitably exercised a strong influence on his development as a mathematician. Evidence of Linfoot's mathematical precocity is provided by his first paper [1] which was written while he was still an undergraduate. It was also as an undergraduate that he developed the technique of preparing notebooks—of a well-nigh lapidary quality—based on the lectures and tutorials he attended, which was to result in a lucid written record of the mathematics of his time. (Many of these notebooks—a list of which is appended to this notice—will be kept in the Archives of the Society.) One of the early notebooks [A12] is especially interesting as it records the tutorials he received during 1925–26 from Besicovitch, who was then visiting Oxford at Hardy's invitation. These tutorials were the result of Besicovitch's desire to improve his English by the natural device of teaching English students. However from Besicovitch's reported remark, "I am not learning enough English—he [Linfoot] understands before I explain", one may conclude that he did not regard Linfoot as being ideal material for this purpose!

Linfoot spent the academic years 1926–28 in Oxford and was an active participant in Hardy's seminar, whose members also included L. S. Bosanquet (later to become his brother-in-law), Mary Cartwright and Gertrude Stanley. (Many of Hardy's lectures of this period were transcribed by Linfoot and appear in his notebooks [A1]–[A11].) In 1928 he obtained his D.Phil. with a thesis on almost periodic functions.

During 1928–29 he spent 2 semesters in Göttingen where he attended lectures and seminars by Landau on number theory, H. Bohr on almost periodic functions and van der Waerden on topological groups: all of these are written up (in English) in his notebooks ([B1]–[B5]). Despite the briefness of his stay in Göttingen, he was able to establish good links with the local society by virtue of the fluent and accurate German he had begun to acquire as a boy (the language being particularly well taught at his school). While at Göttingen it seems that his command of the language developed to a point where a new acquaintance, possibly finding his accent difficult to identify, felt impelled to ask him: "And what part of Germany do you come from?"!

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EDWARD HUBERT LINFOOT
1905–1982

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During 1929–31 he was the recipient of a Jane Eliza Procter Fellowship at the Graduate College of Princeton University. His notebooks of this period ([C1]–[C4]) reveal that he attended courses and seminars by Alexandroff on dimension theory, and Robertson and von Neumann on quantum mechanics. (The notes ([C3]) on von Neumann's 1930 Princeton lectures are of particular interest since they provide a record of von Neumann's Hilbert space formulation of quantum mechanics before the publication of his well-known *Mathematische Grundlagen der Quantenmechanik* in 1932.) In 1931 he returned to England and taught at Balliol until 1932 when he was appointed Assistant Lecturer, and later Lecturer, in Mathematics at Bristol. It was at this time that he came to know Hans Heilbronn and collaborated with him on the well-known paper [**16**].

In 1935 he married Joyce Dancer, herself an able mathematician. Their daughter Margaret was born in 1945 and son Sebastian in 1947.

While at Bristol his scientific activity gradually shifted from pure mathematics to optics. Remarkably, this was due in large measure to his grasp of the political situation in Germany at the time. As early as 1929, when he left Göttingen, he became convinced that another war with Germany was likely, and his views on the matter were confirmed by his conversations with the German refugee scientists arriving in Britain after 1933. Linfoot, a man of delicate physical constitution, knew that he would be unfit for military service, and so he began to consider what he could most usefully do. The presence of Dr. C. R. Burch in the H. H. Wills Physics Laboratory at Bristol stimulated him to begin the study of optics, a subject which he felt was of great practical value and in which he had retained an interest since boyhood (in his teens he had constructed a small telescope for lunar observations). As an accomplished mathematician he was certainly in a position to advance the theoretical aspects of the subject, and his delicacy of touch and visual acuity enabled him to contribute to its practical side as well. His skill in this latter respect developed sufficiently for him to be able to exhibit a microscope of his own construction at the 1939 Annual Exhibition of the Physical Society.

During the second world war, as he had foreseen, optical systems of advanced design were urgently needed for photographic aerial reconnaissance, and, while remaining at Bristol, he did substantial work in this area for the Ministry of Aircraft Production. In this period he was also associated with Professor Mott's research group which was working on a variety of urgent technical problems.

