Taken in 1953.

Snapshot taken by Paul Halmos
in 1973 on a visit to Sheffield.

LEON MIRSKY 1918–1983

OBITUARY

LEON MIRSKY

Leon Mirsky, formerly Professor of Pure Mathematics in the University of Sheffield, died suddenly on 1 December 1983. He was a member of the Society for 40 years.

Leon was born in Russia on 19 December 1918; his father was a doctor, his mother a dentist. The parents were anxious to leave the country, but were prevented from doing so. Nevertheless, when Leon was 8, they succeeded in sending him to live a better life with an uncle and aunt in Germany. The uncle was in the wool trade and, when Hitler came to power, the family moved to Bradford. Leon, to whose education much care had always been devoted, went to Herne Bay College to be prepared for entry to a university.

In 1936, at the age of 17, Leon proceeded to King's College, London, working initially for the Intermediate Examination. The course included physics and he used to recall, with some pride, that he even managed the laboratory work. (More practical aspects of physics, such as the wiring of an electric plug, were ever beyond his reach.) After surmounting the Intermediate hurdle Leon embarked on the degree course in mathematics for which he had gained a university scholarship. His passion was the theory of numbers on which he read extensively, filling many notebooks with items that particularly pleased him. It was then that he conceived his life-long admiration of Landau to which many years later he gave such eloquent expression with his memoir [**80**].

The memories of Leon's contemporaries afford intriguing glimpses of his student days. Peter Gant (who later taught mathematics at Felsted School) recalls Leon's formal dress, complete with rolled umbrella, which went well with his remarkable erudition but contrasted delightfully with his lively sense of fun. Another friend was Ron Clark with whom he went youth hostelling during the vacations and who introduced him to rock climbing. (Leon never understood how anyone could have a fear of heights. Not so long ago a builder was horrified at his wish to accompany him on an inspection of his roof.) These holidays continued well into the 1940s with Leon enjoying not only the climbing, but also the company in the evenings. However, Leon's principal interests outside mathematics were always intellectual, mainly in the areas of philosophy, literature and history. His companion on these explorations was Geoffrey Kneebone with whom he exchanged ideas and impressions for the rest of his life.

After having been awarded a First Class Honours BSc degree, Leon began in 1940 to work for the degree of MSc. To anyone accustomed to present-day conditions it will seem strange that a person of his ability should not have aimed for a PhD. At that time students were, of course, on the whole privately funded, but finance played no part in the decision, for Leon always received ample support from·his uncle. However, Geoffrey Kneebone has no doubt about the explanation: as an alien in wartime Britain Leon felt very insecure and he was therefore anxious to obtain as soon

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as possible a qualification which would make him reasonably eligible for a university post; PhDs were not then so common as to be essential. After the outbreak of war King's College was evacuated to Bristol and the two mathematics departments more or less amalgamated. E. H. Linfoot was then a lecturer in mathematics at Bristol and, though his research interests had already shifted to mathematical optics, his earlier work was sufficiently recent for him to be a natural supervisor for Leon, devoted as ever to the theory of numbers. The thesis was completed in a year and earned its author the degree of MSc with distinction.

Leon's academic career began in 1942 with his appointment to a temporary assistant lectureship in Sheffield, where P. J. Daniell was professor of mathematics. Two years later he moved to a similar position in Manchester, but in 1945 he returned to Sheffield, at last a regular assistant lecturer. Here he was to remain except for the session 1951–52, spent as an exchange lecturer in Bristol, a city for which he always had an affection and which was now endowed with the additional attraction of Heilbronn's presence. As soon as Leon was settled in Sheffield he began to work for the degree of PhD as a staff candidate (consequently without the benefit of supervision) and he was awarded the degree in 1949. He became a full lecturer in 1947, was promoted to senior lecturer in 1958 and to reader in 1961. Finally, in 1971, he was given a personal chair in pure mathematics, an honour even rarer then than it is now. He officially retired in September 1983 at the age of 64, though at the time of his wholly unexpected death he was still giving one course of lectures in order to help his hard-pressed department.

