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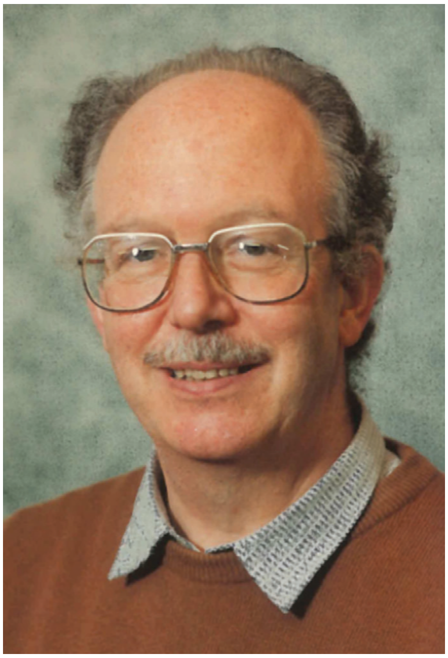
OBITUARY

John David Philip Meldrum, 1940–2018



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1. *Family years*

John David Philip Meldrum was born in Rabat, Morocco, on 18 July 1940 and died in Edinburgh, Scotland, on 9 August 2018 after a battle with the Parkinson disease. His parents were Philip Meldrum, an Anglican clergyman, and Ruth Meldrum, née Abadia, a former nurse. John was the second of two children, the first-born being a sister, Sonia. Not much is known about their years in Morocco, except that John attended the Lyceé Lyautey (Petit Lycée, Grand Lycée) in Casablanca until age 16 when the family returned to England. However, John retained a little of Morocco throughout his life, speaking French whenever the opportunity arose, writing mathematical papers in French journals, and translating French mathematics into English. (See ⟨**5**⟩.)

Upon their return to England in 1956, the Meldrum family settled in Ipswich, Suffolk, where John attended Ipswich School, finishing his A-level examinations in 1958. He then gained an Exhibition to Emmanuel College, Cambridge.

While at Cambridge, John attended St Barnabas Church, where he met his future wife, Patricia (Pat) Sealy. They were married on 30 March 1968. A daughter, Elizabeth (Liz), was born on 26 March 1970; she is now Executive Director, Chaplain Services, Santa Clara County Jail at Correctional Institutions Chaplaincy in Santa Clara, California. She and her husband have three children: Abigail, Emily and Thomas. A son, David, was born on 13 August 1973;

he is now an ordained Anglican priest, serving in South Africa, with three children: Morgan, Keanue and Jenica.

After moving to Edinburgh in 1969 the Meldrum family was very active in St Thomas' Episcopal Church, Corstorphine, serving in many roles, including vestry prayer groups and as small group leaders. Certainly, religion was a large part of their married life.

Both children recall a very happy home life with much joy and laughter; they recall John telling the same jokes many times over. John and Pat loved hosting people for dinner parties, and were generous with hospitality and fellowship with church friends, mathematical colleagues, graduate students, and visitors. Both authors can attest to this.

After the children left for their respective universities, Pat accompanied John to mathematical conferences and visits to universities. Again, both authors can attest to the fellowship of John and Pat at meetings and visits.

Pat died on 4 May 2011. John continued to do a little travelling, but soon had to stop because of the Parkinson disease.

2. *Cambridge years*

John enrolled in Cambridge in the Michaelmas term, 1958, as an Exhibitioner at Emmanuel College (he sat the scholarship examination in December 1957, before his A-levels) and in 1960–1961 he was appointed as a Senior Scholar of Emmanuel College, and a Bachelor Scholar from 1961. In 1959, he took Part I of the Mathematics Tripos, gaining a first class; Part II was taken in 1961 (Wrangler) and Part III in 1962. John was admitted as a research student at Cambridge in October 1962. He received his B.A. from Cambridge in 1961. John was appointed as a Supernumerary Fellow and College Lecturer in Mathematics at Emmanuel in 1964, two years before being awarded his PhD (he received his M.A. in Mathematics in 1965). He remained in post until 1969. From 1965 to 1969, he served as Director of Studies in Mathematics at New Hall and Emmanuel College. In 1964, John was awarded the Rayleigh prize in mathematics.

There was a very strong and active group of algebraists at Cambridge at that time, headed by Philip Hall, who was appointed to the Sadleirian Professorship in 1953. A few years later, D. R. Taunt, D. G. Northcott and D. Rees were appointed to junior positions. Among the students in algebra around the time of John were Trevor Hawkes, Brian Hartley and Derek Robinson. Probably the Part III lectures of Hall inspired John.[†] Thus one sees an active collection of algebraists particular in group theory.

John Meldrum received his PhD in 1966 under the supervision of Derek Roy Taunt (a student of Philip Hall), who had worked as a code breaker at Bletchley Park during World War II. John's dissertation had the title 'Central Series in Wreath Products'.

3. *Edinburgh years*

On 1 October 1969, John Meldrum was appointed as a lecturer in the Department of Mathematics, University of Edinburgh. On 1 October 1982, he was appointed as Senior Lecturer and on 1 October 1996, he was promoted to be a Reader. He retired from the University of Edinburgh on 31 July 2004.

