



L. J. Mordell

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J. W. S. CASSELS

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## 1. *Origins and early life*

Louis Joel Mordell was born on 28 January, 1888, in Philadelphia (Pennsylvania). His parents were recent immigrants from Lithuania. His father Phineas Mordell (1861–1934) had come over in 1881 at the age of 20 and, according to family tradition, his first task was to earn the passage money not only of his bride-to-be (Annie née Feller, 1865–1938) but of his sisters as well. Phineas Mordell had taught Hebrew in Lithuania and was later to acquire a wide reputation as a Hebrew scholar but during his first years in Philadelphia he worked at various trades while industriously pursuing his studies. Later, according to one account, he worked partly as a teacher and partly as a night watchman so as to have time for his researches: it was only in middle age that he could devote himself entirely to teaching and study.

Louis was the third of a family of eight (four sons and four daughters). His eldest brother Albert was to become a member of the Philadelphia Bar and a well-known literary critic. His brother Maurice also became a member of the Philadelphia Bar and was for many years Assistant City Solicitor. The remaining brother, David, never fully recovered from gassing in the 1914–18 war.

The young Louis attended the public schools (in the American sense), primary school from 6 to 10 and the Grammar School from 10 to 14. Towards the end of his time at the Grammar School he tells us he came across some old algebra books in the 5 and 10 cent counters of a second-hand book store. He was fascinated by what he read and was soon reading widely. Several of these books were on his shelves when he died.

At the age of 14 he transferred to the Central High School of Philadelphia, “the oldest high school in the United States outside of New England” (founded 1838).

Here his mathematical talent was recognized and by the end of his sophomore year he had completed the four-year mathematics course. In later life Mordell kept up his contacts with his class-mates "the Class of 1906" although he was seldom able to attend their annual dinner. A "Report of the Class Historian" dated 1940 adds to our meagre knowledge of the period by revealing the nick-name 'X, Y, Z' and by quoting the following from the "Archives class record":

"'An unsoaped individual with neither the imagination of a theologian nor the insight of a politician'.

Entered Freshman; Member Senior Football Team; Teacher in Elective in Mathematics."

To judge from some of the entries against other members of the Class, too much cannot be read into the initial passage. Indeed the portrait in the *Philadelphia Press* of 10 January, 1907, looks almost too tidy.

The young Mordell noticed that many of the examples in the mathematical books he read were taken from Cambridge Scholarship and Tripos papers and, to use his own words, "I conceived what I can only describe as a thoroughly mad and crazy idea of going to Cambridge and trying for a scholarship". He adds "I had no idea of the necessary standards, I was self-taught mathematically and had never participated in a competitive examination".

With the help of his family and with money earned by coaching his fellow students he scraped up enough for a single ticket to Cambridge. He planned to work his passage back if the venture failed. According to the *Philadelphia Press* for Thursday, 10 January, 1907, "he devoted seven hours of every day to coaching and on one occasion stood at a blackboard for forty-eight hours in an endeavour to pull a student through an examination". (Even more incredibly it adds that the student who was subject to this ordeal did pass.)

In those days there were two main Scholarship groups. Mordell entered for the St. John's group because it contained more Colleges and so he thought the chances of success were higher than in the Trinity group. He need not have calculated so finely as he was top of the list by a substantial margin and was elected to a Scholarship at St. John's. His telegram to his father was the single word "Hurrah"—all he could afford.

## 2. Cambridge 1907–1913

So in October 1907 Mordell started to read for the Mathematical Tripos supported by a St. John's Scholarship and a scholarship from Philadelphia High. He has left a description of the very different Cambridge of those days in his *Reminiscences of an octogenarian mathematician* (Ref. 250), which I have already quoted briefly above. As it is readily available and gives a better impression of its author's person-

ality than I could hope to do, I have resisted the temptation to quote *in extenso* in the hope that the reader will be tempted to look it out for himself.†

Mordell's Director of Studies was the geometer H. F. Baker (later Lowndean Professor), whom he found unsympathetic. In later life he felt that he would have been more fortunate with G. H. Hardy at Trinity. He took Part I of the Mathematical Tripos, then the main mathematical examination, in 1909, the last year of the "order of merit". That is, in addition to the now usual subdivision into classes, the names of all the candidates were published in order. The first was the "Senior Wrangler". Interest in his identity spread far beyond academic circles and in LJM's words "reminded one of the Derby". As well as the usual College and University tuition, serious contenders had "private tutors" or coaches. Mordell's was T. J. I'A. Bromwich. The announcement of the results by the senior examiner was described in *The Times* for Wednesday, June 6th, 1909 as follows:

"The list is read at 9 o'clock: but half an hour before that time an enormous crowd gathered on Senate House Hill, and when the doors were opened they thronged into the floor of the house, which quickly filled. The galleries were reserved for the ladies, and these were mainly occupied by students of Newnham and Girton, who stood three or four deep against the rails. The period of waiting was occupied by the undergraduates in making chaffing remarks to friends across the building, and even dons and ladies did not escape their harmless banter. A few minutes before the clock of Great Saint Mary's Church struck nine, the Senior Moderator, Mr. J. M. Dodds, of Peterhouse, took up his position at the front of the East Gallery, and a hush passed over the crowd as he proceeded to unfasten the bundle which contained the lists. On the last stroke of the hour he began to read the list. Hardly had he pronounced the name 'Daniell' when a wild shout went up from the Trinity men, which continued for nearly half a minute. The Second Wrangler's name was greeted with less enthusiasm, presumably owing to the fact of his belonging to the same college as the Senior Wrangler: but Mr. Mordell, who came third, had a splendid reception from the Johnians, . . .". The man at the bottom of the list was traditionally presented with a "wooden spoon" at the subsequent graduation ceremony. The women, of course, were on a separate list.‡

So, in Mordell's own words "I blotted my copy-book and was only Third Wrangler. I think I could have done better". The Senior Wrangler was P. J. Daniell and the Second, E. H. Neville, both of Trinity and both coached by the legendary R. A. Herman.§

† Letters which LJM wrote to his parents at this time have survived and are now in the possession of his son, Mr. Donald Mordell. In one of them, written in 1907, he estimates the total travelling expenses from Cambridge to Philadelphia (and so, presumably, *vice-versa*) at \$27.50.

‡ The results of Part II and III of the Tripos are still read out at 9 a.m. in the Senate House, but the excitement is less intense now that the names in each class are in alphabetical order.

§ Up to the previous year, Trinity and St. John's had produced an equal number of Senior Wranglers. The final score was thus Trinity 56, St. John's 55. Herman coached one or more of the Senior Wranglers (ties for first place were possible) every year from 1903 to 1909. (Obituary by E. H. Neville, *Cambridge Review* for February 10, 1928, pp. 237-239.)