His professional status in mathematical optics was publicly confirmed in 1948 when he moved to Cambridge to become Assistant Director of the University Observatory (and, later, John Couch Adams Astronomer). This was an especially propitious moment for him to arrive in Cambridge since the Mathematics Laboratory had just begun its activities and was engaged in the early stages of the construction of Edsac I, one of the first generation of fast computers. The development of these was of crucial importance for optical design, where the sheer weight of the arithmetical calculations constituted a real barrier to progress. Linfoot began almost immediately to write programs which were well received by the Laboratory personnel since they ran for a reasonable length of time and so enabled the computer's performance to be properly assessed. He developed at this time a keen interest in computers which came to exercise a strong influence on his thinking. I remember him remarking to me, sometime in the late 1970s, in response to my observation that I would be hard put to give a satisfactory definition of mathematics, that he wouldn't find it an easy task either, but of one thing he was sure: mathematics

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is the most efficient means of programming the human central nervous system so far devised!

Despite his professional shift to optics, Linfoot never lost his interest in pure mathematics, and he made resolute efforts to keep abreast of contemporary developments. In the early 1960s he studied Kelley's *General topology* and Loomis' *Abstract harmonic analysis*: I well recall how amused he was at the fact that most of Kelley's book of 300 pages is summarized in the first 12 pages of Loomis'!

Until his retirement in 1970 he produced a stream of papers on optical subjects; in addition, he wrote two books: *Recent advances in optics*, 1955 and *Fourier methods in optical image evaluation*, 1964. This latter work is a striking testimony to Linfoot's mathematical prowess: he applies the methods of Fourier analysis employed in information theory to the problem of evaluating an optical image on, for example, a photographic plate.

Linfoot had a wide range of interests in addition to his professional specialities. Music was especially important to him: his father had been a violinist and, although he did not take up an instrument as a child, he learned to read musical notation and much enjoyed score-reading. In his 30s he taught himself to play the piano and acquired enough technique to play the preludes (but not, to his chagrin, the fugues!) of Bach's '48'. He was also well-read in many branches of literature and had built up a large and diverse personal library. He was a fine chess player (he played for Oxfordshire), and had a keen interest in the Japanese game of *Go*. (One of his Japanese research students gave him a magnificent *Go* board as a gift.) He had a talent for line drawing, revealed in the graphs and diagrams of the many notebooks (to be housed in the Manuscript Section of the Cambridge University Library) which record his thirty years study of optics. His graphic ability was remarkable: left-handed from birth, but right-handed by education, he could write rapidly and legibly with either hand. As his notebooks show, with pen he habitually used his right hand, with pencil, his left.

Linfoot's retiring nature and delicate health caused him to lose contact with many of his colleagues as he got older, and this may have been responsible for his work receiving less than the recognition it deserved. On the other hand those who, like myself, had the good fortune of knowing and receiving instruction from him during his later years benefited greatly from his encouragement and wise counsel. I had the privilege of being the Linfoots' guest at various periods during the 1960s and I shall always remember their kindness to me. In the spacious drawing room of their delightful house in the Cambridge Observatories, immersed in such a stimulating cultural atmosphere, one felt in some way close to the heart of things.

Linfoot's mathematical work

Linfoot's mathematical papers were all written during 1926–39, most of them in collaboration. Although they cover a considerable range of topics—Fourier analysis, number theory, and probability theory *inter alia*—they all exhibit the mastery of the techniques of classical analysis and penchant for detailed calculation that he was later to bring to his work in optics (for an assessment of which, see the forthcoming obituaries in the Journal of the British Astronomical Association and the Quarterly Journal of the Royal Astronomical Society).

In the early paper [1] he uses methods of classical geometry to describe the domains of convergence of Kummer's 24 solutions to Riemann's differential equation known as the *P*-equation.