We have pointed out above that John was active in church life while in Edinburgh. He was also active in the Department life, the University life, and the mathematical community life.

In addition to lecturing to undergraduates, John was active in the Department's algebra seminar, was a member of several course committees in the Department, serving as chair of

[†]Comment by G. Dales: In the lectures of those days, the lecturer wrote notes in chalk on the blackboard; some were not readable; Hall's (and Taunt's) were immaculate and precise.

a working party on third-year courses, 1979–1980, and from 1993 until his retirement, he was the Departmental Director of Studies. At the Faculty of Science level, he was a member of the Postgraduate Studies Committee from 1977 to 1980.

John also served as an external examiner for mathematics at the University of Glasgow, at the University of Aberdeen, and at the University of St Andrews.

At the postgraduate level, John was involved in several ways with students. He served as supervisor or co-supervisor of thirteen research students, the first one while he was still at Cambridge and the final one two years after his retirement. Moreover, John had over twenty appointments as an External Examiner for Postgraduate degrees, including students from India and South Africa.

As an indirect benefit to the Mathematics Department at Edinburgh, let us also include the many mathematical visitors John attracted, some for a month, others for longer stays. These visitors came from all over the world; United States, Canada, South Africa, Estonia, to list a few, attesting to John's mathematical expertise and willingness to share with others. The first author spent an enjoyable and productive visit as an SERC fellow in 1985, and the same for the second author for a four-month visit in 1990.

We mention other involvements in the mathematical community as a whole. Globally, John was a reviewer for international reviewing journals, a referee for four major publishers and a referee for numerous journals. More locally, John was a member of the London Mathematical Society, the European Mathematical Society, and the Edinburgh Mathematical Society, for which he served as Honorary Secretary for several years. In addition, he was active in the British Mathematical Colloquium (BMC), the largest pure mathematics conference to be held annually in the United Kingdom. The BMC has been held every year since 1949 at various venues ⟨**17**⟩. John served as the chair of the meetings committee of the BMC in 1976 and was the chair of the General Meeting in 1977. He was elected by the BMC to serve as the scientific contact to the Edinburgh Mathematical Society for several years.

John Meldrum was internationally known. One indicator of this is the above-mentioned visitors to the Mathematics Department at Edinburgh. Another indicator is the numerous invitations to international meetings that he received. He was a major lecturer at nearly twenty conferences and symposia. He was an invited lecturer or colloquium speaker at several universities, both in the United Kingdom and Europe, as well as South Africa, Canada, and the United States. He also was the organizer of two very well-attended international conferences in Edinburgh in 1978 and 1999. Related to this is the numerous research grants obtained to sponsor visitors, to travel to conferences, and to organize conferences.

John was also a well-liked and pleasant member of his department. We quote here Chris Smyth, a former colleague: 'John was a friendly, supportive and conscientious colleague, well respected in the department. He was famous (notorious?) for his puns — he would pop his head around my door occasionally with his latest one. He and Pat were good to me and my family when we first arrived in Edinburgh.'

In this section we have focussed on two aspects of John Meldrum's academic life, namely teaching and service, indicating his involvement and dedication to each of these areas. In the next section we focus on a third aspect of his academic life, namely his research.

4. *Research*

In this section we shall discuss the mathematical work of John Meldrum. As mentioned above, John received his PhD in 1966 under the supervision of Derek Roy Taunt with a dissertation with the title 'Central Series in Wreath Products'. Almost all of his remaining research is related to group theory, either generalizations to semigroups or the study of self-maps on groups, that is near-rings. To facilitate our discussion, we shall further divide this section into five non-independent subsections: Groups, semigroups, and wreath products; Near-rings in general; Endomorphism near-rings and distributively generated (d.g.) near-rings; Matrix near-rings and group near-rings; Books.

4.1. *Groups, semigroups and wreath products*

Perhaps a definition of wreath products is in order here: Let A and G be groups. In order to construct a (complete) wreath product of the groups A and G, we need two further groups, namely an arbitrary subgroup H of G and a so-called base group B. (All groups are written multiplicatively.) We suppose that an action ϕ of H on A is given: $\phi(h) : a \mapsto a^h$ $(a \in A)$. The group B consists of all functions $b : G \to A$ such that $b(xh) = b(x)^h$ for all $h \in H$ and $x \in G$. If $b_1, b_2 \in B$, then the product of b_1 and b_2 is defined in a pointwise manner by $b_1(x)b_2(x)$ for all $x \in G$, and it follows that this product is in B again. In fact, it can be shown that B is a group with respect to this product. The identity is the function b_0, where $b_0(x) = 1$ for all $x \in G$, and the inverse of b is the function b' that satisfies $b'(x) = b(x)^{-1}$ for all $x \in G$. Finally, we find that there is an action of G on B: for $u \in G$ and $b \in B$, $b^u(x) = b(ux)$ for all $x \in G$, we have that $b^u \in B$, from which the action follows.