In his third year Mordell took Part II of the Tripos and stayed on to do research. It was then, he tells us, that he took up the theory of numbers in earnest. There was then no great interest in the subject in Cambridge, or indeed in Britain, and Mordell always regarded himself as self-taught. There was no Ph.D. degree at that time,† but in Mathematics the Smith's prizes provided a recognition of research achievement. The first Smith's Prize went to Neville; and Mordell got the second with an essay on the integral solution of the diophantine equation

$$y^2 = x^3 + k, \quad (2.1)$$

which was the basis of a long paper in the *Proceedings of the London Mathematical Society* (item 2 of the Bibliography). This was a topic to which he was to return again and again, and the equation is sometimes referred to as "Mordell's equation". It was brought into prominence by Fermat and had been much studied, particularly in the nineteenth century. Mordell reviews the history and gives new or more general conditions under which it can be proved insoluble or all the solutions can be obtained. Almost incidentally he gives a result which was later to be the key to his best known theorem and which, consequently, we must describe in some detail before resuming the historical narrative.

Let

$$f(x, y) = ax^3 + 3bx^2y + 3cxy^2 + dy^3 \quad (2.2)$$

be a binary cubic form with integral coefficients  $a, b, c, d$ . Associated with (2.2) are several invariants and covariants:

the hessian

$$\begin{aligned} H(x, y) &= \frac{1}{36} \begin{vmatrix} \partial^2 f / \partial x^2 & \partial^2 f / \partial x \partial y \\ \partial^2 f / \partial x \partial y & \partial^2 f / \partial y^2 \end{vmatrix} \\ &= (ac - b^2)x^2 + \dots \\ &= Ax^2 + Bxy + Cy^2 \quad (\text{say}); \end{aligned} \quad (2.3)$$

the discriminant

$$D = 4AC - B^2; \quad (2.4)$$

and the cubic concomitant

$$\begin{aligned} C(x, y) &= \frac{1}{3} \begin{vmatrix} \partial f / \partial x & \partial H / \partial x \\ \partial f / \partial y & \partial H / \partial y \end{vmatrix} \\ &= (a^2 d - 3abc + 2b^3)x^3 + \dots \end{aligned}$$

If the variables  $x, y$  are subjected to a unimodular transformation, the discriminant remains unchanged and the hessian and the cubic concomitant of the transformed form are just the transforms of the original ones. Further, there is the identity ("syzygy")

$$C^2(x, y) + 9H^3(x, y) + Df^2(x, y) = 0 \quad (2.5)$$

† It was introduced after the 1914–18 war to provide a qualification to attract Americans who would otherwise have gone to Germany for a Doctorate.

All this was much more familiar at the beginning of the century than it is now. To get the relation (2.1) Mordell puts

$$D = -4k. \quad (2.6)$$

Let  $u, v$  be integers such that

$$f(u, v) = 1. \quad (2.7)$$

Then (2.5) shows that

$$X = -H(u, v); \quad Y = \frac{1}{2}C(u, v) \quad (2.8)$$

gives an integral solution of

$$Y^2 = X^3 + k. \quad (2.9)$$

Now suppose that  $k$  has no square factors (other than 1) and that  $X$  is odd and prime to  $k$ . Then Mordell shows that the solution  $(X, Y)$  of (2.9) can be obtained by the above process, and indeed with  $u = 1, v = 0$ . We have to determine suitable coefficients  $a, b, c, d$  in (2.2). By (2.7) and (2.8) we have  $a = 1$  and  $A = -X$  respectively. By (2.9) we can (in many ways) choose integers  $B_1$  and  $C$  so that

$$B_1^2 - 4AC = k.$$

Then (2.4) holds with  $B = 2B_1, D = -4k$ . But now the values of  $b, c, d$  are uniquely determined by the identity (2.3) together with  $C(1, 0) = 2Y$  (the second half of (2.8)). Clearly  $b, c, d$  are rational. A completely elementary but typically ingenious argument shows that they are integers, as required, provided that  $B_1$  is chosen so that

$$B_1 \equiv -Y \pmod{X};$$

which is possible by (2.9).

Two cubic forms (2.2) which are equivalent under an integral unimodular transformation clearly give the same solutions  $X, Y$  of (2.9) by the formula (2.7). Since the number of equivalence classes of forms with given discriminant is finite, this means that the equation (2.7) has to be solved for only finitely many forms  $f$  (one in each class). In a paper which was little noticed at the time Thue had already shown that the equation (2.7) has only finitely many integral solutions for any cubic form  $f$  (or more generally, for any  $f$  other than a power of a binary quadratic). If Mordell had known of this theorem, he would have deduced at once that  $y^2 = x^3 + k$  has only finitely many integral solutions  $x, y$ ; at least with  $x$  prime to  $2k$ . As a matter of fact he only learned of Thue's result later (reference 15) and at the time he believed that there could be infinitely many solutions for some  $k$  (cf. end of reference 2).

The following year Mordell remained in Cambridge and continued the same line of investigation. By considering the syzygy of the invariants of the binary quartic he showed that all the integral solutions  $(x, y)$  of an equation

$$ey^2 = ax^3 + bx^2 + cx + d \quad (2.10)$$

can be expressed in terms of the solutions  $(u, v)$  of a finite number of equations of the type

$$f(u, v) = n,$$

where  $n$  is a given integer and  $f$  is a given binary quartic. If he had known Thue's theorem he could have deduced that the number of solutions of (2.10) is finite.

He submitted this work in candidature for a Fellowship at St. John's but was unsuccessful. A paper was rejected by the London Mathematical Society but was accepted by the *Quarterly Journal* (reference 3). A revealing note which Mordell must have written at this time on the cover of an offprint reflects his bitter disappointment.† If one attempts to read the paper through their eyes one can perhaps forgive St. John's College and the L.M.S. for their defective prophetic powers. Indeed like nearly all of Mordell's early papers it is written in a narrative style without the conventional enunciation of theorems etc. and it is difficult to disentangle what it is that is actually proved. He would clearly have benefited greatly from advice on how actually to present a piece of mathematics.

### 3. London 1913–1920

When his time at Cambridge came to an end, Mordell wrote to his father that he had had two offers of employment, one in London at £200 per annum and the other in Nova Scotia at £300. In a subsequent letter he told his father that he was taking the lower paid job because he felt that he would have more chance to develop as a mathematician in England than in Nova Scotia. The London job was, presumably, the lectureship at Birkbeck College which he occupied from 1913 until 1920 except

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#### *Preface*

† “This paper was originally sent for publication to the L.M.S. in 1913. It was rejected and certain alterations were suggested, only one of which the author could see his way clear to fall in with (making use of the syzygy of the quartic instead of solving directly for  $a, b, c, d, e$  from  $g_2 = ae - \text{etc.}$   $g_3 = ace + \dots$ ). It was rejected by them again last November. It was then published in the *Quarterly Journal* (edited by the editor of H. Smith's *Collected Works*) without any alteration or correction.

