ROLF NEVANLINNA 1895–1980

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ROLF NEVANLINNA

1. *Background*

Rolf Nevanlinna was born on the 22nd October 1895; he died on 28th May 1980. He came from a Swedish-speaking Finnish family containing soldiers, scientists and engineers. The family name of Neovius was changed to Nevanlinna by his father in 1906.

Rolf's paternal grandfather, Edward Engelbrekt, studied at the cadet school in Hamina (Finland) and the engineering academy in St Petersburg. He taught mathematics and topography at the cadet school and rose to the rank of major-general. Edward's brother, Frithiof, also studied at St Petersburg and taught mathematics and topography in various military schools in Russia. He returned at the age of 40 to head the cadet school at Hamina and had the reputation of being something of a martinet. On one occasion Mannerheim (later Field Marshal Baron von Mannerheim) went A.W.O.L. (leaving a mattress as his substitute). He had to leave the academy and continued his career in St Petersburg.

Rolf's maternal grandfather, Herman Romberg, was an astronomer, who catalogued stars in the observatory in Pulkova.

Rolf's father, Otto Wilhelm, was born in 1867. He went to the cadet school and then studied mathematics, physics and astronomy at Helsinki University. His Ph.D. thesis on spectral lines in oxygen and nitrogen, and some of his research, suggested the existence of the rare gases for which Ramsay was awarded the Nobel Prize in 1904. While studying with Herman Romberg at Pulkova, Otto Wilhelm met Margarete Romberg; they married in 1892 and settled in Joensuu, where Otto was a teacher. The children of the marriage were Frithiof (1894), Rolf (1895), Anna (1896) and Erik (1901).

2. *Education*

When Rolf went to school in 1902 he moved straight into the second class, since he could already read and write. He seems to have been rather bored and played with trains under the desk, consequently getting the low mark of 6 out of 10 for 'care and attention'. So he refused to go on and left school for one-and-a-half years until the family moved to Helsinki. Here things were better. He liked his cousin, Väinö, who was five years older, and together the boys were rather mischievous. On one occasion they dropped paper bags filled with water from the balcony of the flat. They also built paper warships and bombed them, inspired by the Russo–Japanese war.

At the school there were some outstanding and sometimes eccentric teachers, such as the historian, Melander, who wrote about hunting by the ancient Finns, and the Finnish teacher, Koskimies, who wrote poetry. Rolf also learned German and French and laid the basis for his superb gift for languages, although his fluency developed only on his trips abroad later. Perhaps the best teacher was his own father, who taught him mathematics and physics in the final years at the school.

Margarete was an excellent pianist and Frithiof and Rolf would lie under the piano and listen to her playing. At 13 they went to orchestra school and became

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accomplished musicians—Frithiof on the 'cello and Rolf on the violin. Through free tickets from the orchestra school they got to know and love the music of the great composers, Bach, Beethoven, Brahms, Schubert, Schumann, Chopin and Liszt, as well as the early symphonies of Sibelius (1865–1957), conducted by the composer. Rolf first met Sibelius' music in 1907, when he heard his Third Symphony. Although later he met Hilbert, Einstein, Th. Mann and other famous people, Rolf said that none had had such a strong effect on him as Sibelius. The boys played trios with their mother and their love of music—in particular of chamber music—lasted all their lives.

Rolf had a wonderful feeling for music: he belonged to a quartet in Helsinki and became chairman of the Sibelius Academy. The distinguished musicologist, Erik Tawaststjerna, told me how Rolf could remember whole scores by heart, discuss the music in detail—"... the drums come in here" he would say. Tawaststjerna was very glad to discuss his authoritative work on Sibelius with Rolf and paid tribute to the fine understanding he displayed. Kristiina, Rolf's daughter from his second marriage, is a pianist and music teacher; so the tradition continues in the next generation.

Rolf's sister, Anna, became a drawing teacher, and one of her sons, Heikki Haahti, became Professor of Mathematics at Oulu. The youngest son, Erik, was engaged on decoding in the Second World War. Veikko Nevanlinna, Frithiof's son, and Olavi Nevanlinna, Frithiof's grandson, also became professors of mathematics at Jyväskylä and Helsinki University of Technology, respectively.