The semidirect product W of B and G with respect to the action of G on B (explained above) is called the *twisted wreath product*, denoted by $W = A \operatorname{Wr}_H G$. In the case where $H = \{1\}$, this twisted wreath product is called the *complete wreath product*, denoted by $A \operatorname{Wr} G$. It can be shown that B is isomorphic to the complete (unrestricted) direct product $\prod_{\alpha\in\Lambda} A_\alpha$, where Λ denotes the set of left cosets of H in G, and, for each $\alpha \in \Lambda$, $A_\alpha \cong A$. If we use, instead of B, the restricted direct product, we obtain the *wreath product* of A and G, denoted by $A \operatorname{wr} G$.

One important application of wreath products is that it can be used to construct interesting examples. For example, there is an infinite p-group whose centre is $\{1\}$ ⟨**22**⟩. Also, there is an infinite p-group which contains a proper subgroup equal to its normalizer ⟨**23**⟩.

John Meldrum's first published paper is based on his dissertation 'Central Series in Wreath Products'. We mention here some of the significant results.

A group G is said to be of type (h, d) if the upper central series terminates after h steps and the lower central series terminates after d steps. A lemma of Grün ⟨**8**⟩ states that, if G is of type (h, d), then $h \geqslant 2$ implies that $d \geqslant 1$. Using intricate constructions with wreath products, John in [**2**] exhibited groups of type (h, d) for all groups in which both series are finite and in some cases where h and d are infinite. In [**4**] he completed this work by finding groups of type (h, d) for all ordinals $h, d \geqslant 1$. Once again very complicated constructions of wreath products were used.

In [**1, 3**], John investigated the nilpotent class of nilpotent wreath products of the form $G = A \operatorname{wr} B$. Baumslag ⟨**1**⟩ showed that $G = A \operatorname{wr} B$ is nilpotent if and only if A is a nilpotent p-group of finite exponent p and B is a finite p-group for the same prime p. Liebeck ⟨**10**⟩ computed the nilpotent class of nilpotent $G = A \operatorname{wr} B$ when A and B are abelian. John in [**3**] found the class of nilpotent G when B is abelian, thus generalizing the result of Liebeck.

Other papers devoted to John's work on wreath products are [**7, 10, 16, 57, 74**]. More will be said about [**74**] in the next section.

In the late 1980s and early 1990s John became interested in the semigroup of endomorphisms, $\operatorname{End}(A)$, of an algebra A. In 1983 he considered semigroup properties of $\operatorname{End}(G)$ where G is a group, relating these semigroup properties to properties of the group. In [**48**] he related group properties to semigroups that are semilattices of groups, such as Clifford semigroups (strong semilattices of groups). In [**68**], with his student M. Samman, he characterized the endomorphisms, $\operatorname{End}(S)$, where S is a Clifford semigroup, in terms of the associated endomorphisms and homomorphisms of the underlying groups in S. In [**58**] he returned to arbitrary algebras and investigated for which algebras, A, does $\operatorname{End}(A)$ have certain semigroup properties.

4.2. *Near-rings in general*

A *near-ring* is a triple $(R, +, \cdot)$, where R is a non-empty set and $+$ and $\cdot$ are two binary operations on R, such that all the axioms for a ring are satisfied, except perhaps the commutative law for addition, and the left distributive law. More specifically, such a near-ring is referred to as a *right* near-ring, since the right distributive law holds. In a similar way, a *left* near-ring is a near-ring in which the left distributive law holds, and not necessarily the right distributive law. All results that are true for right near-rings are true for left near-rings (after a left–right switch), and conversely. As a matter of fact, in the books by Pilz ⟨**19**⟩ and ⟨**20**⟩, all near-rings are supposed to be right near-rings, while in John Meldrum's book [**37**], all near-rings are supposed to be left near-rings.

It is well known that, if $(A, +)$ is an abelian group, then $\mathrm{End}(A)$ is a ring under pointwise addition and composition of endomorphisms of A. In the same way, if $(G, +)$ is an additively written, but not necessarily abelian, group, then $M(G) = \{f : G \to G\}$ is a near-ring under pointwise addition and compositions of self-maps of G. Any ring is a subring of the ring $\mathrm{End}(A)$ for a suitable abelian group $(A, +)$, and, likewise, any near-ring is a subnear-ring of the near-ring $M(G)$ of all self-maps of a suitable (not necessarily abelian) group $(G, +)$. An important subnear-ring of $M(G)$ is

$$M_0(G) = \{f : G \to G \mid f(0) = 0\},$$

the near-ring of all zero-preserving self-maps of G. Any (right) near-ring R with the property that $x \cdot 0 = 0$ for all $x \in R$ (such as $M_0(G)$) is referred to as a *zero-symmetric near-ring*.

In a right near-ring R the right distributive law holds, but, if $d \cdot (x + y) = d \cdot x + d \cdot y$ for all $x, y \in R$, then d is called a *distributive element* of R. The set of all distributive elements forms a semigroup under multiplication, and if it happens that $(R, +)$ is generated (as a group) by a semigroup $(S, \cdot)$ of distributive elements of R, then R is said to be a *d.g. near-ring*. A d.g. near-ring is always zero-symmetric.

A module G of a near-ring R is called a *tame module* if each R-subgroup of G is an R-ideal of G. John made significant contributions to the study of tame modules, including the seminal paper [**19**], which contains various important series of R-ideals of a tame R-module G, such as the socle series of G, that have proved to be important in the study of both G and R.