“Indeterminate equations have never been very popular in England (except perhaps in the 17th and 18th centuries); though they have been the subject of many papers by most of the greatest mathematicians of the world: and hosts of lesser ones; and we could literally give hundreds of references.

“Such results as, that we can find the general solution of  $f(x, y, z) = 0$  ( $f$  = ternary cubic) when we know one solution and the general solution of  $y^2 = 4x^3 - g_2 x - g_3$ , marks the greatest advance in the theory of indeterminate equations of the 3rd and 4th degrees since the time of Fermat; and it is all the more remarkable as it can be proved by quite elementary methods. Again the author's solution of  $z^2 = ax^3 + bx^2 y + \dots$ , is practically the only instance of the complete integer solution of an indeterminate equation of degree greater than the second.

“We trust that the author may be pardoned for speaking thus of his results. But the history of this paper has shewn him that in his estimation, it has not been properly appreciated by English mathematicians.”

*Note.* The reference to “last November” shows that this must have been written in 1914. “H. Smith” is H. J. S. Smith. In an expository paper written somewhat later (reference (11)) he states that, apart from Smith and Sylvester, English mathematicians had taken practically no part in the great advances in the Theory of Numbers in the last century.

for two years as a statistician at the Ministry of Munitions. One of his favourite anecdotes shows that for some of this period at least he still lived in Cambridge. He travelled by a certain train which should have got him to Birkbeck in time. But frequently the train arrived late. He pointed out the discrepancy between promise and performance to the Railway Company, who said that they would do something about it. And so they did: they adjusted the advertised time of arrival and, in consequence, the train now always arrived as advertised, but always too late for him.

In May 1916 he married Mabel Elizabeth (1896–1971), the only daughter of Rosa and Joseph Cambridge, a small farmer who, so his granddaughter recalls, pastured his stock on the greens of the eponymous town. There were to be two children of the marriage, a daughter, Frances Kathleen, now Mrs. Smith, and a son, Donald Louis, who reversed his father's Atlantic crossing and is now President of the Ryerson Polytechnic Institute, Toronto.

During his Birkbeck period Mordell's main interest appears to have been the theory of modular functions and its application to the theory of numbers. In 1917 he proved a conjecture of Ramanujan about his “ $\tau$ -function”—the coefficients of a certain modular form (reference 4). He does not appear to have pursued the question but later Hecke was to re-discover the argument:† it is now perhaps the most fundamental tool in the area (the “Hecke operator”) and, as Hecke showed, Ramanujan's property of the  $\tau$ -function is just a special case of a very general phenomenon.‡

Mordell was, apparently, the first to treat the representation of integers as the sum of a fixed number  $n$  of squares of integers by using the finite dimensionality of the space of modular forms of given dimensions to establish identities,§ thereby unifying the existing mass of results for individual values of  $n$ . He treated first even values of  $n$  in reference (13). It turns out that there is a certain modular form which Mordell called  $\chi$  which gives the principal part of the formulae for the number of representations; the other terms being much smaller can be regarded as “error terms”.|| As it happened Hardy was attacking the same problem at the same time from the point of view of the “circle method”. This does not in general give precise

† I have been told that foreshadowings can also be found in the work of Hurwitz.

‡ cf. E. Hecke, “Über Modulfunktionen und die Dirichletschen Reihen mit Eulerscher Produktentwicklung I”, *Math. Ann.*, 114 (1937), 1–28 = *Werke*, 644–671, especially Satz 28.

§ The fact that modular function identities involving theta functions give formulae for numbers of representations as sums of squares goes back to the origins of the theory in Jacobi's *Fundamenta Nova*. What is new is that the finite dimensionality gives a machinery for finding and proving identities. This method was later used with great effect by Hecke for representations by definite quadratic forms in general. (E. Hecke, “Analytische Arithmetik der positiven quadratischen Formen”, *Kgl. Danske Videnskabernes Selskab, Math.-fys. Meddelelser*, 13 (1940), No. 12, 134 pp. = *Werke* pp. 789–918.) For the ascription of the method to Mordell, see G. H. Hardy, “On the representation of a number as the sum of any number of squares, and in particular of five or seven”, *Proc. Nat. Acad. Sci. Washington*, 4 (1918), 189–193 = *Collected Papers I*, pp. 340–344, and Hardy, Ramanujan, Chapter 9.

|| We would now say that they are given by cusp forms. But Mordell had no general theory of cusp forms and had to use explicitly given functions.

formulae but produces a principal term, the "singular series" and an estimate of the error. The "singular series" turned out† to be Mordell's  $\chi$ . But Hardy's work produced a "singular series" for odd  $n$  as well and this enabled Mordell to extend his treatment to odd  $n$  (reference 16).

In the same circle of ideas are the works on class number relations‡ (references 6, 7, 19, 21), the evaluation of

$$\int_{-\infty}^{\infty} \frac{e^{at^2+bt}}{e^{ct}+d} dt$$

(references 18 and 68) and the paper on identities (reference 25).

#### 4. Manchester College of Technology 1920–1922

In 1920 Mordell "decided that a change of scene would be welcome" and was appointed Lecturer at the Manchester College of Technology. His brief stay there saw the publication of his most important individual theorem.

Reference has already been made to the relevance of Thue's theorem to Mordell's early work. He had already published a brief note (reference 15) drawing attention to the connection but now he gave the details (reference 27 and 29) to show that

$$ey^2 = ax^3 + bx^2 + ex + d \quad (4.1)$$

has at most finitely many integral solutions  $x, y$ . It was in an attempt to extend this result to the case where there is a quartic on  $x$  on the right-hand side§ that he was led to the proof of his "finite basis theorem" (reference 28).|| This is concerned with the rational points  $(x, y)$  on a curve

$$y^2 = 4x^3 - g_2 x - g_3 \quad (4.2)$$

where  $g_2, g_3$  are given integers. These rational points can be given the structure of an abelian group in a natural way (e.g. by using the Weierstrass parametrization). The

† Subsequently Siegel's work on the analytic theory of quadratic forms was to produce an interpretation of  $\chi$  as a weighted sum of the generating functions of the number of representations by all the forms in the genus of  $x_1^2 + \dots + x_n^2$ .

‡ These are the linear relations between the class numbers of imaginary quadratic number fields, or, what is essentially the same, of definite binary quadratic forms. He was to return to this topic much later (reference (175)).

§ The authority for this statement is reference (31). As a matter of fact the result Mordell was trying to prove is true. Siegel has characterized all those Diophantine equations in two variables which have an infinity of integral solutions. A basic tool in Siegel's proof is Mordell's finite basis theorem as generalized by Weil.

|| The logic of Mordell's proof betrays its origin. The first step is an unnecessary transformation of (4.2) into

$$y^2 = \text{quartic.}$$

(For the simplest direct proof using (4.2), see A. Weil, "Sur un théorème de Mordell", *Bull. Sci. Math.*, 54 (1930), pp. 182–191.)

finite basis theorem states that this group is finitely generated. The theorem is much more general than it sounds, since Poincaré had shown that the problem of finding all the rational points on any curve of genus 1 reduces to (4.2), at least if one rational point can be found. In his paper Mordell mentions this only at the very end, as an afterthought (and does not mention at all that Poincaré had conjectured the theorem).