When Leon came to Sheffield Richard Rado was a lecturer in the department. He and his wife Luise took Leon under their wing: they provided him with the affectionate home atmosphere that he craved as a newcomer to a strange city, and they enormously influenced his intellectual development as well. Leon greatly admired Richard's power as a mathematician and often paid glowing tribute to the benefit he derived from their association, not only in those days but also throughout his later life when contact was mainly by correspondence. Richard Rado always kept a mathematical diary and Leon, ever disposed towards the keeping of written records, also adopted this habit. For the first three years entries were sparse, with only 21 pages covered. However, from January 1946 onwards (as a result of a New Year resolution?) there is a torrent of material. During that year Leon filled three volumes totalling nearly 600 pages. Though the pace later slackened, he left altogether 35 volumes with 6652 pages. The diaries contain isolated results he had met and which he had found pleasing or striking and also a great deal of original work, much of which was later used in published papers. Leon found the diaries extremely helpful in his research and often urged his less strong-minded colleagues to follow his and Richard Rado's example. Luise Rado encouraged Leon's literary interests, particularly by reading English and German poetry with him. Jointly with Richard she opened the world of music to him and he became especially fond of the 19th century German songs which Luise used to sing to him. The friendship between Leon and the Rados was given a lasting monument with the publication in 1971 of the volume *Studies in pure mathematics*. This is a collection, edited by Leon, of 27 individual papers (one by Leon himself [**69**]) and presented to Richard Rado on his 65th birthday. A record of the ceremony, at which both Leon and Richard made memorable speeches, appears in volume 7 of the Bulletin of the IMA.

In 1953 Leon acquired a home of his own, and developed yet another scholarly interest, when he married Aileen Guilding who was then a lecturer in Biblical Studies and subsequently became professor and head of her department. They shared common tastes in literature and music as well as an abiding love of the countryside. The practice of taking long walks in Derbyshire and elsewhere became an important ingredient of their lives.

During the 1940s and early 50s Leon seemed to be immutably attached to the theory of numbers. However, when A. G. Walker, who was appointed to Daniell's former chair in 1947, asked him to give a lecture course in linear algebra (which had hardly figured in his own undergraduate curriculum) he immediately became fascinated by this novel subject. The result was his textbook on linear algebra, followed by some 35 papers spread over roughly ten years. Then, in the mid 1960s, Leon's research smoothly slid into the area of combinatorics, as is explained in the final section of this article. But his earlier loves never lost their charm for him and the breadth of his interests was one reason why he was such a stimulating colleague. Even more important, though, was his whole attitude to research. He was always asking questions, posing problems and anxious only to know the answers, unconcerned as to who supplied them. No-one adhered more steadfastly than he to his maxim that research should be a cooperative rather than a competitive activity. The number of his joint papers bears witness to this. What the list of his publications does not show is the extent to which he stimulated others to tackle problems that had been suggested by him or had, at least, arisen in the course of discussion or correspondence with him. The great expansion of the department in Sheffield during D. G. Northcott's headship gave him particular personal pleasure by creating ever increasing opportunity for the interplay of minds; and it is significant that four members of the department, whose principal work is in other fields, have published single-author papers on combinatorics. The dedication to Leon, on the occasion of his 60th birthday, of several papers written by friends and colleagues was a fitting tribute to his influence.

Leon was a born teacher. He welcomed the challenge of presenting a whole theory or just one proof in a logical, efficient, clear and elegant manner. Unsparingly self-critical, he stinted no effort to make his writing live up to his exacting standards; and his research papers, expository articles and textbooks all bear the unmistakable stamp of his style. The same meticulous care was devoted to the preparation of his lectures, which were highly individual performances. There was never any hint of familiarity with his audience and Leon always wore a gown to emphasize the formality of the occasion. On the other hand, the alert student could spot a succession of jokes all made with an entirely straight face and no change of tone. Though students, at first, found Leon intimidating, they always warmed to him and were known to reward him with applause. Of course Leon took particular trouble over his public lectures which he usually delivered entirely without notes. A professor of applied mathematics has said that one such lecture almost converted him to pure mathematics.