Rolf did well at secondary school and matriculated near the top of his class. His chief interests were classics and mathematics, in that order. Between school and university he read Lindelöf's *Introduction to higher analysis* and did all the problems. In 1913 he went to Helsinki University, where Lindelöf (who was a cousin of Rolf's father) was the outstanding scientist. His lectures (in Swedish) went to the heart of things and his audience listened with rapt attention. Lindelöf was a very warm person with an attractive impulsiveness and he helped Rolf with advice and criticism. His highest praise for a piece of work was "this defends its place". Lindeberg and Johansson were other professors and F. Iversen was an assistant.

In 1915 Rolf felt he ought to go to Germany to get military training in a battalion established there for Finnish freedom fighters. His parents would have accepted this with great reluctance, feeling that his talents lay in other directions, and he was finally dissuaded. (If he had gone, the Russians might well have taken reprisals against his parents.) In 1918 Mannerheim defeated the Red troops and Finland became independent. In this civil war, Rolf was saved from conscription by his weight, which at 50 kilos was too low. He played a small part as a clerk and refused to take part in the execution of the Red guards, some of whom he rescued.

In 1918/19 he wrote his thesis. He put a great deal of effort into presenting his work in an optimal way, and then and afterwards he wrote and re-wrote his manuscripts. Many years later Carathéodory expressed his own feeling when he said, "Leading ideas came in minutes, but the development and foundation took decades".

3. *Family*

In 1911 Rolf and Frithiof travelled to Joensuu to recall their childhood and stayed for a few days with their aunt, Elise, in Wiborg. Her daughter Mary made a very deep impression on Rolf. They became engaged six years later and were married on 4 June 1919, the day Rolf got his doctorate. They spent the summer of that year in

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Maarianhamina, on Aaland, an island in the Baltic. Rolf and Mary had four children: Kai, who was born in 1920, became a doctor and died in 1950 from the after effects of wounds sustained in the Second World War; Harri, born in 1922, who also studied medicine and became head of the blood transfusion service of the Finnish Red Cross, which until recently supplied all the world's Interferon; Aarne, born in 1925, who became an architect; Sylvi, 1930–1981, who married Robert Austerlitz, Professor of Fenno-ugrian languages at Columbia University. Sylvi was also a linguist. She died recently of cancer, just as her father had done.

Rolf and Mary used to speak Swedish to each other and with the older boys, Kai and Harri. With the younger children, Aarne and Sylvi, and the grandchildren, they spoke Finnish. When his children asked him what he was doing all by himself for so long, Rolf used to say, "I am counting". Once Kai was quiet for a bit and then said "You must have got a very long way by now".

Rolf was always an enthusiastic teacher. When Harri was studying genetics, Rolf taught him mathematics. They went through Lindelöf's *Introduction to analysis* together. When he heard that his barber's son needed coaching in mathematics, he offered to teach him too.

Rolf was very proud of Harri's success and used to like to tell how he was once asked by a medical man whether he was related to the famous Dr Nevanlinna.

Rolf and Mary got a summer house in Lohja in 1937 and Rolf worked in a study next to the sauna. There were many mathematicians, such as Ahlfors, Cramer, Fueter, the Pólyas, Speiser and Strebel, who visited him there. Strebel said once, "I am never going to go away again". The last part of the journey to Lohja had to be made by horse and cart. Once when Fueter made the journey it rained and Rolf walked by his side all the way, holding an umbrella.

In 1945, while Rolf was helping to organise a chamber music society, he met Sinikka Kallio-Visapää, an authoress and distinguished translator, particularly of Thomas Mann. Rolf's first marriage was dissolved and he and Sinikka were married in 1958 by the Finnish Ambassador in Paris. A second daughter, Kristiina, was born to Sinikka in 1946 to join Rolf's four children by his first marriage.