Mordell's finite basis theorem was generalized by André Weil. The curve (4.2) is an abelian variety of dimension 1. Weil proved the corresponding theorem for abelian varieties of any dimension and with any algebraic number-field as ground field (instead of the rationals). This generalization has played a fundamental role in many areas of number theory and is usually cited as the "Mordell-Weil theorem". More recently it has been yet further generalized by Néron and Lang. Mordell himself, however, played no part in these developments. The modern approach to this circle of ideas is very different from what was available to Mordell. It is noteworthy, however, that when Birch and Swinnerton-Dyer had to find the groups of points on elliptic curves with a computer they adopted a technique close to Mordell's original one.†

The proofs of both the results mentioned in this section have the rather curious logical property of being non-effective; that is that although they demonstrate the finiteness of the object studied they give no procedure for finding it, even in theory. Thus in the first case for any given integers  $a, b, c, d, e$  in (4.1) there was no infallible procedure for finding all the integral solutions  $(x, y)$ . It thus remained a challenging problem to find all the integral solutions in given special cases, and Mordell was to return to this problem much later (references 121, 202, 208, 209, 213, 214). It is only in the last few years that Alan Baker has given an effective procedure (Contributions to the Theory of Diophantine Equations, *Phil. Trans. Roy. Soc.*, 263 (1968), 173–208). None of the proofs of the finite basis theorem available to date is effective.

### 5. Manchester University 1922–1945

In 1922 Mordell was appointed to a Readership in the University of Manchester and in 1923 he was elected to the Fielden Chair of Pure Mathematics, which he was to hold until his translation in 1945 to Cambridge.

His reputation was now established and in 1924 he was elected F.R.S. while still an American citizen.‡

Mordell's colleague in applied mathematics was Sydney Chapman. He soon left for Imperial College, being succeeded by E. A. Milne, but Mordell and Chapman remained lifelong friends. They were born only one day apart and there was a friendly rivalry as to who would survive the longest.

† "Notes on elliptic curves", *J. Reine Angew. Math.*, 212 (1963), 7–25 and 218 (1965), 79–108.

‡ He was granted a certificate of naturalization as a British subject on 10 December 1929 (Letter M288235 from the Home Office to me, dated 20 June, 1973).

Mordell tells us in his *Reminiscences* (ref. 250) how he was introduced to rock climbing by (Sir) Robert Robinson in 1925. About the same time and encouraged by the existence of a modern swimming pool in Manchester he took up diving. "I was looked upon as a great man, not for so trivial a reason as being an F.R.S., but because I used to dive off a five-metre board." In later life failing eyesight was to restrict his activities and he gave up rock climbing deliberately after someone he knew was killed, but fell walking and swimming were to remain favourite recreations until the end of his life. At the London Mathematical Society's memorial meeting, Professor R. A. Rankin enlarged on this facet of Mordell's activities as follows:

"As is well known, Mordell was an enthusiastic mountaineer and swimmer and I knew him in both capacities. His favourite footgear for mountain walking was gymshoes. I remember Mordell joining Dennis Babbage and myself on an excursion to the top of Fairfield in the Lake District in 1949. Babbage and I had a very brief dip of a few seconds in the icy water of Grisedale Tarn, which is nearly 1800 feet above sea level, but Louis spent nearly ten minutes splashing about in the water and gave no sign of finding it at all cold.

"He knew the mountains of North Wales, the Lake District and Scotland well and had climbed them in his younger days together with colleagues such as the late C. T. R. Wilson. Very much later, after he had retired from his chair in Cambridge, he came up to Glasgow to give a talk and asked me to arrange a climb. With a group of younger members of the department we went up the Cobbler (Ben Arthur), a mountain of a rather interesting shape. He climbed very slowly and managed to get up the steepest part to the saddle just below the top, but declined to go further; he had, incidentally, been to the summit many years before. However, on the way down he put us all to shame, as he was the only member of the party tough enough to bathe in one of the pools of the Cobbler burn.

"One reason for his incredible physical toughness, whether shown in climbing, swimming or dancing, may have been his great capacity for sleep. It was no use taking him a run in a car to admire the local scenery; he just fell asleep and didn't see anything, except in the brief intervals when the car stopped, when he might open his eyes to identify some landmark known to him."

This is perhaps the appropriate place to mention the other major recreation, bridge. As a non-player, I can only record the verdict of a fellow addict that Mordell was the worst good player he knew. Dr. Smithies adds the comment that "he often redeemed some very rash bidding by superb play of the hand".

An amusing example of Mordell's gift of sleep alluded to by Rankin occurred much later at the British Mathematical Colloquium at Greenwich, when he was in the chair for a lecture of Davenport's. He rapidly fell asleep in full view of the audience. In the course of his lecture, Davenport told a certain relevant anecdote. At the end, roused perhaps by the applause, Mordell was, as usual, instantaneously restored to full consciousness. He congratulated the speaker on his fine lecture, and was unable to understand the mirth of the audience when he went on to add that the one thing

which puzzled him was that the speaker had failed to tell the story of how . . . (and he went on to recount the same anecdote in much the same way).

But to return to Mordell's tenure of the Fielden Chair in Manchester. During the later 1920's his international reputation and contacts increased. His papers are more numerous but lack the significance both of what had gone before and what was to come; perhaps the most important theme was the Poisson Summation Formula and its applications (references 49, 50, 53, 57).

During the thirties Mordell built up a strong school of mathematics at Manchester and one which attracted many visitors. Amongst those who were active there for a substantial period in some capacity or other one might mention:† R. Baer, G. Billing, C. Chabauty, H. Davenport, P. Erdős, H. Heilbronn, Chao Ko, D. H. Lehmer, K. Mahler, B. Segre, J. A. Todd, P. Du Val, L. C. Young, G. Žilinskas. As Davenport rightly says "When one recalls the very small scale of mathematical activity in that age, both in England and in the world at large, as compared with the activity today, one realizes that Mordell at Manchester exercised a notable influence".

Davenport adds "Those who served under him as junior members of staff found him an admirable head of department. He was very conscious of his responsibilities, and made us very conscious of ours, but at the same time he did everything possible to encourage us in our researches, and this independently of whether their subject matter interested him personally or not". His usual morning greeting was "What's your news?", so that they almost felt obliged to produce some new mathematical result for him. For undergraduate courses he made the rule that after each lecture the lecturer had to make a record of the material covered. This was not merely useful in constructing the syllabus next year. If for some unavoidable reason the lecturer was unable to continue then someone else was detailed to give the missing lecture from the point reached. Dr. Todd recalls, however, that when he and Du Val were invited to present some of their results to the London Mathematical Society and approached him diffidently because the day clashed with their lecture commitments, he insisted at once that it was important for their careers that they should both go.