The polynomial near-ring of a group G consists of the maps on G generated by the identity and constant maps of G using the group operations of G. John co-authored three papers [**27, 32, 38**] dealing with polynomial near-rings of groups. A polynomial near-ring of a group is a special case of an important type of near-ring called a *compatible near-ring*. In 2009 a group of individuals, which included John, gathered in Linz, Austria, and began an attempt to determine the unit groups of compatible near-rings satisfying the descending chain condition on right ideals that led to [**73**], the first of a series of five papers on this topic. The final paper of the series [**74**], which is also John's final publication, completes the description of these unit groups. In a sense, [**74**] completes a circle of John's research as the description of these unit groups requires a more general form of a wreath product of groups, called a *Linz wreath product*, named after its place of origin.

Let G be a group, written additively, with identity 0, and let S be a semigroup of endomorphisms of G. Then

$$M_S(G) = \{f : G \to G \mid f(0) = 0 \text{ and } f\alpha = \alpha f \text{ for all } \alpha \in S\}$$

is a zero-symmetric (centralizer) near-ring with identity. Centralizer near-rings are general in that every zero-symmetric near-ring with identity is isomorphic to a centralizer near-ring for a suitable pair (S, G) [**37**, Theorem 2.8]. Moreover, centralizer near-rings are the building blocks of near-rings since for suitable pairs (S, G), $S \subseteq \mathrm{End}(G)$, they give rise to primitive near-rings ⟨**4**⟩. This leads to the problem of characterizing classes of centralizer near-rings in terms of the pair (S, G). John co-authored several papers on centralizer near-rings and we mention some of the significant results. In [**26**] John and Zeller investigated relationships between simplicity and 2-primitivity of $M_A(G)$, where $A \leqslant \mathrm{Aut}(G)$ is a group of automorphisms of G, and properties of (A, G). Here, the group theoretic concept, $\mathrm{stab}(g) = \{\alpha \in A \mid \alpha(g) = g\}$ played a prominent role. First, one has Betsch's Lemma ⟨**4**⟩: Let $x, y \in G$. There exists $f \in M_A(G)$ with $f(x) = y$ if and only if $\mathrm{stab}(x) \subseteq \mathrm{stab}(y)$. Secondly, they found the finiteness condition (F.C.) to be useful: If $\mathrm{stab}(x) \subseteq \mathrm{stab}(\alpha(x))$ then $\mathrm{stab}(x) = \mathrm{stab}(\alpha(x))$.

In a joint paper with Oswald [**17**], necessary and sufficient conditions on the pair (S, G), where $S \leqslant \mathrm{Aut}(G)$, for $M_S(G)$ to be a regular, or unit regular, or equivalence near-ring, were studied. This theme is continued in [**30**], in which the two-sided invariant subnear-rings of $M_S(G)$ $(S \leqslant \mathrm{Aut}(G))$ were investigated. Let $S \leqslant \mathrm{Aut}(G)$. The pair (S, G) is said to be *regular* if each $\mathrm{stab}(g)$ for $g \in G \setminus \{0\}$ is maximal in $\{\mathrm{stab}(x) \mid x \in G \setminus \{0\}\}$. From [**17**], the near-ring $M_S(G)$, $S \leqslant \mathrm{Aut}(G)$, is regular if and only if (S, G) is regular. The main result of [**30**] is the following: For $S \leqslant \mathrm{Aut}(G)$, if (S, G) is regular and there are only a finite number of conjugacy classes of stabilizers, then all two-sided invariant subnear-rings of $M_S(G)$ are determined. When $S = \{id\}$, this result had been given by Berman and Silverman ⟨**3**⟩.

The group U of units in $M_S(G)$, $S \leqslant \mathrm{Aut}(G)$, was investigated in [**36**]. An equivalence relation is defined on the orbits Sx, $x \in G$, for S acting on G, and to each equivalence class E_j, $j \in J$, of orbits, a multiplicative group B_j, $j \in J$, is found. The characterization $U \cong \prod_{j \in J} B_j$ is then established. Moreover, the groups B_j are isomorphic to certain wreath products, pointing out, once again, John Meldrum's group theory knowledge on near-ring theory.

In [**39**] the multiplicative subsemigroup N_d of distributive elements of the centralizer near-ring $N = M_S(G)$, S a subsemigroup of $\mathrm{End}(G)$, was investigated. In fact, $id \in N_d$, and this multiplicative monoid contains

$$\overline{S} = \{\alpha \in \mathrm{End}(G) \mid \alpha\sigma = \sigma\alpha \text{ for all } \sigma \in S\}\,.$$

This work generalized work of Betsch ⟨**4**⟩, in which he took S to be a group of automorphisms of G. One wishes to know when $N_d = \overline{S}$. It was established here that, if G is a free group and S is one of the semigroups $\mathrm{Inn}(G)$, $\mathrm{Aut}(G)$, $\mathrm{End}(G)$, then $N_d = \overline{S}$ if and only if G is the infinite cyclic group.