4. *Career*

When Rolf graduated in 1919 there were no jobs open in universities; so he became a school teacher, while Frithiof had joined the insurance company, Salama, as a mathematician. Rolf joined Frithiof there while continuing to teach 18 lessons a week at the school. In 1920 Landau invited Rolf to join him in Göttingen and he went there in 1924.

During these years Rolf started to develop the theory which bears his name. For the potential theoretic approach in particular he collaborated with Frithiof. Rolf became a Docent at the University of Helsinki in 1922 and Professor in 1926, and it was only then that he stopped teaching in school. Evenings and Sundays were excellent times for research, and Nevanlinna theory was greatly influenced by Rolf's discussions with Frithiof, which continued all their lives. They would walk up and down each side of a big square, talking mathematics.

In 1922 Rolf attended the 5th Scandinavian Mathematical Congress at Helsinki and there he met Mittag-Leffler, who had for a short time been Professor at Helsinki, as well as the brothers Bohr, T. Carleman and H. Cramer. He strongly supported international congresses because of the opportunities they give to younger

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mathematicians to appreciate the total mathematical situation. Rolf would point out how many fruitful personal contacts are made on these occasions and how international understanding is promoted and hostilities diminish when people meet face to face in this way.

Rolf always enjoyed lecturing and teaching on a personal basis. He prepared his lectures in outline and felt that there should be some room for improvisation around a fixed theme. Sometimes at the blackboard a connection with other areas would occur to him. Later on he would write up his lectures and sometimes turn them into books.

In 1924 Rolf visited Göttingen. Here he met Hilbert, Landau, Courant and Noether. After a lecture by Rolf there Hilbert said, "You have opened a hole in the wall of mathematics; soon other researchers will come and close it". However the wind of change continues to blow through that hole. Later Rolf met Alexandroff and Urysohn and also Carathéodory in Munich.

Landau at that time divided his days into 6 hours work alternating with 6 hours rest. When work started again at midnight, Landau used to call his assistant to help him work. When Rolf later told this story in Zürich he was immediately nicknamed Landau.

Rolf's French contacts started in 1926 when Lindelöf arranged for him to go to Paris, where he met Hadamard and Montel. He also visited Bloch in the mental hospital but this visit ended when guards took Bloch away. His first visit to Zürich took place in 1928, where he was accompanied by Lars Ahlfors, to whom Rolf suggested the Denjoy conjecture. Ahlfors' proof of this led to one of the first two Fields medals in 1936. Rolf refused the offer to succeed Weyl at Zürich. In 1936–37 Rolf was again in Göttingen as visiting Professor. Here he had an assistant for the first time, namely H. Wittich. He also met Herglotz, whom he described as a "most interesting person".

During the Second World War Rolf developed a method for reviewing ballistic tables. Kai went to the front and Harri volunteered for the army and went to the N.C.O. school.

In 1941 Rolf became Rector of Helsinki University and discussed with Mannerheim how soldiers could continue to study during quiet times at the front, and what could be done after the war. In 1944 a bomb hit the building he was working in, and he also found an unexploded bomb. Suddenly he was a hero.

Returning from Stockholm once with Vice-Rector Erik Lönnroth they were attacked by two men who tried to steal their luggage, which was being pushed by a sergeant. Together they fought off the robbers.

In 1945 Rolf was asked to resign his post as Rector, no doubt because of his pro-German sympathies, but the staff loved him and one of the porters thanked him "for not being a bureaucrat".

In October 1946 Rolf went again to Zürich. There he met many mathematicians and also the physicist, Pauli, of whom he said afterwards "Pauli was one of the very few men in whose company I immediately felt the presence of genius".

In 1948 he became one of the 12 members of the newly established Finnish Academy, encouraged by the president, A. I. Virtanen, the 1945 Nobel Laureate in Chemistry, for at first he had refused. (At that time academicians received a salary, but that is no longer the case.) Rolf continued to be Guest Professor at Zürich for the next 15 years. Among his former students are O. Lehto, L. Sario, H. Keller, A. Steiner, K. Strebel and G. Elfving.