He was enlightened, too, in his relations with undergraduates. When a member of staff reported a student for some act of unruliness in class, Mordell accepted the student's word of honour that he was not in fact the culprit, and, despite protest, declared the matter closed. I cannot help noting, however, one respect in which Mordell was more in period. When, very much later, I became a head of department he would, from time to time, give me advice. On more than one occasion he urged that, if I should have occasion to reprimand a female student or member of staff, I should ensure that there was a third person present who would be able to testify as to what happened. I am sure he was not pulling my leg.

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† There was a group photograph of some of the young Manchester mathematicians of this time which LJM loved to produce in later years. The point he made was that they had all become professors, and those of us who knew it well dubbed the photograph "Professors All".

During the Nazi troubles Mordell was very active in assisting refugee mathematicians, for some of whom he managed to obtain temporary or permanent support in Manchester. Professor Mahler, whom Mordell invited for the session 1933–34 writes “ [He] helped me greatly with my English, both ordinary and mathematical, and he made me supervise an M.Sc. student on work I suggested to him. Both Mordell and his wife were most hospitable and helped me much on the personal side ”.

In his later years in Manchester Mordell’s research was very fruitful and new themes occurred which were to occupy him throughout his life.

(i) *Estimation of trigonometric and character sums*

This problem was suggested to him by Davenport to whom it was put by Littlewood. The estimates he obtained at this time (references 63, 64, 65) were superseded by the proof of the so-called Riemann Hypothesis for function fields.† Much later he was to use similar techniques on a wider class of problem (references 190, 197, 200, 202, 205, 206, 229); in particular in the proof of particular cases of his conjecture that, apart from specified exceptions, the number  $N$  of solutions of a congruence

$$f(x, y, z) \equiv 0 \pmod{p},$$

where  $f$  is a cubic polynomial, satisfies‡

$$N = p^2 + O(p).$$

(ii) *Cubic surfaces and hypersurfaces*

This is a natural extension of his earlier work on cubic curves. Some of his work discussed elsewhere could equally have been discussed under this head.

Mordell conjectured that an equation

$$f(x, y, z) = 0,$$

where  $f$  is a cubic polynomial with integral coefficients, has infinitely many integral solutions  $(x, y, z)$  if it has one. He was later to find counter-examples (reference 139) but gave a wide variety of cases for which the conjecture is verified (references 100, 102, 125, 135, 137, 146, 147, 148). He mentions the special case

$$x^3 + y^3 + z^3 = 3$$

for which it is still not known whether there are infinitely many integral solutions or not.§

† But see B. J. Birch, “ How the number of points of an elliptic curve over a fixed prime field varies ”, *J. London Math. Soc.*, 43 (1968), 57–60.

‡ This conjecture was later proved by Davenport and Lewis, *Quart. J. Math.*, 14 (1963), 154–159.

§ The only known solutions are  $x = y = z = 1$ ,  $x = y = 4$ ,  $z = -5$  and the permutations of the latter.

Another problem to which he was to recur at the end of his life is whether every integer  $n$  is the sum of four integral cubes, in particular whether a finite set of parametric solutions exists which will cover all  $n$  (references 72, 75, 246, 253).

Mordell also considered rational points on cubic surfaces especially in collaboration with Segre, a geometer with particular interest in cubic surfaces. Segre was the first to show that (with appropriate exceptions) a cubic surface with one rational point has infinitely many, but Mordell produced a simpler and less geometric proof† (106). For a special surface see reference (101). Mordell's conjecture that there is a "Hasse Principle" for cubic surfaces was disproved by Swinnerton-Dyer‡: Mordell gave further counter-examples (reference 212).

In the same circle of ideas is Mordell's "Waring's problem for cubic forms" (references 71, 224).

### (iii) *Geometry of Numbers*

In the late 1930's and early 1940's the work of Mordell, Mahler and Davenport in this subject saw the greatest development since its initiation by Minkowski. After Davenport had determined the lattice constants for the product of three ternary linear forms, Mordell§ devised a general method for reducing an  $n$ -dimensional problem to an  $(n-1)$ -dimensional one (references 94, 98, 110). In order to apply this to Davenport's result he found the lattice constants of the region

$$|f(x, y)| < 1,$$

where  $f$  is a binary cubic form. This region is non-convex and so required entirely novel techniques|| which he then systematized and applied to other problems (references 90, 93, 114, 116, 117).

Among the other papers in this area, reference 81 is especially noteworthy. The Soviet mathematician Khintchine (= Hinčin) had already proved transference theorems by more complicated methods but Mordell would repeat with delight his verdict that the "method of the additional variable" introduced here was sheer witchcraft.

## 6. Cambridge 1945–1953

In 1945 Mordell returned to Cambridge¶ as Hardy's successor in the Sadleirian Chair and was elected to a Fellowship of his old College, St. John's. For his

† For another proof of the crucial case see R. F. Whitehead, "A rational parametric solution of the cubic indeterminate equation  $z^2 = f(x, y)$ ", *J. London Math. Soc.*, 19 (1944), 68–71.

‡ "Two special cubic surfaces", *Mathematika*, 9 (1962), 54–56.

§ There was some earlier work on the geometry of numbers (references 37, 51, 59, 66, 76).

|| Davenport later found more algebraic proofs: "The reduction of a binary cubic form I, II", *J. London Math. Soc.*, 20 (1945), 14–22, 139–157.

¶ He was still a B.A., technically a junior member of the University. It was typical that he had not paid the fee which, apart from the lapse of a few years, was the only additional pre-requisite for a Cambridge M.A. His early poverty had made him careful with money and he never lost the habit. This sometimes gave an appearance of meanness, though he was often generous.

inaugural lecture he returned to the subject of his earliest researches, the equation  $y^2 = x^3 + k$  (reference 120).

In those days there was no physical focus of mathematical life in Cambridge.† Most of the staff were Fellows of Colleges and either lived there or, if like Mordell they were married, had a College room as an office. His was in the Wedding Cake‡ with a view over the Backs, and he was to retain it until the end of his life.

He soon attracted a large flock of research students of whom I was one. There was a weekly seminar and at least once a term when there was an outside speaker we were invited to meet him afterwards at the Mordells' flat in Belvoir Terrace (or, later, at his house in Bulstrode Gardens). We felt a little subcommunity and this was particularly valuable for those, like the present writer, who came from other universities and so found it difficult to make contacts in the diffuse Cambridge milieu. Most of us worked in various aspects of the Geometry of Numbers. I think he must have found it difficult to think of enough suitable topics: whenever I asked him, all he told me was that I would certainly get a Ph.D. for proving that Euler's constant is transcendental§—but perhaps he thought it would be better for me to find problems of my own.