In [**35**] conditions were found on (S, G) such that $M_S(G)$ is a near-field. Thus a large collection of near-rings were determined from special pairs (S, G). We give a representative result for the case in which $S \leqslant \mathrm{Aut}(G)$: Let S be a group of automorphisms of G. Then the following are equivalent:

(i) $M_S(G)$ is a near-field;
(ii) $G = \{0\} \cup Sx$ for $x \in G \setminus \{0\}$ and G satisfies F.C. (defined above);
(iii) $G = \{0\} \cup Sx$ for $x \in G \setminus \{0\}$ and (S, G) is regular.

It was remarked in [**35**] that no example of a pair (S, G), $S \leqslant \mathrm{Aut}(G)$, was known such that $G = \{0\} \cup Sx$, but (S, G) does not satisfy F.C. When G is finite, F.C. is always satisfied, so $M_S(G)$ is a near-field if and only if $G = \{0\} \cup Sx$. Hence the above result generalized a result in ⟨**15**⟩. The general situation for S a semigroup of endomorphisms was also given, but is more technical because of the lack of stabilizers. As is well known, the additive group of a near-field is abelian. However, the authors gave an example of a near-field arising from a pair (S, G), $S \leqslant \mathrm{Aut}(G)$, with G non-abelian.

Once again one sees the flow of information between groups and near-rings in the hands of John Meldrum.

4.3. *Endomorphism near-rings and distributively generated* (d.g.) *near-rings*

Most of John Meldrum's interest in near-ring theory was manifested in his investigations of connections between groups $(G,+)$ and their various near-rings of mappings. Properties of these groups were used to obtain information about the structure of the corresponding near-rings — in particular those near-rings generated by semigroups of endomorphisms of G, that is, d.g. near-rings. It is no wonder that more than half of the material in his book [**37**] is devoted to these so-called *endomorphism near-rings*. John's first paper on near-rings [**5**] showed that the variety of near-rings generated by d.g. near-rings is the class of zero-symmetric near-rings. This extended a result of Malone ⟨**14**⟩.

One would expect a d.g. near-ring R to be 'closer' to a ring (in some sense), as all its elements are sums and differences of distributive elements of R. In fact, it is not too difficult to see that, if the addition of a d.g. near-ring is commutative, then it is a ring; see [**39**]. Other highlights in this paper by Carl Maxson and John are:

(1) if R is a d.g. near-ring generated by a semigroup A of automorphisms of a group $(G,+)$, then R is a ring if and only if each inner automorphism of the group $(R,+)$ is an endomorphism that commutes with all $\alpha \in A$;
(2) if α is an idempotent endomorphism of the group $(G,+)$, then the d.g near-ring R generated by the semigroup $S=\{id_G,\alpha\}$ is a ring if and only if $\operatorname{Im}\alpha$ is a normal subgroup of G;
(3) suppose that $(G,+)$ is finitely generated, where $X=\{x_1,x_2,\ldots,x_n\}$ is a minimal set of generators of G. Suppose that n is even, say $n=2m$, and that $z_i=[x_{2i-1},x_{2i}]$ for $1 \leqslant i \leqslant m$, are the only non-trivial commutators of elements in X. Then, if $\{z_1,\ldots,z_m\}$ is an independent set of generators of G', the derived group of G, and the semigroup S of endomorphisms of G that generates the d.g. near-ring R, contains the identity automorphism id_G of G, then R is a ring if and only if, for each $\sigma \in S$, it follows that $\sigma + id_G = id_G + \sigma$.

Three types of endomorphism near-rings are of particular importance: (i) the subnear-ring of $M_0(G)$ generated by $\operatorname{Inn}(G)$, the group of inner automorphisms of G, and denoted by $I(G)$; (ii) the subnear-ring of $M_0(G)$ generated by $\operatorname{Aut}(G)$, the group of all automorphisms of G, and denoted by $A(G)$; (iii) the subnear-ring of $M_0(G)$ generated by $\operatorname{End}(G)$, the semigroup of all endomorphisms of G, and denoted by $E(G)$. It follows that $I(G) \subseteq A(G) \subseteq E(G)$, and several authors have tried over many years to classify classes of groups for which equality occurs between these types of near-rings, and to characterize these near-rings. For example, in 1958 Fröhlich showed in ⟨**7**⟩ that when G is a finite simple non-abelian group, then

$$I(G) = A(G) = E(G) = M_0(G)\,.$$

Many years later, in 1987, John and Yuen Fong studied the endomorphism near-ring $E(H)$, where H is a direct sum of n copies of a finite simple non-abelian group G, thus generalizing the work of Fröhlich. They showed, amongst other results, that $E(H)$ contains the direct sum of n copies of $M_0(G)$, and that $E(H)$ is a 2-primitive near-ring; see [**45**].

John played a central role in continuing to investigate these three types of endomorphism near-rings for various classes of groups $(G,+)$. In 1977, the case where G is the infinite dihedral group was investigated, and John determined the near-rings $I(G)$, $A(G)$, and $E(G)$ in [**12**], after an elaborate attempt to determine all the endomorphisms of G. These near-rings determined by the finite dihedral groups were already described in the early 1970s; see ⟨**12**⟩ and ⟨**13**⟩. In [**18**] a similar analysis was carried out for the case where G is a finite general linear group. In all cases $(I(G), A(G), E(G))$, the structure of the near-ring is given modulo a nilpotent ideal contained in the nilpotent radical (where the nilpotency class of the radical is also determined).