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After the war Rolf's interest began to turn to Calculus of Variations and applications to physics. He was also concerned in getting the first computer to Finland, and to establish Computer Science as a university subject. I first met him at the Harvard Congress in 1950 and again in 1953 in Ann Arbor, and we met frequently after that. He always seemed interested in what was going on and in the many ramifications of his theory. We would play sonatas together and he and my wife played the Bach double concerto. Once at Ann Arbor Rolf met some Finns, one of whom asked him what he was doing. "I am at the University of Michigan", said Rolf. "How old are you?" Fifty-seven!" "Won't you get your degree soon?". This illustrates Rolf's modesty very well. In 1953 I wrote a paper in which I rediscovered some of Rolf's results. He was the referee, but never said a word about it. Rolf's priority was pointed out to me many years later by A. A. Gol'dberg.

From 1959 to 1962 Rolf was President of the I.M.U. At that time the Secretary of the Union was Rolf's close friend, K. Chandrasekharan, who in 1971 became President himself.

Nevanlinna was fairly conservative in his views. He did not think that sets formed the best introduction to mathematics at school. He did not like to see children spoilt, feeling that they would find life hard later. "Immature and unrealistic radicalism will change in the hard school of life and reality" he used to say. But he took an optimistic view in general of the future of Finland and the world.

Rolf never needed much sleep. In the late 1930's he used to rise at 4, join his family at 10 till lunch time, then work again till 7, and spend the evening with his family. In later life I remember him bright and full of zest at 11 or 12 p.m. when all I wanted to do was go to sleep. The Helsinki congress took an enormous amount out of Olli Lehto, who organised it with his usual care. Afterwards Rolf said "I must go to the Joensuu conference because Olli is so tired".

Finally when he was dying he asked his doctor, "Can I still work?" When he heard that he could not he refused to eat any more and took only fluids. He was calm and peaceful at the end. Afterwards his doctor said "I have had hundreds of patients, but he is the first who has taught me something".

Rolf is not forgotten by his descendants. One boy told his mathematics teacher "My great grandfather is Rolf Nevanlinna". When the teacher said, "And who is he?" the boy replied, "if you don't know that, you don't know much mathematics". Christian Burr, another small mathematician, settled a long arithmetical problem in his head and was disappointed at getting less than full marks. His teacher said, "You should not have done it in your head". "What do I use my head for then", said Christian.

5. *Honours*

Considering the tremendous importance of Nevanlinna theory, recognition came relatively slowly to Rolf, but his last 30 years were very full of honours. He had honorary doctorates from Heidelberg (1936), Bucharest (1942), Giessen (1952), Berlin (1955), Jyväskylä (1969), Glasgow (1969), Uppsala (1974) and Istanbul (1976).

He received the International Wihuri Prize for Scientists and Artists in 1958 and the Henrik Steffens Prize for Nordic culture in 1967.

He was elected to Honorary Membership of the London Mathematical Society in 1959. He was also an honorary member of the following institutions: Finnish Academy of Science and Letters (1975); Deutsche Akademie (1938); Finnish Mathematical Society (1955); Swiss Mathematical Society (1962); Society of

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Actuaries of Finland (1965); Teachers of Mathematics and Physics (1965); Göttingen Academy (1967); Royal Swedish Academy (1967) (Foreign Member); Danish Academy (1967) (Foreign Member); Leopoldina (1967); Hungarian Academy (1970); Correspondent of the Institut de France (1967). He was an Honorary Fellow of Göttingen University (1937) and Honorary Professor of Zürich University (1948). He was an Honorary member of the Sibelius Academy (1978) and Honorary President of the Finnish Cultural Foundation.

He had the Grand Cross of the Order of the White Rose of Finland and he was Commander, First Class, of the Order of the Lion of Finland. He had the Cross of Liberty, Second Class without Swords, for merit during the war 1939–40.

6. *Mathematical work*

Nevanlinna's thesis [1919] and its sequel [1922*b*] are concerned with regular functions $f(z)$ satisfying $|f(z)| < 1$ in $|z| < 1$, which assume, at pre-assigned points z_1 to z_n, pre-assigned values w_1 to w_n. The problem is to find whether such functions exist and if so what is the range of variation of $f(z_{n+1})$, where z_{n+1} is a further point. The problem is solved completely by means of successive algorithms, and a number of previously known special cases arise as consequences. The theory was developed independently by Pick [1915] a little less completely.