When one of us produced a manuscript Mordell would read it over with him. If it did not make complete sense at the end of a sentence, he stopped. He would not take the plea that it would become clearer if he read on. When this happened one had to produce another draft. Sometimes he would stop at a semicolon: but that I regarded as unfair. The process continued until he could read right through. I learned a lot about the art of exposition in this way, partly from explicit comments (he was particularly fond of Pólya's advice "If you have two things to say, say them one at a time") but much more from observing his difficulties. I personally am very grateful to him for this discipline, though it is doubtful whether it always had the desired effect. A legend has it that one pupil, tired of five successive approximations to the ideal, produced his original draft as his sixth. Splendid, said the Master, having by now absorbed the argument, "Why did you not write it like this the first time?"

He gave courses for Part III of the Mathematical Tripos and these were also attended by Research Students. The topics were usually Geometry of Numbers, Algebraic Numbers or Diophantine Equations. These courses were popular and well attended, partly, I fear, because they had the reputation of being less demanding than some of the others and he set easy questions for the examination. He lectured slowly and carefully and wrote everything on the board in his usual illegible hand.

† There was also, of course, no clerical assistance of any kind. When I became a Lecturer in 1950 the then Secretary of the Faculty Board wrote me a long letter in long-hand about the details of my appointment but in that year a single typist was appointed for the whole of the Faculty.

‡ As the 19th century transpontine court of St. John's is known to all except Johnians.

§ It would be worth an F.R.S.! When I taxed him with this later he denied ever having made the suggestion. Dr. Smithies tells me that Hardy offered to resign his Chair in favour of anyone who could even prove Euler's constant irrational.

(He is the only person I have ever known who wrote "r" and "s" so that they were completely indistinguishable: and they were a favourite choice of variable.) One can get an impression of the style and general feel of his lectures on Diophantine Equations from the book which he wrote near the end of his life.

As was then customary for a Cambridge Professor, he took no part in general mathematical teaching. He was *ex officio* on the Faculty Board but he does not seem to have played an active role in Faculty affairs except in connection with appointments, when he pushed very hard for the appointment of people whose work he thought highly of.

So far as Mordell's own mathematical activity is concerned, his tenure of the Sadleirian Chair saw the end of the period of great activity in the Geometry of Numbers and the beginning of the final period when he published papers on a wide variety of problems. It will be more convenient to deal with these all together in the next section.

### 7. *Grand Old Man* 1953–1972

On his retirement from the Sadleirian Chair Mordell retained his house in Cambridge but spent long periods as Visiting Professor on the North American continent. For 1953 he had originally accepted an invitation to the United States but was refused a visa† and went to Canada instead. An attraction of Canada was that his son Donald was Professor at McGill. On his return to Cambridge LJM would dilate with pleasure on the discovery that over there "Professor Mordell" was naturally taken as referring to Donald rather than to himself. As a Visiting Professor he lectured on number theory, of course, but also got great enjoyment from giving calculus courses and the like. His interest in transatlantic undergraduate teaching is reflected in his paper on the Putnam Competitions (reference 198).

He was greatly in demand as a visiting speaker and kept a list, arranged geographically, of the institutions at which he had spoken. Ultimately it had about 190 entries. There was some sort of competition with Paul Erdős about this, and Mordell claimed to be still in the lead, though resigned to losing in the end. He took an unfeigned interest in the life and mathematical activities of all the places he visited. To quote Davenport again "There can be few who have done so much in recent years to spread a love of mathematics in the world at large by their personal influence, and it is safe to say that Mordell's zest for his subject, and his love of learning in general, will have served as an inspiration to many young people, especially those in places that are remote from important scientific centres".

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† No reason was given despite appeals from his family in Philadelphia, from members of "the class of 1906" and from the British and American mathematical and scientific establishment. Subsequent visa applications were rejected on the incontrovertible ground that earlier applications had been; and it was only in 1959 that he was allowed to re-enter the land of his birth. It subsequently transpired that one of the many tenants of his Manchester house (which he had kept and let through agents) was believed by the competent American authorities to be a Communist.

All his life Mordell seems to have had a great unsatisfied desire for recognition. In his old age he patently enjoyed the role of "grand old man" and characteristically did his best to do it justice. He took considerable pains in the preparation and delivery of his lectures. Although occasionally he used French or German† (his accent was execrable), he normally spoke in English, making a point of talking especially slowly and distinctly for a foreign or international audience.

With his wide circle of mathematical friends and acquaintances all over the globe, Mordell enjoyed conferences large and small. He was particularly in his element in the big four-yearly International Congresses. At each of these he organized a dinner. At first they were comparatively modest, but that at Nice was particularly memorable when he was host to about a hundred mathematicians and wives.

As he grew older, Mordell spent more of his time in Cambridge, though he still frequently made lecture trips abroad. (He was on the point of leaving for an extensive trip to the U.S. when he died.) It was remarked, however, that he always arranged things so as to be back for the St. John's May Ball—he was an enthusiastic if not noticeably skilful dancer.

When in Cambridge he worked in his room in the Wedding Cake most mornings and often for the rest of the day. Any mathematician who dropped in was sure of a warm welcome. He took a keen interest in the work of the number theorists in the Department and was a most assiduous attender of the number-theory seminar. He was a regular speaker and when I took over after Davenport's death I knew I could depend on him to step into the breach if there was an unexpected gap. He would not infrequently give an extended commentary after other peoples' seminar talks and set their work in relation to his own and that of others. If he felt that there were too many talks in areas such as those bordering on algebraic geometry in which he was not interested, he would complain to me that it was not number-theory. He came all the same, but slept through it. He also liked to keep in touch, as far as he could, with mathematical undergraduates, particularly those of St. John's.

In his later years Mordell worked in a wide variety of problems in the general area of the theory of numbers; how wide can be judged by glancing through the list of his publications. Indeed he published almost as many papers after retirement as before it. Some of his friends thought that some of the papers were unworthy of him and tried to urge moderation, but in vain. The truth is that he was a much worse judge of his own work than of that of others. When the editors of one journal tried to soften the rejection of a MS with the comment that they did not think it would enhance his reputation he replied that he was the best judge of that, and insisted on publication. But right to the end he continued to display an extraordinary virtuosity in the manipulation of comparatively elementary techniques and to produce papers of which anyone might be proud. He was particularly pleased when he could obtain

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† Presumably the Italian text of reference 153 was translated for him.

or improve results which had first been derived by more sophisticated or complicated means (e.g. references 133, 237, 242).