By talent and inclination Leon was well fitted for editorial work. In this capacity he gave many years of service to the *Journal of Linear Algebra and Applications* and to the *Journal of Mathematical Analysis and Applications*. He was also an editor of *Mathematical Spectrum*, a magazine designed for sixth formers and undergraduates. His three articles for the magazine [**64**, **79**, **82**] admirably convey his own enthusiasm for mathematics.

Within the University of Sheffield Leon conscientiously accepted his share of committee work. Though he professed to find ordinances and regulations unintelligible, he was always interested in larger issues and he had the reputation of unfailingly going to the heart of things when others might still be bogged down on the periphery.

However, it was in the university senate that he made his real mark. Passionately devoted to the concept of a university as a community of (mature) scholars, he abhorred the aims and methods of the student agitations in the 1970s. When the official policy was appeasement, he time and again courageously opposed it. His stand earned him respect and admiration, but only rarely support.

Amongst scientists the awe-inspiring spread and depth of Leon's learning in the humanities must have been almost unique. In philosophical, historical and literary discussions he could hold his own with professionals. A phenomenal memory and an exceptional reading speed made this possible, but enthusiasm was the driving force. Literature, particularly poetry, gave him most pleasure. He was fluent in German and Russian, read comfortably in French and taught himself enough Italian to appreciate Dante in the original. But Leon's tastes reflected the lighter side of his nature as well. He was captivated by P. G. Wodehouse and he knew the Gilbert and Sullivan operas by heart. At one time he also read great quantities of detective novels—at great speed, of course. A somewhat unexpected interest of his was cricket. Friends were apt to regard this as an amusing pose, but they were wrong, for he found the spectacle of the game aesthetically pleasing and the tactics appealed to his intellect.

Leon was not only a remarkable personality, he was also a complex one. Despite all his achievements, he was extremely modest. On the other hand, he had very firmly held and vigorously expressed opinions, though he was always open to argument and sometimes even changed his mind. His view of the world could loosely be called 'progressive', but he was innately conservative, as was indicated by many outward signs, including his dress. He had a very strong sense of justice and fair dealing, together with the courage to speak out against violations of his code of conduct. Occasionally, when the issue was a minor one, less principled observers might feel some embarrassment. Generosity, in all senses of the word, was another of Leon's characteristics. He was a fascinating and often delightful companion and, ever eager to help, he would devote any amount of time to his friends' affairs. Indeed, there are many for whom his name principally conjures up his extraordinary capacity for friendship.

We are grateful to Leon's many friends who have written or spoken to us about him. Our greatest debt is to Mrs Aileen Mirsky, Professor and Mrs Richard Rado and Dr G. T. Kneebone whose help in the preparation of the foregoing account has been invaluable.

Leon's work in his principal areas of mathematical interest—the theory of numbers, linear algebra, and combinatorics—is discussed under separate headings below.

Theory of numbers

One of Mirsky's main interests in the theory of numbers was the subject of r-free numbers. An r-free number is an integer (usually positive) that is not divisible by the rth power of any integer other than 1, or, what is equivalent, by the rth power of any prime. Thus r-free numbers are like primes in that they can be isolated by an exclusion process analogous to the sieve of Eratosthenes. The restriction is obviously of a less demanding nature than that of primality so that problems about r-free numbers are usually more tractable than their counterparts involving primes. Nevertheless, such questions can present considerable difficulties, there being several to which we still await an answer.