In [1921] the author obtains the sharp coefficient estimates for starlike univalent functions. However Rolf Nevanlinna's immense international reputation is based on the value distribution theory, which bears his name. A most interesting account of the birth of this theory and its background has been given by Lehto [1982] and I am greatly indebted to that account for what follows.

Picard [1880] had proved his celebrated theorem that an entire function assumes every value with at most one exception. The question naturally arose whether anything further could be said about the roots of the equation $f(z) = a$ for different values of a. Let $n(r, a)$ be the number of these roots in $|z| < r$. The exponent of convergence, or order, $\rho(a)$ of the roots is defined by

$$(1) \qquad \rho(a) = \overline{\lim_{r\to\infty}} \frac{\log n(r, a)}{\log r}.$$

Hadamard [1893] showed that if

$$M(r) = M(r, f) = \max_{|z| = r} |f(z)|$$

and the order of f is defined as

$$(2) \qquad \rho = \limsup_{r\to\infty} \frac{\log\log M(r)}{\log r},$$

then $\rho(a) \leqslant \rho$ for every a. Borel [1897] notably extended Picard's theorem by proving that $\rho(a) = \rho$ for all a with at most one exception and that such an exceptional a can occur only if ρ is a positive integer or $+\infty$. For infinite order the corresponding result indicated by Borel was proved by Blumenthal [1910].

The theory obtained so far was based on the product decomposition of entire

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functions of finite order due to Hadamard [1893]. It had a number of disadvantages. The theory lacked precision and did not work well for functions of infinite order or meromorphic functions. For the latter the maximum modulus $M(r, f)$ could not be used satisfactorily as an indicator of growth since $M(r, f)$ is infinite whenever $f(z)$ has a pole on $|z| = r$.

The situation was revolutionised by Nevanlinna in the 1920's. He used a result of Jensen [1899], namely

$$(1) \qquad \log|f(0)| = \frac{1}{2\pi}\int_0^{2\pi} \log|f(re^{i\theta})|d\theta + \sum \log\frac{r}{|b_\nu|} - \sum \log\frac{r}{|a_\mu|},$$

where a_μ are the zeros and b_ν the poles of the meromorphic function $f(z)$ in $|z| \leqslant r$. Following Valiron [1913] he wrote

$$(2) \qquad \sum \log\frac{r}{|a_\mu|} = \int_0^r \log\left(\frac{r}{t}\right) dn(t,0) = \int_0^r \frac{n(t,0)dt}{t} = N(r,0);$$

$$(3) \qquad \sum \log\frac{r}{|b_\nu|} = \int_0^r \log\left(\frac{r}{t}\right) dn(t,\infty) = \int_0^r n(t,\infty)\frac{dt}{t} = N(r,\infty).$$

He then wrote $\log^+ x = \max(\log x, 0)$, so that

$$\log x = \log^+ x - \log^+ \frac{1}{x}, \quad x \geqslant 0,$$

and

$$\frac{1}{2\pi}\int_0^{2\pi} \log|f(re^{i\theta})|d\theta = \frac{1}{2\pi}\int_0^{2\pi} \log^+|f(re^{i\theta})|d\theta - \frac{1}{2\pi}\int_0^{2\pi} \log^+\left|\frac{1}{f(re^{i\theta})}\right| d\theta.$$

This was the apparently simple but vital new step. Writing

$$m(r,\infty) = \frac{1}{2\pi}\int_0^{2\pi} \log^+|f(re^{i\theta})|d\theta$$

and, for any finite complex a,

$$(4) \qquad m(r,a) = \frac{1}{2\pi}\int_0^{2\pi} \log^+\left|\frac{1}{f(re^{i\theta})-a}\right| d\theta,$$

we obtain Jensen's formula in the form [1924c]

$$m(r,\infty) + N(r,\infty) = m(r,0) + N(r,0) + \log|f(0)|.$$

Ahlfors [1976] rightly wrote that this was the moment when Nevanlinna theory was

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born. On applying this result to $f(z)-a$ instead of $f(z)$, we see that $N(r,\infty)$ is unaltered and $m(r,\infty)$ is changed by at most $\log^+|a|+\log 2$. Thus we obtain The First Fundamental Theorem

$$m(r,\infty)+N(r,\infty) = m(r,a)+N(r,a)+\log|f(0)-a|+\varepsilon(a),$$

where $|\varepsilon(a)| \leqslant \log^+|a|+\log 2$.