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In the papers [2] and [3] he sharpens and extends Poisson's formula for the deviation of a discrete random variable from its mathematical expectation. In [5] he gives a new sufficient condition for the truth of the formula ("Poisson's summation formula")

$$\sqrt{\alpha}\left(\tfrac{1}{2}\phi(0)+\sum_{n=1}^{\infty}\phi(n\alpha)\right)=\sqrt{\beta}\left(\tfrac{1}{2}\psi(0)+\sum_{n=1}^{\infty}\psi(n\beta)\right)$$

where $\alpha > 0$, $\alpha\beta = 2\pi$, which relates a given real function $\phi(x)$ to its Fourier cosine transform $\psi(x)$.

The papers [4] and [6] are concerned with almost periodic functions. The main result of [4] is a strengthening of a theorem of Bohr asserting the identity of almost periodic functions having the same Dirichlet series expansions, while in [6] he strengthens another theorem of Bohr giving sufficient conditions on the coefficients of a Bohr–Fourier series $\sum_{n=1}^{\infty} a_{\lambda_n} e^{i\lambda_n t}$ for it to converge absolutely.

The six papers [7]–[12] on number theory, written jointly with C. J. A. Evelyn, contain an investigation of the behaviour of N^{th}-power-free numbers, for any given number N. In [7] an estimate is obtained for the number of ways of decomposing a large number into s such numbers, for a given s. In the five succeeding papers [8]–[12] these basic estimates are refined and extended, culminating in a treatment of the case in which the members of an N^{th}-power-free decomposition are required to belong to a given arithmetic progression.

The papers [13] and [14], written in collaboration with L. S. Bosanquet, are concerned with summability of Fourier series. In [13] they formulate and investigate the behaviour of a logarithmic refinement of the notion of (C, α) summability of series; in [14] this new scale of summability is applied to Fourier series, where it is shown to yield rather detailed information about their convergence. In [15] they prove the claims made in [13] about the behaviour of certain generalizations of the functions

$$g_\alpha(t) = \int_0^1 (1-u)^{\alpha-1}\cos tu\,du\,, \qquad \bar{g}_\alpha(t) = \int_0^1 (1-u)^{\alpha-1}\sin tu\,du$$

first studied by W. H. Young.

The important paper [16], written in collaboration with Hans Heilbronn, is concerned with complex quadratic extensions of the rational field. Numerical evidence strongly suggests that there are at most nine positive integral values of m for which the fundamental theorem of arithmetic holds in the quadratic extension $Q(\sqrt{-m})$, namely

$$m = 1, 2, 3, 7, 11, 19, 43, 67, 163\,.$$

In [16] it is established using analytical methods that there can be at most one more such m. It was not in fact shown until the 1960s that the additional m does not exist: for a more detailed discussion, see Cassels and Fröhlich [1977].

Papers [17]–[19], written jointly with his Bristol colleague W. M. Shepherd, deal with some problems arising in connection with Fourier series. In [17] they show that

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the trigonometric series equations

$$\sum_{n=0}^{\infty} \alpha_n \cos n\theta = \cos m\theta$$

$$\sum_{n=1}^{\infty} \alpha_n \sin n\theta = -\sin m\theta$$

have at most 2 linearly independent solutions $\langle \alpha_0, \alpha_1, \ldots \rangle$. In [**18**] they consider the infinite system of equations

$$(1) \quad \begin{cases} \dfrac{\alpha_0}{\lambda} + \dfrac{\alpha_1}{\lambda+1} + \dfrac{\alpha_2}{\lambda+2} + \ldots = 0 \\[2ex] \dfrac{\alpha_0}{\lambda-1} + \dfrac{\alpha_1}{\lambda} + \dfrac{\alpha_2}{\lambda+1} + \ldots = 0 \\[2ex] \dfrac{\alpha_0}{\lambda-2} + \dfrac{\alpha_1}{\lambda-1} + \dfrac{\alpha_2}{\lambda} + \ldots = 0 \\ \qquad \vdots \qquad\qquad\qquad \vdots \end{cases}$$