Many of the results regarding these endomorphism near-rings were given modulo their various radicals. For example, in [**13**], John and co-author Carter Lyons characterized these radicals in terms of series of subgroups of G. In fact, they did this for the d.g. near-ring generated by any subsemigroup S of $\mathrm{End}(G)$, as long as $\mathrm{Inn}(G) \subseteq S$.

Next in line was the symmetric group $G = S_n$. During the late 1970s, John and his PhD student Yuen Fong embarked on the huge project of determining the endomorphism near-rings of the symmetric groups. The case $n \geqslant 5$ was treated in [**22**], where it was found that

$$I(S_n) = A(S_n) = E(S_n) = R\,,$$

and where R has an ideal N such that $N^2 = \{0\}$, and $R/N \cong M_0(A_n) \oplus \mathbb{Z}_2$. The one-sided ideals of R were also determined in this paper. In another (related) paper [**25**], John and Yuen Fong studied also the case where $n = 4$, the first time that the endomorphism near-ring on a solvable group of class three was studied. Due to the fact that, in S_4, there exists a non-trivial normal subgroup (Klein's four-group) different from the alternating group A_4, the approach here was quite different from the approach used for $S_n,\ n \geqslant 5$. It was shown that all the Jacobson radicals of the near-ring $E(S_4)$ coincide (denoted by J), and that J is a nilpotent ideal of nilpotency class 3. Furthermore, $E(S_4)/J \cong \mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus M_2(\mathbb{Z}_2)$, and it was shown that $I(S_4) = A(S_4) = E(S_4)$ is indeed an enormous near-ring with

$$2^{35} \cdot 3^3 = 927\,712\,935\,936$$

elements. This is quite a big jump from $I(S_3) = A(S_3) = E(S_3)$, a near-ring with a mere 54 elements.

One might start to think that $I(G) = A(G) = E(G)$ for all groups G, but, for example, $I(A_4) \subsetneq A(A_4) = E(A_4) \subsetneq M_0(A_4)$, while these strict containments all become equalities for the groups $A_n,\ n \geqslant 5$. For $n = 3$ we have

$$I(A_3) = A(A_3) = E(A_3) = \mathrm{End}(A_3) = GF(3) \subsetneq M_0(A_3)\,.$$

See ⟨**21**⟩.

In 1997, further work on endomorphism near-rings by John was published in [**64**]. This time, links between the (group)nilpotence of $(G, +)$, and (near-ring) nilpotence of an ideal of the near-ring $I(G)$ were investigated. Roughly ten years later, John and co-author Gordon Mason published a paper [**71**] where, in contrast to earlier work, the emphasis here was to recapture information of the group G from the near-ring $I(G)$. For example, they proved the following: Let G and H be two groups, and let R and S be the d.g. near-rings generated by semigroups of endomorphisms $S_G \supseteq \mathrm{Inn}\, G$ and $S_H \supseteq \mathrm{Inn}\, H$, respectively. Suppose that G and H are monogenic R- and S-modules, respectively, and that the minimal condition on annihilators hold for both R and S. If R is isomorphic to S (as near-rings) and G and H are both perfect, then it follows that $G/Z(G) \cong H/Z(H)$. This result generalized a result by Gary Peterson ⟨**18**⟩.

In later years, some further substantial papers on endomorphism near-rings (such as ⟨**21**⟩) appeared, and, without exception, it is difficult not to detect the influence of the foundational work that John Meldrum (and his co-authors) have established during earlier years.

4.4. *Matrix near-rings and group near-rings*

In 1964 Beidleman ⟨**2**⟩ tried to define matrix near-rings by considering the set $M_n(R)$ of $n \times n$ square arrays with entries from a given near-ring R. It turned out that, with the usual operations of addition and multiplication of matrices, and, if R has an identity, $M_n(R)$, $n > 1$, is a near-ring if and only if R is a ring. The problem is, for example, that the existence of non-distributive elements in R gives rise to a non-associative multiplication in $M_n(R)$.

The only interesting case left was therefore to consider near-rings without identity. In subsequent papers by Heatherly and Ligh (⟨**9**⟩, ⟨**11**⟩), those near-rings R for which $M_n(R)$ is also a near-ring (under the familiar operations of addition and multiplication) were completely characterized, namely that $M_n(R)$, $n > 1$, is a near-ring if and only if R is n-distributive. (A near-ring R is called *n-distributive*, n a positive integer, if for all a, b, c, d, r, a_i and b_i in R: (i) $ab + cd = cd + ba$, and (ii) $(\sum_{i=1}^{n} a_i b_i)r = \sum_{i=1}^{n}(a_i b_i r)$.)

It is interesting to note that there are, for example, near-rings R (without identity) such that $M_3(R)$ is a near-ring, but neither $M_2(R)$ nor $M_4(R)$ is a near-ring, under the usual addition and multiplication of matrices (see ⟨**6**⟩ for an example of a near-ring which is 3-distributive, but neither 2- nor 4-distributive).