Nevanlinna wrote for $f(0) \neq \infty$

$$T(r,f) = m(r,\infty)+N(r,\infty) \tag{5}$$

and called $T(r,f)$ the characteristic function of $f(z)$. Then the First Fundamental Theorem shows [1925*g*] that for every a in the closed plane other than $a = f(0)$

$$m(r,a)+N(r,a) = T(r,f)+O(1), \tag{6}$$

as r varies. For $f(0) = a$ or ∞ a slight modification is required which we ignore here.

The function $T(r)$ gives an excellent description of the growth of any meromorphic function $f(z)$ either in a finite disk or in the whole plane. It is a convex increasing function of $\log r$. If f is entire it has roughly the same growth rate as $\log M(r)$ so that the order ρ can be defined, by replacing $\log M(r)$ by $T(r)$ in (2), but the new definition applies to any meromorphic function in the plane. Now (6) shows that f has in a certain sense the same affinity for every complex value a as measured by the sum of the terms m and N. The latter measures the number of roots of the equation $f(z) = a$ in $|z| \leqslant r$, while the former measures the average closeness of $f(z)$ to a on $|z| = r$. We deduce at once Hadamard's inequality, namely $\rho(a) \leqslant \rho$ for every a.

However, Nevanlinna's aim was to obtain a sharper version of Borel's inequality. He did this by showing that in general it is the term $N(r,a)$ which dominates in (6). More precisely he showed [1925*g*] that if $q \geqslant 3$ then

$$(q-2)T(r,f) \leqslant \sum_{\nu=1}^{q} N(r,a_\nu)-N_1(r)+S(r). \tag{7}$$

This is the Second Fundamental Theorem. Here $N_1(r)$ measures the zeros of f' and the multiple poles of f, that is, the totality of the multiple roots of all equations $f = a$, a root of multiplicity p being counted $p-1$ times. Also $S(r)$ is a term which is in general much smaller than $T(r)$. For instance, for a meromorphic function in the plane,

$$S(r) = O(\log\{rT(r)\}).$$

In the case of infinite order, certain intervals of finite total length must be excluded here.

It should be said that Nevanlinna only proved (7) in the case $q = 3$. This was enough to obtain a very much stronger form of Borel's result. If f is meromorphic and transcendental in the plane then

$$\overline{\lim}\,\frac{N(r,a)}{T(r)} \geqslant \frac{1}{3}$$

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except for at most 2 values of a. Also $\rho(a) = \rho$ except for at most 2 values of a.

The extension of (7) to general q was obtained almost simultaneously by Littlewood in a letter to Nevanlinna and by Collingwood [1924]. Nevanlinna immediately saw the importance of this extension and used it in an appendix to [1925 *g*] to obtain the deficiency relation

$$\sum \delta(a) \leqslant 2$$

where

$$\delta(a) = \varliminf_{r\to\infty} \frac{m(r,a)}{T(r)} = 1 - \varlimsup \frac{N(r,a)}{T(r)}.$$

The result implies that $\delta(a) = 0$ except for a countable set of values of a. A stronger version results if we count multiple roots only once and write $\bar{N}(r,a)$ for the corresponding counting function. Then if

$$\theta(a) = 1 - \varlimsup \frac{\bar{N}(r,a)}{T(r)}$$

we obtain [1926], by using the term $N_1(r)$ in (7),

$$\sum \theta(a) \leqslant 2. \qquad (8)$$

Nevanlinna also defined [1929 *a*] the ramification index (Verzweigungsindex)

$$\mu(a) = \varliminf \frac{N(r,a) - \bar{N}(r,a)}{T(r)}.$$

Thus $\theta(a) \geqslant \mu(a) + \delta(a)$ and we obtain

$$\sum \delta(a) + \sum \mu(a) \leqslant 2. \qquad (9)$$

The complete sharpness of (9) has only recently been shown by Drasin [1977] who has, for pre-assigned $\delta(a)$ and $\mu(a)$ subject to (9) and $0 \leqslant \delta(a) + \mu(a) \leqslant 1$, constructed a meromorphic function having precisely those values $\delta(a)$ and $\mu(a)$ for every a.