His work on

$$y^2 = x^3 + k$$

and its generalisations, on cubic forms and on trigonometric sums has already been mentioned in relation to his earlier work on those topics. In reference 170 he uses the theory of elliptic curves to show that if  $A, B, C, D$  are any four points in the plane then there are points  $A', B', C', D'$  arbitrarily close to  $A, B, C, D$  such that all the six lengths  $A'B', A'C', A'D', B'C', B'D', C'D'$  are rational. Other papers that might be mentioned are 158 on the simultaneous solution of two quadratics<sup>†</sup> and paper 204 on the least integer represented (or, more difficult problem, the least integer not represented) by a polynomial<sup>‡</sup> modulo  $p$ .

As Davenport remarked "For a mathematician of his standing he has written an unusually large number of expository articles, and of book reviews, which are in effect essays on various mathematical subjects". It is the more remarkable that he wrote his first and only full-scale book when he was already 80. In it he gives a masterly survey of the field of diophantine equations from a lowbrow viewpoint and drawing on his unequalled knowledge of the byways of that most unsystematized area.

It is now fashionable to classify mathematicians into "problem solvers" and "system builders". Mordell was an archetypal problem solver. Even when the system lay at his feet asking to be built (as in his early work on modular forms or in his proof of "Mordell's finite basis theorem") he passed on to other problems and left the building for other hands. His work has left an indelible impress on the subject he loved so well.

Mrs. Mordell died in 1971. For some years previously her memory had completely disappeared and it was touching to see them together. Mordell's last years were also saddened by the premature death of Harold Davenport, with whom he was very close, and whom he looked on as his spiritual heir. While Davenport was still alive and after I had succeeded LJM at one remove in the Sadleirian Chair, he would boast that he was unique in having two pupils simultaneously in Cambridge chairs, though strictly speaking Davenport was not his pupil, but Littlewood's.

Mordell retained his vigour and zest to the end. In September 1971 he attended the International Conference on Number Theory in Moscow, gave a paper and spoke for the guests at the final meeting. After the conference, participants were offered a choice of a short visit to Soviet Central Asia or to Leningrad. Mordell decided that he would do both, so he wangled an invitation from Linnik to speak in Leningrad after the Asian tour. I had chosen the less strenuous option and was in Leningrad when he arrived by the overnight train from Moscow, having flown the

<sup>†</sup> See also Swinnerton-Dyer, *Acta Arithmetica*, 9 (1964), 261–270.

<sup>‡</sup> This problem was subsequently taken up by Bombieri and Davenport, *Amer. J. Math.*, 88 (1968), 61–70.

3,000 odd miles from Tashkent the previous day. He seemed none the worse and duly took part in a tour of Pavlovsk, though by the middle of the afternoon he thought he ought to take a rest.

In December 1971, at the end of term, he gave a party to the mathematics Scholars of St. John's and a few other guests to celebrate the 65th anniversary of his success in the Entrance Scholarship Examination and spoke about his early experiences.

The end came suddenly. On Monday, 6 March, 1972, he came into the Department and had a mathematical discussion. In the evening he dined, as usual, in St. John's. Feeling unwell, he was taken home but insisted on being left. In the morning he rang for help but had already collapsed when it arrived. He lost consciousness in the afternoon and died on Sunday, 12 March. At his request the cremation ceremony was non-religious.

#### 8. Honours, etc.

Mordell was elected to the Royal Society in 1924 and awarded the Sylvester Medal of the Royal Society (1949). He was President of the London Mathematical Society (1943–45) and received both its De Morgan Medal (1941) and Senior Berwick Prize (1946). He was a Foreign Member of the Academies of Oslo, Uppsala and Bologna, and received honorary doctorates from the Universities of Glasgow, Mount Allison and Waterloo. He was an editor of both *Acta Arithmetica* and the *Journal of Number Theory* from their foundation. Volume 9 of *Acta Arithmetica* (1964) and Part I of Volume 43 of the *Journal of the London Math. Soc.* (1968) were dedicated respectively to his 75th and 80th birthdays.

Mordell took a considerable interest in the affairs of this Society and, in addition to his period as President mentioned above, he served on the Council for many years. During his life-time he endowed a Mordell Publications Fund to further the publications of the Society and he augmented this fund in his will.

#### 9. Biographical sources

Mordell has given a characteristic account of his early career in a lecture which he gave on many occasions in differing forms and which is published as *Reminiscences of an Octogenarian Mathematician* (item 250). *The Reflections of a Mathematician* (item 167) also contains some autobiographical material. Davenport wrote an excellent biographical memoir for the Mordell volume of *Acta Arithmetica* (vol. 9 (1964), 9–12).

There are not totally concordant articles on Phineas Mordell (LJM's father) in *The Universal Jewish Encyclopaedia* (KTAV, New York, 1969) and in *Encyclopaedia Judaica* (Jerusalem, 1971) and one on Albert Mordell (LJM's elder brother) in the former.

I have used the article in the *Philadelphia Press* for 10 January, 1907, when the St. John's Scholarship was announced, of which there were several clippings amongst Mordell's papers, and also the accounts in *The Times* for 16 June and 23 June, 1909, of the ceremonies attending the final Tripos "order of merit". Mordell completed his Royal Society "Personal Record" (in which Fellows are urged to record biographical details for the edification of posterity) only in the summer of 1970, so it is especially complete. I had access to his personal papers which, apart from some family items, are deposited with the library of St. John's College.

I should also like to acknowledge the help of colleagues and friends, in particular of Professor K. Mahler, Dr. J. A. Todd, Dr. F. Smithies and Mrs. Davenport, and of Mordell's two children, Mrs. Kathleen Smith and Mr. Donald Mordell.

This memoir is adapted by permission from that prepared for the Royal Society. The latter is accompanied by a portrait which is believed to date from the election in 1924. There is a picture with the *Philadelphia Press* article mentioned above, and the *Illustrated London News* for 19 June, 1909, shows LJM together with the first two Wranglers (p. 882). There are more recent portraits in Volume 9 of *Acta Arithmetica* dedicated to him, in the issue of the *Journal of the London Mathematical Society* in honour of his 80th birthday (volume 43 (1968), Part I) and in the *Reflections of a Mathematician* (reference 167). There is a particularly good one on p. 38 of the *Strand Magazine* for January 1950 (in an article about Oxbridge dons), which shows him in the familiar context of his College room.

*Note added in proof.* It is hoped that a similar photograph will appear in volume 23 (1973) of *Acta Arithmetica* with a list of Mordell's recent publications.