in the infinitely many unknowns $\alpha_0, \alpha_1, \ldots$, where λ is real and nonintegral. They show that if $\lambda > 0$ the equations have only the trivial solution $\alpha_0 = \alpha_1 = \alpha_2 = \ldots = 0$ while if $\lambda < 0$, the vector space of solutions has dimension $1+$ the integral part of $|\lambda|$. This result is applied to prove a generalization of the theorem in [**17**], namely, that given functions $f(\theta)$, $g(\theta)$, the trigonometric series expansions

$$\sum_{n=0}^{\infty} \alpha_n \cos n\theta = f(\theta) \qquad (|\theta| < \tfrac{1}{2}\pi)$$

$$\sum_{n=1}^{\infty} \alpha_n \sin n\theta = g(\theta) \qquad (\tfrac{1}{2}\pi < \theta < \pi)$$

can have at most 2 linearly independent sets $\langle \alpha_0, \alpha_1, \ldots \rangle$ of coefficients. Finally, in [**19**] they investigate the solutions of the more general system of equations in which the right hand sides of (1) do not necessarily vanish.

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1. 'The domains of convergence of Kummer's solutions to the Riemann *P*-equation', *Messenger of Math.*, 56 (1926), 17–25.
2. 'On the "Law of large numbers" I', *Proc. London Math. Soc.* (2), 28 (1928), 347–367.
3. 'On the "Law of large numbers" II', *Phil. Trans. Roy. Soc.* (A), 227 (1928), 417–451.
4. 'Generalisation of two theorems of H. Bohr', *J. London Math. Soc.*, 3 (1928), 177–182.
5. 'A sufficiency condition for Poisson's formula', *J. London Math. Soc.*, 4 (1929), 54–61.
6. 'A remark on Bohr–Fourier series', *J. London Math. Soc.*, 4 (1929), 121–123.
7. (With C. J. A. Evelyn) 'On a problem in the additive theory of numbers, I', *Math. Z.*, 30 (1929), 433–448.
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13. (With L. S. Bosanquet) 'Generalized means and the summability of Fourier series', *Quart. J. Math.*, 2 (1931), 207–229.
14. (With L. S. Bosanquet) 'On the zero-order summability of Fourier series', *Quart. J. Math.*, 6 (1931), 117–126.
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16. (With H. Heilbronn) 'On imaginary quadratic corpora of class-number 1', *Quart. J. Math.*, 5 (1934), 293–301.
17. (With W. M. Shepherd) 'On Fourier series with mixed coefficients', *Quart. J. Math.*, 9 (1938), 121–127.
18. (With W. M. Shepherd) 'On a set of linear equations', *Quart. J. Math.*, 10 (1939), 1–10.
19. (With W. M. Shepherd) 'On a set of linear equations, II', *Quart. J. Math.*, 10 (1939), 84–98.

Linfoot's books

Recent advances in optics (Clarendon Press, Oxford, 1955).
Fourier methods in optical image evaluation (The Focal Press, London and New York, 1964).

List of Linfoot's mathematical notebooks to be kept in LMS Archives

A. Oxford 1924–28

1. *Theory of functions* (Hardy, 1925)
2. *Theory of functions* (Hardy, 1926)
3. *Theory of functions* (Hardy, 1927)
4. *Theory of functions* (Hardy, undated)
5. *Theory of numbers* (Hardy, 1924)
6. *Theory of numbers* (Hardy, 1925)
7. *Transfinite numbers* (Hardy, 1925)
8. *Distribution of primes* (Hardy, 1926)
9. *Fourier series & summation of series* (Hardy, undated)
10. *Dirichlet series* (Hardy, 1926)
11. *Miscellaneous lectures* (Hardy, 1926–28)
12. *Real functions* (Besicovitch, 1925–26)

B. Göttingen 1928–29

1. *Continuous groups* (van der Waerden);
   Class field theory (Artin: transcribed by Linfoot in 1932)
2. } *Almost periodic functions* (H. Bohr)
3. }
4. *Waring's Problem* (Landau)
5. *Schlicht functions* (Landau)

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C. PRINCETON 1929–30

1.
2. } *Wave mechanics and quantum theory* (H. P. Robertson, J. von Neumann)
3.

4. *Dimension and Menger's Theorem* (P. Alexandroff)

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