It would take us too far to give the many other beautiful applications made by Nevanlinna himself and others of the Second Fundamental Theorem (7) and its consequence (8). Here is one [1926]. Suppose that f_1, f_2 are transcendental meromorphic functions that assume 5 values a_ν at exactly the same points possibly with different multiplicities. Then $f_1 \equiv f_2$. If the multiplicities are the same we obtain the same conclusion for 4 values a_ν, except when a_1, a_2, a_3, a_4 form a harmonic range and both functions do not take two of the values a_2 and a_4. In this case $f_2 = S(f_1)$ where S is the bilinear transformation which permutes a_2 and a_4 and has a_1, a_3 as fixed points. The corresponding result for entire functions of finite order and 3 values a_ν had previously been obtained by Pólya [1921].

The theory, almost complete now, was put together by Nevanlinna in his famous monograph [1929 *a*]. Much of it had been published in [1925 *g*]—according to Lehto [1982] Nevanlinna's most important work—the appearance of which Weyl [1943] described as "one of the few great mathematical events of our century".

The proofs so far had been entirely analytic in character and were based on a

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generalisation of Jensen's theorem which the inventor Nevanlinna called the Poisson–Jensen theorem. This has played a fundamental role in function theory ever since, to the extent that Boas once said to me (perhaps with slight exaggeration) "anything about meromorphic functions which cannot be deduced from the Poisson–Jensen theorem is not true". The Poisson–Jensen formula is obtained from Jensen's formula (1) by making a bilinear transformation of $|z| < r$ onto itself which sends the origin to a specified point ζ and thus yields a representation of $\log|f(\zeta)|$ in terms of the boundary values $\log|f(re^{i\theta})|$ and the zeros a_μ and poles b_ν of f in $|z| < r$. The analytic technique made it very easy to compare the characteristics and value distribution properties of sums, products, derivatives and other combinations of functions. However in the early 1930's two techniques came along which shed a powerful new light on the theory, namely the geometric approach of Ahlfors [1929] and to some extent Shimizu [1929] and the potential theoretic approach of Frostman [1935]. The first led to an exact form of the First Main Theorem 1, namely (Ahlfors [1929], Shimizu [1929]):

$$m_0(r, a) + N(r, a) = T_0(r) + m_0(0, a).$$

Here $T_0(r)$ is the logarithmic integral of the area $A(r)$, counting multiplicity, of the image of $|z| < r$ by $f(z)$ when this image is regarded as lying on the Riemann sphere. Also

$$m_0(r, a) = \frac{1}{2\pi}\int_0^{2\pi} \log\frac{1}{k\{f(re^{i\theta}), a\}}\, d\theta,$$

where

$$k(w_1, w_2) = \frac{|w_2 - w_1|}{\sqrt{(1+|w_1|^2)(1+|w_2|^2)}}$$

is the chordal distance of two points in the metric of the Riemann sphere. Using these ideas Ahlfors [1935] also obtained a second main theorem comparing $A(r)$ with the number of islands over mutually disjoint domains D_ν, that is, regions in $|z| = r$, which are mapped p to 1 onto D_ν, where p is the multiplicity of the islands.

