SIR WILLIAM DAVIDSON NIVEN.

WILLIAM DAVIDSON NIVEN was born at Peterhead in 1843, and died at his residence in Sidcup in May 1917. After a brilliant career at the University of Aberdeen, he passed on in 1862 to Trinity College, Cambridge, of which he became a Fellow in 1867. In the Mathematical Tripos of January 1866 he had come out as third wrangler, bracketed with his friend James Stuart of Trinity, who was soon to be prominent in the starting of the system of University extension. Later, in 1875, Stuart was elected, by the suffrages of the Senate at large, to a newly founded chair of Mechanism and Applied Mechanics—the nucleus of the present School of Engineering—which he held until 1890. Subsequently he devoted himself in a Parliamentary career mainly to political and social problems, and after some years of retirement and occupation with business affairs, predeceased Niven only by a short time. The senior wrangler of the year was another Scotsman, Morton of Peterhouse, who, coming from the University of Glasgow, was in succession a Fellow of that College and of Christ's, but died young.

It was a time of distinction in the Mathematical Tripos. In the previous year the first two wranglers were Lord Rayleigh and the economist Alfred Marshall. In the following year his brother Charles Niven, now Professor of Natural Philosophy at Aberdeen, was Senior, the mathematician and philosopher W. K. Clifford was second, and Osborne Reynolds, physicist and engineer, was seventh: while in the next year Lord Moulton was senior, Sir George Darwin, late Plumian Professor of Astronomy, second, and Sir W. Christie, formerly Astronomer Royal, fourth.

A third brother James Niven, eighth wrangler in 1874, became Fellow of Queens'; and, having graduated M.B., is now head of the Department of Public Health for Manchester, and a well-known authority on that subject.

Soon after obtaining his fellowship at Trinity, W. D. Niven took up the appointment of Professor of Mathematics at the Royal Indian Engineering College, Coopers Hill. This he presently exchanged for the probably more congenial Chair directing the Mathematical work of the advanced class of Artillery Officers at Woolwich; here he came into practical contact with the rapidly advancing science of gunnery and ballistics, and initiated improvements in the theory of trajectories which maintained their place for many years, until at length the modern high velocities had made the trajectories so flat as to greatly simplify the practical calculations. The new developments of the present war have probably reinstated, and doubtless improved on, the older investigations.

In 1874 Niven accepted an invitation to return to Cambridge, as a lecturer on the staff of Trinity. Here he came in contact with Clerk Maxwell, who had been persuaded by scientific pressure to come back to the University in 1871 as the first Cavendish Professor of experimental physics; the Cavendish laboratory, presented by the Chancellor, the Duke of Devonshire, was completed under his direction in 1874. A close friendship seems to have struck up at once between the two men, congenial in every way, which went on with increasing force until Maxwell's untimely death in 1879 at 48 years of age. Niven charged himself largely with winding up the scientific affairs of his friend: the main formal memorial, two volumes of Scientific Papers, of quarto size now inconveniently large, was prefaced with an introductory biography which his busy life after leaving Cambridge too long delayed. He also completed for press the second edition of the *Electricity and Magnetism.* This period of his work in Cambridge was marked by a great expansion of the mathematical studies of the place in the direction of physical science. As was natural, the new developments of Maxwell in the theory of electricity excited keen interest, at a time when they were largely misunderstood or not understood at all elsewhere. The public lectures of Niven were on the subject of electricity and magnetism; and his class-room was a focus for the theory, in which congregated practically all the mathematicians of the University. The relations thus formally begun were never allowed by him to drop; and in later years he could count nearly all the active developers of electrical science on the mathematical side as his friends and former pupils.

In 1882 Niven was again tempted to leave Cambridge, in order to take up the important office of Director of Studies at the Royal Naval College, Greenwich, which involved the general direction and supervision of education in connection with the Royal Navy. His talent for friendship and quiet social amenities made his beautiful residence, on the river front of the Palace of Greenwich, a centre of the life of the staff; and here also he won and retained through life the friendship of successive admirals who came for a term of office as Governors of the College. He induced some of his Cambridge friends of high mathematical eminence to undertake posts on the staff: by his hospitality he made the position of external examiner so attractive that he could command the services of the best qualified men of the time. On his retirement at the age of sixty in 1903 his official services were recognised by the conferment, then very unusual, of a Knight Commandership of the Bath. By his continued interest in the scientific societies, notably the Royal Society and the London Mathematical Society, he became a considerable stimulus on the social side to the activity and intercourse of scientific men.