During a visit by John Meldrum to Andries van der Walt in Stellenbosch in 1984, they attempted to find a definition according to which a matrix near-ring over an arbitrary near-ring R can be viewed in a natural and sensible way. Their inspiration came from a combination of facts in the instance when R happens to be a ring with identity:

(1) an $n \times n$ matrix is a linear mapping $R^n \to R^n$ (where R^n denotes the direct sum of n copies of the abelian group $(R, +)$); and
(2) all $n \times n$ matrices over the ring R can be generated (using matrix addition and multiplication) by the elementary matrices rE_{ij} (with $r \in R$ in position (i, j) and zeros elsewhere).

Let then R denote any near-ring with identity 1 and n a positive integer. Using the fact that the set $M(G) = \{f : G \to G\}$ of all self-maps of an additively written group $(G, +)$ (not necessarily abelian) forms a near-ring under pointwise function addition, and composition of functions, $n \times n$ matrices over R will be considered as certain mappings from R^n to itself (where, in this case, R^n denotes the direct sum of n copies of the not necessarily abelian group $(R, +)$). These mappings are generated by the *elementary matrix maps*, which are in their turn defined as follows: for each $r \in R$ and $1 \leqslant i, j \leqslant n$,

$$f_{ij}^r : R^n \to R^n$$

is the mapping given by $f_{ij}^r\langle x_1, x_2, \ldots, x_n\rangle := \langle 0, 0, \ldots, rx_j, \ldots, 0\rangle$, with rx_j in the ith position and zeros elsewhere. Note that if R happens to be a ring, then f_{ij}^r corresponds to the $n \times n$ matrix with r in position (i, j) and zeros elsewhere, that is, rE_{ij}. We are now ready to state the formal definition (see [**41**]):

Let R be a right near-ring with identity 1. The *near-ring of $n \times n$ matrices over$R$*, denoted by $M_n(R)$, is the subnear-ring of $M(R^n)$ generated by the set $\{f_{ij}^r \mid r \in R,\ 1 \leqslant i, j \leqslant n\}$.

It follows that $M_n(R)$ is a right near-ring with identity element $I = f_{11}^1 + f_{22}^1 + \cdots + f_{nn}^1$. Moreover, if R happens to be a ring, then $M_n(R)$ is isomorphic to the well-known full ring of matrices over R. It therefore seemed to be the correct approach to generalization of matrix rings to matrix near-rings.

One huge advantage of the development of matrix near-rings was that they serve as a rich source of examples. Here is one such success story.

First of all, let us quote two results in [**37**] on 2-primitive near-rings with minimal left ideals (originally due to Betsch ⟨**4**⟩):

THEOREM A [**37**, Theorem 4.5]. *Let R be a 2-primitive near-ring with minimal left ideal K. If G is an R-module of type 2, then*

(1) $G \cong_R K$;
(2) *there is an idempotent $e \in K \setminus \{0\}$ such that $K = Re = Ke$;*
(3) *every non-zero element of K has rank 1.*

As a partial converse, we have

THEOREM B [**37**, Theorem 4.12]. *Suppose that R is 2-primitive on the R-module G. If R contains an idempotent e of rank 1, then R has a minimal left ideal, or else we have*

$$e \neq 1 \quad \textit{and} \quad \mathrm{Ann}_R(G \setminus eG) = \{0\}. \tag{$*$}$$

These two theorems were proved in 1971, and it was not known whether the exceptional case $(*)$ in Theorem B can actually occur. In 1986, however, an example, using the theory of matrix near-rings, was constructed in ⟨**16**⟩ to show that $(*)$ is indeed possible.

It was only natural that this functional point of view would be utilized to define and study similar generalizations from ring theory to near-ring theory. In 1989 John Meldrum co-authored a paper [**51**] in which the groundwork was laid in order to establish the notion of a *group near-ring*.

As in the case of matrix near-rings, a group near-ring for any near-ring R and (multiplicative) group G can be defined by using certain elementary functions from the group $(R^G, +)$ into itself as generators, where R^G denotes the direct sum of $|G|$ copies of the group $(R, +)$. The idea is to define, for each $r \in R$ and $g \in G$, a function $\langle r, g\rangle$ which will take the element, say x, indexed by $g^{-1}h$ in R^G, and put rx in position h for each $h \in G$. More formally, $\langle r, g\rangle : R^G \to R^G$ is the function defined by

$$(\langle r, g\rangle(\mu))(h) = r\mu(g^{-1}h)$$

for all $\mu \in R^G$ and $h \in G$. The *group near-ring* $R[G]$ constructed from R and G is the subnear-ring of $M(R^G)$ generated by the set $\{\langle r, g\rangle \mid r \in R, g \in G\}$.

Some years before this general notion of a group near-ring was coined, John initiated a more direct approach specifically for the case where R is a d.g. near-ring; see [**11**]. Let us denote this group near-ring by $R(G)$. In the 1989 paper [**51**], a natural link between $R[G]$ and $R(G)$ was established, namely that in case R is d.g., then $R[G]$ is a homomorphic image of $R(G)$.