Further the concept of capacity due to Frostman [1935] enabled Nevanlinna to show in [1936*a*] that given $\alpha > \frac{1}{2}$ we have, for any function of unbounded characteristic in $|z| < R$,

$$m(r, a) = O\{T(r)^\alpha\} \quad \text{as } r \to R \tag{10}$$

for all a outside a set E of capacity zero. On the other hand if E is any compact set of capacity zero, there exists $f(z)$ of unbounded characteristic in $|z| < 1$ and assuming none of the values on E. Thus for every $a \in E$

$$N(r, a) = 0, \quad m(r, a) = T(r) + O(1) \quad \text{as } r \to 1.$$

Related results making stronger assertions, with larger exceptional sets, had been obtained earlier by Littlewood [1930] and Ahlfors [1931]. Nevanlinna had already shown in [1925*g*] that $f(z)$ has bounded characteristic in the plane only if it is constant, and in a finite disk $|z| < R$ only if $f = f_1/f_2$ there, where the f_j are regular and satisfy $|f_j| < 1$ in $|z| < 1$. Thus (10) can be regarded as a far-reaching

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generalisation of Weierstrass's theorem concerning the behaviour of functions near an isolated essential singularity. Nevanlinna [1936*c*] also discussed the size in terms of Hausdorff measure of sets of capacity zero and gave a complete characterisation for generalised Cantor sets.

Other achievements during this period included the invention of harmonic measure [1935*a*] and the associated principle, a strong generalisation of Hadamard's 3-circles Theorem. Nevanlinna had used a related idea to find in [1933*e*] the exact form of an inequality of Milloux [1924], a result independently obtained by Beurling [1933] and a little earlier in a weaker form by Schmidt [1932]. The Milloux–Schmidt inequality, as it is now somewhat unjustly called, has proved to be a fundamental tool in the study of entire functions. These and a number of other matters were put together in [1936*a*], a book which has inspired function theorists ever since it was written and which, like most of the later books, has been translated into Russian and English as well as having several German editions.

Compared with the achievements described above Nevanlinna's other work comes almost as an anticlimax. He continued to write a large number of papers and books throughout his life on many different topics ranging from ballistics [1943*a*] to education [1966*d, e*]. His book [1953*d*] on Riemann surfaces is one of the best accounts of the subject and O. Pretzel has told me how interesting he found the book [1959*f*] written with Frithiof on coordinate free analysis. He wrote an excellent elementary textbook with Paatero [1965*c*]. However his survey lecture [1966*b*] on complex analysis mentions nothing that happened after 1936. Perhaps after that time Rolf did not follow actively the further refinements and applications of his theory being created by a growing circle of younger admirers all over the world. An account of some of the achievements of this theory up to 1975 will be found in Ahlfors [1976].

During the last year or two a breakthrough has been created by S. Rickman with his proof of Picard's theorem for quasiregular mapping from R^n to R^n, the nearest analogue of the concept of a regular function when $n > 2$. Nevanlinna's ideas play a key role in Rickman's theory; there is, for instance, a deficiency relation. No doubt the theory will continue to flourish for many years to come.

Acknowledgments

When I was asked to write this obituary I realised that I ought to go to Finland at some stage. This visit was made possible by Ilpo Laine who kindly invited me to teach in the Nordic summer school at Jyväskylä. During the preceding weekend Sinikka Nevanlinna very graciously entertained me at her home and introduced me to Harri. I also had the privilege of having two long and interesting talks with Mary on the next two days. The conversations I was thus enabled to have with members of Rolf's family were most helpful, and I hope I have not abused their hospitality by misrepresenting what they said.

Sinikka also kindly lent me her copy of Rolf's autobiography [1976*b*], which is now out of print. Aimo Hinkkanen spent many hours skilfully translating selected portions of this work for me and my account of Rolf's life is mainly based on these. I am also greatly indebted to Olli Lehto who smoothed my path everywhere and, together with Sinikka, provided me with a representative set of Nevanlinna's offprints. Sinikka read a preliminary version of the life and suggested a number of changes. I have already indicated my debt to Lehto [1982]. In addition I have consulted the account [1966] by Künzi and Louhivaara of Rolf's career, and the

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later account by Lehto and Louhivaara [1976]. The list of publications at the end of this memoir follows the list prepared by Louhivaara [1976 and an unpublished appendix to it]. Finally I have profited by the splendid account of Rolf's work given by Ahlfors [1976].

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