From 1877 he devoted sustained attention (writing mainly in the *Messenger of Mathematics*) to the development of Maxwell's beautiful method expounded in the *Electricity and Magnetism* (1872), perhaps adapted from the treatment in Thomson and Tait's *Natural Philosophy* (1868), of defining the general spherical harmonic in terms of its poles. The expression

$$(-1)^i \frac{d^i}{dh_1 dh_2 \ldots dh_i} \frac{1}{r}, \quad \text{or shortly} \quad (-1)^i \left[\frac{d}{dh}\right]^i \frac{1}{r},$$

in which r is the distance of a point from the origin, and $\delta h_1, \delta h_2, \ldots$ are displacements of the origin in the directions of the poles, can represent any solid harmonic of degree $-(i+1)$, and also, when multiplied by r^{2i+1}, its conjugate of degree i. Niven proves the theorem, and makes it fundamental, that

$$(-1)^i \left[\frac{d}{dh}\right]^i \frac{1}{r} = \frac{1}{r^{i+1}} \left[\frac{d}{dh}\right]^i Q_i r^i,$$

where $Q_i(\mu)$ is the zonal surface harmonic of order i having r as its axis, and $r\mu = z$. He employs this theorem to obtain rapidly Maxwell's development of the general harmonic in terms of variables such as μ_s representing cos (rs) and μ_{st} representing cos (st), where s, t denote vectors to the poles: thus for the first few orders

$$Y_1 = \mu_1, \quad 2!\, Y_2 = 3\mu_1\mu_2 - \mu_{12}, \quad 3!\, Y_3 = 3(5\mu_1\mu_2\mu_3 - \mu_{23}\mu_1 - \mu_{31}\mu_2 - \mu_{12}\mu_3),$$

which take of course the well-known zonal forms when the poles all coincide with the axis, and the tesseral forms when some of them coincide with the axis and the others are equally distributed around its equator. This method is illustrated by application to Laplace's expansion of a function in a series of harmonics.

Incidentally he notes that

$$1.3.5 \ldots (2n-1) \left[\frac{d}{dh}\right] \frac{1}{r^{2n+1}} = (-1)^i \frac{1}{r^{2n+i+1}} \left[\frac{d}{dh}\right]^i \left(r^i \frac{d^n}{d\mu^n} Q_{n+i}\right),$$

which points to wide though not relevant analytical extensions.

It had been formally established by Sylvester (*Phil. Mag.*, 1876), from simple considerations of geometrical representation, that the system of poles which prescribes a given solid harmonic is unique. It may be remarked that a harmonic may have uniform relations with a set of axes, without those axes being its polar axes: for two or more poles may coincide, and any symmetric function of the polar operators will generate such a harmonic: in this way the harmonics possessing the symmetry of regular or other solids might be expressed in a direct manner.

In a following paper he notes that any finite function V can be expressed in terms of gradients of various orders of its value V_0 at the origin, by the formula

$$V = \exp\left(x\frac{d}{dx}+y\frac{d}{dy}+z\frac{d}{dz}\right)V_0 = \Sigma\frac{1}{i!}\left(x\frac{d}{dx}+y\frac{d}{dy}+z\frac{d}{dz}\right)^i V_0,$$

in which, when V is r^{-1}, the term of order i represents the general solid harmonic of that degree: and he employs a symbolic analysis based on this formula to evaluate directly the various surface integrals of products of harmonics over a sphere.

These papers were preparatory to a very elegant memoir of high analytical power, *Phil. Trans.*, May 1879, pp. 379–416, in which he exhibits the scope of this method by various applications, especially to the evaluation of the integrals over a sphere of products of three or more harmonics, and of products of them with other functions, a subject in which some striking results had previously been elaborated by laborious special methods. The symbolic exponential expressions led him directly to the result from which these developments flow by expansion, in the form that, if V is any function expressible within the sphere of radius R by a Taylor series, then over that sphere

$$\iint V\,dS = 4\pi R^2 \sum_0^\infty \frac{R^{2i}}{2i+1!}\left(\frac{d^2}{dx^2}+\frac{d^2}{dy^2}+\frac{d^2}{dz^2}\right)^i V_0,$$

where the operators refer to the value V_0 at the centre. At the end the method is extended to the evaluation of integrals over an ellipsoid, including the potentials of ellipsoids and circular electric currents, starting from the related expression for $\iint Vp\,dS$.