Mirsky, in a series of papers, obtained theorems concerning r-free numbers that, inter alia, include the counterparts of (i) Vinogradov's theorem concerning the representation of large odd integers as the sum of three primes; (ii) the Goldbach conjecture on the representation of even numbers as the sum of two primes; (iii) the prime twin conjecture concerning the infinitude of pairs of primes differing by 2. As well as deriving new results, he substantially improved upon the work of earlier writers, for example ⟨7⟩. Indeed, it is a measure of his precision that some of his results were keener than those obtained later by others who had overlooked his work. Perhaps his most impressive contribution in this area is his theorem **[11]** on the occurrence of groups of l integers $n, n+a_1, \ldots, n+a_{l-1}$ in which all members are r-free. This work, which has frequently been cited in the later literature, for example ⟨**6**, **12**⟩, is a fine example of the techniques that Mirsky introduced into number theory and that have had an important influence on other writers. It is also noteworthy because it enables one to study the occurrence of intervals of given length between consecutive r-free numbers.

Among the other topics in number theory that occupied Mirsky, we single out the subject of the divisor function $d(n)$ and its distribution. Defined as the number of divisors of the positive integer n, the function $d(n)$ can be shown to be $O(n^{\epsilon})$ and therefore the number $D(x)$ of values taken by it in the range $n \leqslant x$ must be small compared with x. The more detailed behaviour of $D(x)$ is of considerable interest but its full study is beset with difficulties. The striking asymptotic formula

$$\log D(x) \sim \frac{2\pi(2)^{1/2}(\log x)^{1/2}}{(3)^{1/2}\log\log x} \qquad (x \to \infty)$$

proved by Mirsky and Paul Erdös in 1952 **[20]** was therefore a significant development in the theory of $D(x)$ and related sums.

In 1977 Mirsky published an article **[80]** to commemorate the centenary of the birth of Edmund Landau. He wrote it, he said, partly as a tribute to Landau and partly 'to discharge an almost personal obligation.' Modestly he called it 'an extended footnote' to the obituaries by Hardy and Heilbronn and by Knopp, but it is a considerable piece of work on which Mirsky lavished much time and effort. After a biographical sketch it traces the later development of a number of topics on which Landau had worked, and in such a way that the non-expert is kept wholly absorbed.

Linear algebra

Mirsky published 35 papers on algebra, dealing mostly with linear algebra and its applications, and a substantial text-book entitled *An introduction to linear algebra* (Oxford University Press, 1955).

The book, in fact, came first. For in 1950 he was asked to give a course on linear algebra at Sheffield University. He became absorbed in the subject, and the book was the outcome of his profound reading and creative thinking about it. In the 1950s a few books were already available which embodied the words 'linear algebra' in their title. But undergraduate courses on this subject were not so common or uniform as they are now.

It will be remembered that what we now call linear algebra has a long history which went through three distinct phases. These may be briefly described as dominated in turn by (1) Determinants, (2) Matrices, (3) Vector Spaces. The first era lasted more than a hundred years. Indeed during the nineteenth century determinants were a popular subject in which algebraists could display their virtuosity in formal manipulations. Next came the epoch of matrices; initiated by Cayley and others around 1850, matrix algebra was brought to a high degree of perfection by Frobenius towards the end of the last century. Yet matrices were slow to gain general acceptance; it was not uncommon to meet mathematicians who had, around 1930, obtained a respectable degree at a prestigious university, but felt uncomfortable when confronted with matrices. When at last the power of matrix algebra was appreciated, it became fashionable to discount the importance of determinants, and students received but scanty training in their use. The final shift to the contemporary point of view was completed by 1960: it is now held that linear algebra is chiefly concerned with vector spaces and the linear transformations (or, rather, homomorphic maps) between them; matrices in their turn have been relegated to an inferior position.