#### *Publications of L. J. Mordell*

1. "Note on irregular determinants", *Messenger of Math.*, 42 (1912), 124.
2. "The Diophantine equation  $y^2 - k = x^3$ ", *Proc. London Math. Soc.*, 13 (1913), 60–80.
3. "Indeterminate equations of the third and fourth degrees", *Quart. J. of Pure and Applied Math.*, 45 (1914), 170–186.
4. "The inversion of the integral  $u = \int \frac{y \, dx - x \, dy}{\sqrt{(a, b, c, d, e)(x, y)^4}}$ ", *Messenger of Math.*, 44 (1914), 138–141.
5. "Theta functions in the theory of the modular functions and groups of linear substitutions arising therefrom", *Quart. J. of Pure and Applied Math.*, 46 (1915), 97–124.
6. "Note on class relation formulae", *Messenger of Math.*, 45 (1915), 76–80.
7. "On class relation formulae", *Messenger of Math.*, 46 (1916), 113–135.
8. "On the class number for definite ternary quadratics", *Messenger of Math.*, 47 (1917), 65–78.
9. "The class number for definite binary quadratics", *Messenger of Math.*, 47 (1917), 138–142.
10. "On the solutions of  $x^2 + y^2 + z^2 + t^2 = 4m_1 m_2$ ", *Messenger of Math.*, 47 (1917), 142–144.
11. "The theory of numbers", *Science Progress*, 45 (1917), 127–131.
12. "An arithmetical proof of a class relation formula", *Messenger of Math.*, 45 (1916), 177–180.
13. "On the representations of numbers as a sum of 2r squares", *Quart. J. of Pure and Applied Math.*, 48 (1917), 93–104.
14. "On Mr. Ramanujan's empirical expansion of modular functions", *Proc. Camb. Phil. Soc.*, 19 (1917), 117–124.

15. "A statement by Fermat", *Proc. London Math. Soc.*, 18 (1919), v.
16. "On the representation of a number as a sum of an odd number of squares", *Trans. Camb. Phil. Soc.*, 22 (1919), 361–372.
17. "On the representation of algebraic numbers as sums of four squares", *Proc. Camb. Phil. Soc.*, 20 (1921), 250–256.
18. "The value of the definite integral  $\int_{-\infty}^{\infty} \frac{e^{at^2+bt}}{e^{ct}+d} dt$ ", *Quart. J. of Pure and Applied Math.*, 48 (1920), 329–342.
19. "On the generating function of the series  $\sum F(n) q^n$ , where  $F(n)$  is the number of uneven classes of binary quadratics of determinant  $-n$ ", *Messenger of Math.*, 50 (1920), 113–128.
20. "On a simple summation of the series  $\sum_{s=0}^{n-1} e^{2s^2\pi i/n}$ ", *Messenger of Math.*, 48 (1918), 54–56.
21. "On some series whose  $n$ -th term involves the number of classes of binary quadratics of determinant  $-n$ ", *Messenger of Math.*, 49 (1919), pp. 65–72.
22. "Mathematicians and their work", *Math. Gazette*, 10 (1920), 321–323.
23. "Highways and byways in the theory of numbers", *Science Progress*, 16 (1921), 3–8.
24. "On the reciprocity formula for the Gauss's sums in the quadratic field", *Proc. London Math. Soc.*, (2) 20 (1921), 289–296.
25. "Note on certain modular relations considered by Messrs. Ramanujan, Darling and Rogers", *Proc. London Math. Soc.*, (2) 20 (1921), 408–416.
26. "Three lectures on Fermat's Last Theorem", Cambridge University Press, 1921. French trans. by A. Sallin, Presses Universitaires de France, Paris, 1929.
27. "Note on the integer solutions of the equation  $Ey^2 = Ax^3+Bx^2+Cx+D$ ", *Messenger of Math.*, 51 (1922), 169–171.
28. "On the rational solutions of the indeterminate equations of the 3rd and 4th degrees", *Proc. Camb. Phil. Soc.*, 21 (1922), 179–192.
29. "On the integer solutions of the equation  $Ey^2 = ax^3+bx^2+cx+d$ ", *Proc. London Math. Soc.*, (2) 21 (1923), 415–419.
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32. "On the number of solutions in positive integers of the equation  $yz+zx+xy = n$ ", *American J. of Math.*, 45 (1923), 1–4.
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34. "The approximate functional equation for the theta-function", *J. London Math. Soc.*, 1 (1926), 68–72.
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36. "On power series with the circle of convergence as a line of essential singularities", *J. London Math. Soc.*, 2 (1927), 146–148.
37. "Minkowski's theorem on the product of two linear forms", *J. London Math. Soc.*, 3 (1928), 19–22.
38. "The present state of some problems in the theory of numbers", *Nature*, 121 (1928), 138–140.
39. "A summability convergence theorem", *J. London Math. Soc.*, 3 (1928), 87–89.
40. "The magnitude of the derivative of a function", *J. London Math. Soc.*, 3 (1928), 119–121.
41. "The convergence of series summable  $(C, r)$ ", *J. London Math. Soc.*, 3 (1928), 170–172.
42. "Some applications of Fourier series in the analytic theory of numbers", *Proc. Camb. Phil. Soc.*, 24 (1928), 585–596.
43. "Review of Berwick's *Integral bases*", *Math. Gazette*, 14 (1928–29) 244–245.

44. "On a circular region enclosing a root of an algebraic equation", *J. London Math. Soc.*, 4 (1929), 202–204.

45. "Kronecker's fundamental limit formula in the theory of numbers and elliptic functions and similar theorems", *Proc. Royal Soc. A* 125 (1929), 262–276.

46. "Poisson's summation formula and the Riemann zeta-function", *J. London Math. Soc.*, 4 (1929), 285–291.

47. "On an arithmetical formula", *J. London Math. Soc.*, 4 (1929), 291–296.

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49. "Poisson's summation formula in several variables and some applications to the theory of numbers", *Proc. Camb. Phil. Soc.*, 25 (1929), 412–420.

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52. "Note on Kapteyn's and Bateman's integrals involving Bessel functions", *J. London Math. Soc.*, 5 (1930), 203–208.

53. "The zeta functions arising from quadratic forms, and their functional equations", *Quart. J. Math.*, 1 (1930), 77–101.

54. "Review of Dickson's *Studies in the theory of numbers*", *Math. Gazette*, 15 (1930–31), 361–362.

55. "Note on some linear diophantine inequalities", *Proc. Camb. Phil. Soc.*, 28 (1930), 489–490.

56. "On a new Waring's problem with squares of linear forms", *Quart. J. Math.*, 1 (1930), 276–288.

57. "Hecke's modular functions and other analytic functions in the theory of numbers", *Proc. London Math. Soc.*, (2) 32 (1930), 501–556.

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62. "Note on the diophantine equation  $ax^2+by^2+cz^2+dt^2=0$ ", *Bull. American Math. Soc.*, 38 (1932), 277–282.

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### Book

*Diophantine Equations* (Academic Press, 1969), xii+312 pp.

The pamphlets "Three lectures on Fermat's last theorem" and "A chapter in the theory of numbers" (items 26 and 120) are being reprinted by the VEB Deutscher Verlag der Wissenschaften with an introduction by O. Neumann under the title "Two papers on number theory".

A Russian translation of "Reflections of a mathematician" (item 167) has appeared under the title: *Размышления Математика* (Издательство Знание, Москва, 1971).