He returned to the subject, and generalised it much further, in October 1890, in a memoir "On Ellipsoidal Harmonics", *Phil. Trans.*, 1891, pp. 231–278, stimulated by reading Green's Memoir on Ellipsoids of varying densities; and his work retains some of the direct and forceful power of its source of inspiration. The adaptation of the analysis, including a correlation between spherical and internal ellipsoidal solid harmonics, starts with the relation, adapted from Maxwell, that if $f(x, y, z)$

and $F(x, y, z)$ are two solid spherical harmonics of degree n then

$$\iint f(x, y, z)\, F(x, y, z)\, dS = 4\pi \frac{2^n n!}{\overline{2n+1}!} f\left(\frac{d}{dx}, \frac{d}{dy}, \frac{d}{dz}\right) F(x, y, z),$$

and proceeds from the general function expressible in a Taylor series as before. External ellipsoidal harmonics are treated as differential derivates from the expression for the potential of a uniform ellipsoid. The formulæ are developed in detail for cases of axial symmetry: also for the cases of cylindrical and paraboloidal harmonics. The fundamental harmonic expression for the reciprocal of the distance between two points is developed. Many expressions giving solutions of physical problems occur incidentally.

The electrostatic capacity of a lens-shaped conductor is readily expressed in finite terms as an example of Kelvin's theory of electric images, when the internal angle a at which the bounding spherical surfaces meet is $2\pi/n$, where n is integral. For other values of a the capacity must therefore be equal to this expression together with some function $lF(a)$, where l represents the linear scale of the system, and $F(a)$ has the property of vanishing when $a = 2\pi/n$ and so has roots which become infinitely numerous near the value zero. A property of this type is not unusual: the function $\sin a^{-1}$ is a case in point.

The problem of determining the distribution of free charge, and the capacity, for general values of a was attacked by Niven in two papers (*Proc. London Math. Soc.*, 1894 and 1896) employing a method remarkable as being purely abstract. He constructs a functional equation which the potential must satisfy, somewhat after the manner of Poisson's discussion of the electrostatic distribution on two adjacent complete spheres, and effects a solution, subject to conditions imposed by the problem, in the form of a definite integral. This unusual procedure excited considerable interest and perhaps some hesitation as to the sufficiency of the argument. When H. M. Macdonald attacked the problem by the standard method of Bessel functions, he reached results at first sight different; but he soon had the satisfaction of establishing their identity with those of Niven. The results, of course, were a considerable enlargement of the few previously known solutions for a conductor partially enclosing a hollow space: for they extended Lord Kelvin's classical case of a thin spherical bowl to a bowl bounded by segments of any two intersecting spheres. A striking corollary, easily verified for special cases, is that the difference of the charges on the two faces of the bowl is equal to the potential multiplied by a differential capacity

$$a \frac{\pi}{\gamma} \cot \frac{\pi}{\gamma} a,$$

when the two surfaces intersect at an interior angle α, and one of them intersects the plane of the circular edge, whose radius is a, at an interior angle γ. Previous papers in *Proc. Math. Soc.* were a reconstruction of the analytical theory of electric images applied to two influencing charged spheres (Vol. VIII, 1876, pp. 64–83), and an extension of the same method to intersecting spheres (Vol. XII, 1880, pp. 27–36). There is also an elaborate paper on the harmonics of an anchor ring (Vol. XXIV, 1893), in which the expressions are built up from a system of infinitely thin rings, and applied to the electrostatic influence between two solid rings, or a sphere and a ring, on a common axis.

For many years W. D. Niven was one of the chief supporters of the London Mathematical Society, serving on the Council for fourteen years, at various times beginning with the year after his election into the Society in 1873. He was President for the Sessions 1908 and 1909. On retiring from the chair in November 1910 he delivered the customary address, choosing as his subject "The Relations of Mathematics to Experimental Science": with characteristic modesty he seems to have withdrawn it from publication.

He was elected into the Royal Society in 1882, and was on its Council for the periods of 1892–4 and 1904–6, serving as Vice-President for the latter two years.

A few years ago his scientific friends and past pupils, of Cambridge, London, and Aberdeen, combined to have his portrait painted in oils. It was presented to the University of Aberdeen, where it hangs in King's College.

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