Mirsky's text-book appeared when the three strands of linear algebra were all alive simultaneously, albeit in different age groups of algebraists. It is one of the great merits of this book that it furnishes a lucid and unbiased account of the different approaches to the subject. In accordance with the historical development the book opens with a chapter on determinants which includes such classical gems as Laplace's expansion formula and Jacobi's ratio theorem. Next, vectors are introduced, a distinction being made between 'vector spaces' (n-tuples) and 'linear manifolds' (abstract vector spaces). Then follows a down-to-earth account of matrix algebra standing on its own feet. However, in subsequent chapters Mirsky clearly brings out the modern view that we are chiefly interested in 'invariant', that is basis-free, properties of linear transformations. Mirsky intended his book to be an elementary text for undergraduates. Therefore he omitted the Jordan canonical form, evidently with regret, because this gap prevented him from tidying up the useful discussion of matrix power series. However, there is plenty of material to compensate: numerous inequalities for the characteristic roots (he eschews the ugly term eigenvalues), and bounds for a determinant, including Hadamard's celebrated inequality. The generous supply of exercises and problems at the end of each chapter contains a wealth of information covering a wide field of applications. After several reprints this book rightly deserved to be included in the Dover Collection, together with other classics.

Mirsky's published papers on linear algebra range from short notes to elaborate survey articles. They reflect his preference for concrete problems to which he gives neat answers, often in the 'best possible' form. Limitation of space does not allow us to summarize here each of his algebraic papers; instead we select a few areas which were typical of his research interests.

(i) In several of his papers Mirsky uses the concept of majorization applied to finite sequences of real numbers: following G. H. Hardy, J. E. Littlewood and G. Polya ⟨**10**, p. 45⟩ the notation $(a_1, a_2, \ldots, a_n) \prec (b_1, b_2, \ldots, b_n)$ signifies that

$$a_1 \geqq a_2 \geqq \ldots \geqq a_n;\; b_1 \geqq b_2 \geqq \ldots \geqq b_n;$$

$$a_1+a_2+\ldots+a_n = b_1+b_2+\ldots+b_n;$$

$$a_1+a_2+\ldots+a_r \leqq b_1+b_2+\ldots+b_r \quad (1 \leqq r \leqq n).$$

Mirsky was interested in the question whether there exist matrices with prescribed diagonal elements and characteristic roots or characteristic equation. This is one of the results [**30**]: Let $\omega_1, \omega_2, \ldots, \omega_n$, $a_1, a_2, \ldots, a_n$ be real numbers. Then $(a_1, a_2, \ldots, a_n) \prec (\omega_1, \omega_2, \ldots, \omega_n)$ is a necessary and sufficient condition for the existence of a real symmetric $n \times n$ matrix with $\omega_1, \omega_2, \ldots, \omega_n$ as its characteristic roots and $a_1, a_2, \ldots, a_n$ as its diagonal elements. Similar problems are posed about orthogonal, Hermitian, and doubly-stochastic matrices. This work is related to papers by A. Horn ⟨**13**, **14**⟩.

(ii) Several authors have used techniques of linear algebra to obtain information about the zeros of a polynomial. Mirsky has made some interesting contributions to this topic. Here are two examples in both of which f is the polynomial $x^n + a_1 x^{n-1} + \ldots + a_n$ with zeros $\xi_1, \xi_2, \ldots, \xi_n$. [**47**]: Let

$$A_k = \max\{|a_{i_1}| + \ldots + |a_{i_k}| : 1 \leqq i_1 < \ldots < i_k < n\}.$$

Then $|\xi_1\xi_2\ldots\xi_k| \leqq \max(1 + A_k, |a_n|)$. This sharpens a result of K. Mahler ⟨**15**⟩. [**48**]: Let $s(f)$ be the largest partial sum of the zeros of f, that is, $s(f) = \max|\sum_{k\in N}\xi_k|$, where N ranges through all subsets of $\{1, 2, \ldots, n\}$, and let

$$m(f) = \max(n, |a_1|^2, \ldots, |a_n|^2).$$

Then, for every $\varepsilon > 0$, there exists a number $K(\varepsilon)$ such that $s(f)/m(f) \leqq \pi^{-1} + \frac{1}{2} + \varepsilon$ whenever $m(f) > K(\varepsilon)$.