Since these two seminal articles [**41, 51**] on matrix and group near-rings appeared, numerous others on these topics have followed by several authors, including some co-authored by John. One very interesting result appeared in a joint paper [**66**] by John and the second author, where a finite abelian zero-symmetric near-ring R was found for which the J_0-radical of $M_2(R)$ is a so-called 'intermediate ideal' (a property that can never be satisfied by the J_2-radical). This specific example was discovered while the second author and John and his wife Pat spent a night in a B&B in Lochinver during a trip to the northern parts of Scotland. Ever since, this example was nicknamed 'the Lochinver example'.

Once again, during these endeavours, it was difficult not to realize the vision that John had when investigating the strong links between groups and their near-rings of mappings.

4.5. *Books*

a) *Near-rings and their links with groups* [**37**]. This is the third book to appear on near-rings, following the two editions of *Near-rings* by Pilz ⟨**19, 20**⟩, but with a different purpose and direction. Pilz's book is encyclopaedic in nature and a valuable reference while Meldrum's book is more for the beginning graduate student in mind. In fact, the first author has used it successfully on several occasions to introduce students to near-ring research. In the first part of the book, background and basic material for research in near-rings is presented. In the second part, John focusses specifically on endomorphism near-rings and d.g. near-rings. This book was indeed the right book at the right time in the development of the study of near-rings.

b) *Wreath products of groups and semigroups* [**62**]. This is a very useful book for those seeking to learn about wreath products as examples and counter-examples. Much of the work in this book had only appeared in various journal articles and several of the results are improvements of the original works. As with the near-ring book above, this book is written for the person wanting to learn about the theory, the construction, and the applications of wreath products of groups and semigroups.

c) *The elements of the history of mathematics* ⟨**5**⟩. This is a direct translation into English of the French book with the same title. It consists of a compilation of the historical notes from the Bourbaki series. Interesting to note here, is that John actually added a footnote in his translation to fix an incorrect statement in the original book; see ⟨**5**, page 196, footnote 17⟩.

5. *Research students*

John Meldrum had thirteen research students:

(1) J. C. Lennox, 1966–1969, PhD
(2) P. Munro, 1970–1973, PhD
(3) M. Shafi, 1972–1974, MPhil
(4) A. Scott, 1972–1975, PhD
(5) S. J. Mahmood, 1976–1979, PhD
(6) Y. Fong, 1976–1979, PhD
(7) P. Egerton, 1977–1982, PhD
(8) I. G. T. Roberts, 1977–1983, MPhil
(9) A. Ljeskovacs, 1978–1981, PhD
(10) A. Qureshi, 1978–1981, PhD
(11) S. J. Abbasi, 1986–1989, PhD
(12) M. Samman, 1994–1998, PhD
(13) A. Miah, 2003–1906, PhD

J. C. Lennox obtained his degree from the University of Cambridge, while all the others obtained their degrees from the University of Edinburgh. In two cases (A. Scott and A. Qureshi), John was the second supervisor.

S. J. (Suraiya) Mahmood writes: ‘John David Philip Meldrum was my supervisor for my Ph.D. in near-rings at Edinburgh University from 1976-1979. An inspiring teacher and supervisor, who nurtured his students to not only excel academically but also enabled a holistic sense of well being. As a mother of three young children following a challenging and academically rigorous regimen of study and research, I found him to be always helpful and kind with an intrinsic understanding of the challenges of a young student family. His helpful and understanding nature enabled us to quickly settle into our new environment. Besides our obvious connection on Mathematics we shared a passion for Sports, especially cricket and table tennis. Our research together continued even beyond the completion of my Ph.D. and we participated in many conferences together. The bond formed between both families has continued over the years, with my last meeting with J. D. P. Meldrum a few weeks before his passing away in 2018. He was a guide, a fellow mathematician, a mentor and a lifelong friend, who greatly enriched our lives over a span of 42 years.’

A note from Y. (Yuen) Fong: ‘J. D. P. Meldrum was my Ph.D. supervisor. I finished my Ph.D. dissertation in 17 months. He was a great teacher. He taught me the concept of group action that opened many new avenues for my research. After I returned to Taiwan (National Cheng Kung University in Tainan), I have established a strong algebra research group that consists of many algebraists of international stature. These include Kostia Beidar who stayed in Taiwan for 12 years, M. A. Chebotar who visited for six years, M. Bresar (several months), E. R. Puczylowski (several months), L. A. Bokut (five years), K. Kaarli (one year), R. Wiegandt (one year), L. van Wyk (one year), M. A. Mikhalev (several months) and his son A. V. Mikhalev (two years), and J. R. Clay (six months). All of this was possible due to the influence of John Meldrum on my mathematical career. I honour him as my everlasting teacher and master.'

Finally, M. (Mohammad) Samman comments: 'I met John first in 1994 when I arrived to Edinburgh University to do my Ph.D. under his supervision. Apart from my Ph.D. work in which he was a source of aspiration for me, I cannot describe the way and the impact of his sincere behaviour on me and on others. On one occasion, I travelled with him to attend a near-ring conference in Hamburg, he helped me in determining the direction of Qiblah at which Muslims should pray. He is really a man of very high morals, modest, who left a fingerprint after he left our world. We, near-ringers, and Math society miss John, but his candle light did not turn off.'

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