(iii) The algebraic topic which he treated most extensively is that of doubly-stochastic matrices. Jointly with H. K. Farahat [**43**] he succeeded in refining Birkhoff's result by proving that every doubly-stochastic $n \times n$ matrix can be represented as a convex combination of at most $n^2 - 2n + 2$ permutation matrices, and that this number cannot be diminished. In collaboration with Hazel Perfect [**53**, **54**] he made discoveries about the spectral properties of doubly-stochastic matrices and investigated interesting applications to convexity. In a very readable survey article [**50**] he gives an account of the theory of doubly-stochastic matrices up to about 1960.

(iv) Lest it should be thought that Mirsky worked exclusively within the framework of classical algebra, mention should be made of the important article with Farahat [**33**]; it deals with the embedding of groups in a ring and it contains applications to number theory and linear associative algebras. The topic is related to earlier papers by A. Ranum ⟨**20**, **21**⟩.

Combinatorics

The major part of Mirsky's contribution to combinatorics was in transversal theory. He was one of the pioneers in this branch of combinatorial mathematics, and many of the methods which he developed have had a considerable influence on the direction of growth of this relatively new discipline. The pivot on which transversal theory turns is a deceptively simple theorem of Philip Hall ⟨**9**⟩ which states that *a (finite) family of sets has a system of distinct representatives, or a transversal, if and only if the union of every k sets of the family contains at least k elements.* The content of this theorem is implicit in earlier literature but (as the authors remarked in [**57**]) 'it is precisely Hall's formulation that has provided the key to numerous problems and has stimulated a great deal of subsequent research'. Mirsky's first paper in this field [**55**], a joint one with Hazel Perfect, contains a characterization of the 'pattern' of elements of a doubly-stochastic matrix. The link with his earlier work in linear algebra is evident, for he had long been interested in doubly-stochastic matrices and their connection with convexity. Now, their more combinatorial aspects, and in particular the ways in which these can be handled by the use of Hall's theorem, began to attract his attention. Related problems concerning the patterns of multidimensional stochastic matrices have more recently been investigated by, for instance, J. Csima and R. A. Brualdi ⟨**2**, **3**⟩.

A common theme can be detected in several of Mirsky's papers, notably

[**56**, **60**, **61**, **68**, **75**]. This is his method of 'elementary constructions'. In these papers Mirsky displays the strongly self-refining nature of Philip Hall's basic theorem by showing that the theorem (and its infinite analogue) lead directly to a number of far-reaching generalizations. The technique which he uses is to apply Hall's theorem to a single new family obtained by such constructions as 'adjunction', 'extension', 'replication' etc. from the usually much more complicated system applying in the generalization. In [**56**], for example, a transparently simple proof of an infinite analogue of a theorem of Philip Higgins ⟨**11**⟩ on pairwise-disjoint transversals with prescribed defects is provided by these methods. Particularly noteworthy is Mirsky's insertion theorem for common transversals established in [**61**] by such a technique. *Let* $\mathfrak{A}$, $\mathfrak{B}$ *be two finite families of sets and let* $\mathfrak{A}' \subseteq \mathfrak{A}$, $\mathfrak{B}' \subseteq \mathfrak{B}$. *Then the following statements are equivalent.* (1) *There exist* $\mathfrak{A}_0, \mathfrak{B}_0$ *with* $\mathfrak{A}' \subseteq \mathfrak{A}_0 \subseteq \mathfrak{A}$, $\mathfrak{B}' \subseteq \mathfrak{B}_0 \subseteq \mathfrak{B}$ *which possess a common transversal.* (2) $\mathfrak{A}'$ *and a subfamily of* $\mathfrak{B}$ *possess a common transversal, and so do* $\mathfrak{B}'$ *and a subfamily of* $\mathfrak{A}$. This result is closely related to the symmetric supply-demand theorem of Fulkerson in the theory of network flow (see ⟨**8**, p. 42⟩). It is the precursor of important generalizations in the context of linkages, notably by J. S. Pym ⟨**17**, **18**⟩. This kind of insertion theorem is required in the proofs of many of those theorems in which upper and lower bounds are *simultaneously* prescribed for (say) the components of a generalized transversal of a given family and its intersections with the components of a given partition. In matrix terms, this problem is virtually equivalent to the derivation of necessary and sufficient conditions for the existence of an integral matrix whose elements, as well as whose row- and column-sums, conform to prescribed upper and lower bounds (see [**62**]). Theorems of this type have been established by other writers by use of linear programming techniques and flows in networks; generalizations to infinite matrices have since been investigated by R. A. Brualdi ⟨**1**⟩.

The interaction between transversal theory and matroid (abstract independence) theory has had profound and beneficial consequences for both disciplines. Mirsky was one of the first to perceive and exploit the interrelationship between the two theories. The basic result that the partial transversals of a family of sets satisfy the matroid axioms (though anticipated in the work of Edmonds and Fulkerson ⟨**5**⟩) was established and transversal independence thoroughly investigated in a joint paper [**59**]; and the central role of Rado's generalization of Hall's theorem to independent transversals ⟨**19**⟩ was recognized and applied in 'first-order' and 'second-order' transversal theory (see also [**65**]). One of the central areas of study in independence theory is the so-called 'representation problem' of Whitney ⟨**23**⟩: namely, to obtain necessary and sufficient conditions for an independence space to be linearly representable over a field or a division ring. (P. Vámos ⟨**22**⟩ has shown that this problem cannot be solved 'within' independence theory itself.) A notable theorem in [**59**] asserts that *transversal independence is linearly representable over a transcendental extension of the rationals*; it has more recently been considerably strengthened by other researchers.

Mirsky's influence on the development of transversal theory stems not only from his original research but also from his perceptive expository papers and from his book. Ever eager to chart and to codify, he perceived a unifying theme through the maze of the many and varied ramifications of Hall's theorem, and in doing so he recognized the emergence of a new field of study. As early as 1967 he was shaping it in the long expository paper [**57**] written jointly with Hazel Perfect, which appears now as a precursor of his book *Transversal theory*. One difference between the paper and the book is that abstract independence features in [57] only in a minor way and its relation to transversal theory was not then fully appreciated. *Transversal theory* appeared in 1971 and has remained a standard work of reference ever since. Its list of unsolved problems, suggestions for future research and very full bibliography enhance its value as a text for combinatorialists in this field. The largely expository paper [74] treats more comprehensively than the book the role of a remarkable theorem of Nash-Williams ⟨16⟩ on sums of independence spaces in the context of certain covering and packing theorems, and it provides an admirable conspectus of this area of research.

Aspects of combinatorial mathematics outside transversal theory have also, but to a lesser degree, claimed Mirsky's attention. Ramsey theory interested him greatly over a number of years though the volume of his published work does not fully reflect this. In 1977 he wrote the mathematical introduction [83] to a book on F. P. Ramsey. He also contributed a short paper [76] to the Bulletin of the IMA to commemorate the centenary of the birth of Issai Schur. In this paper he describes a theorem of Schur which is closely akin to Ramsey's theorem. A brief contribution to the theory of partially ordered sets [70] provides an example of Mirsky's discernment. Here he observes that, if the roles of chains and antichains in the statement of Dilworth's decomposition theorem ⟨4⟩ are formally interchanged, then another valid decomposition theorem is obtained. It is a shallower result than Dilworth's, but its interest lies in the fact that many corollaries of Dilworth's theorem also follow from this 'dual' formulation. An interesting general class of problems is explored in two papers [73, 81] written jointly with H. Burkill. Let M be a set of (square) matrices and M^* a proper subset of M with the understanding that both contain matrices of every size; and further let $\psi(n) = \psi(n; M, M^*)$ denote the largest integer k such that every $n \times n$ matrix in M possesses a $k \times k$ submatrix in M^*. Estimates are sought for $\psi(n)$, when n is large, for various specifications of M and M^*. From among many results we mention just one. *Let M and M^* consist of all matrices whose elements take at most r (resp. s) different values ($r > s$). If $\varepsilon > 0$, then for large n*

$$(1-\varepsilon)\log n/\log(r/s) < \psi(n) < 2\log n/\log(r/s).$$

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