JULES HENRI POINCARÉ.

JULES HENRI POINCARÉ was born on the 29th of April, 1854, at Nancy, where his father was a physician held in great respect among the citizens. He received his school education at the Lycée of his native city, but this was interrupted by the war of 1870–71, during which he acted as secretary to his father in his attendance upon the sick and wounded. It is recorded that he was so eager for news of the progress of the war that at this time he learnt German in order to read the German newspapers. At school, although he early showed a special aptitude for mathematics, and it was always expected that he would become a mathematician, he nevertheless outclassed the other boys of his age in all the subjects of instruction. He proceeded to Paris in 1873, and was admitted "first" to the École Polytechnique, proceeding in 1875 to the École des Mines, and becoming finally qualified as a mining engineer in 1879, the same year in which he was created Docteur ès Sciences mathématiques in the University of Paris, and was appointed to give instruction in Analysis in the Faculté des Sciences at Caen. In 1881 he returned to Paris to take up an appointment as Maître de Conférences d'Analyse in the Faculté des Sciences of the University. In 1885, he began to give instruction in Mechanics in the same institution, becoming Professor of Mathematical Physics in 1886 and Professor of Mathematical Astronomy and Celestial Mechanics in 1896, a title he continued to hold till the time of his death on July the 17th, 1912. From 1883 to 1897 he acted as Répétiteur d'Analyse, and from 1904 to 1908 as Professor of Astronomy, at the École Polytechnique. In 1902 he was appointed Professor of Theoretical Electricity at the École des Postes et des Télégraphes in Paris. During the last few years of his life he occupied so many positions of dignity and eminence, and his pen was so constantly in request for articles, prefaces, addresses, and reports, that it is surprising that he could continue to find time for research work.

Poincaré's original investigations cover a wider range in Pure Mathematics, Theoretical Physics, Celestial Mechanics, and the Philosophy of the exact sciences than those of any other mathematician of the age. A mere catalogue of his writings would occupy more space than would be suitable in this Notice, and it will be desirable to select for mention and description some of the more important. His earliest writing, occupied with functions that are defined by differential equations, is dated 1878, and among his first pieces of research work was an essay submitted in competition for the Grand Prix offered in 1880 by the Académie des Sciences for an improvement in the theory of differential equations. His great discoveries in this theory began to be published in a series of notes inserted in the *Comptes Rendus* for 1881. Here he first propounded his construction of Fuchsian and Kleinian groups and the corresponding automorphic functions. A brief general account of this theory was published in the *Mathematische Annalen*, Bd. 19, 1892, and the developed theory was expounded as a coherent whole in a series of memoirs published in the *Acta Mathematica*, tt. 1–5, 1882–84. The elliptic functions of a complex argument are uniform functions which have the property of taking the same value at corresponding points in a doubly infinite set of parallelograms. These form a lattice covering the whole plane of the complex argument, the corresponding points being determined by a linear substitution which is equivalent to the result of two compounded simple translations, parallel to the sides of one of the parallelograms, through distances which are numerical multiples of the lengths of the sides. Poincaré's idea was to substitute for one of these parallelograms a curvilinear polygon, formed in general by arcs of circles, and for the others a set of related polygons, the whole set covering the plane, or a prescribed portion of the plane, and corresponding points being determined by the linear substitutions belonging to an infinite discontinuous group. His automorphic functions, or as he called them "Fuchsian" functions, are uniform functions which have the property of taking the same value at the corresponding points of all the polygons. He showed how expressions for such functions could be constructed, and how by means of them it is possible to construct the integrals of any linear differential equation with algebraic coefficients. He also showed that by means of the same functions a pair of variables connected by any algebraic equation can simultaneously be expressed as uniform functions of a single parameter. The great generality and power of these researches won for him instant recognition as an analyst of the first rank.

While this work was still in course of publication, Poincaré's attention was attracted to questions of Mechanics, in particular to the integration of the differential equations of the problem of three bodies, and he began to write on this subject as early as 1882. It may be surmised that it was in the process of assimilating the work of others on analytical dynamics that he became acquainted with Thomson and Tait's "Natural Philosophy," some passages in which, dealing with the equilibrium of rotating masses of fluid, appear to have set him thinking about the theory of stability and to have given rise to the memoir "Sur l'équilibre d'une masse fluide animée d'un mouvement de rotation," published in the *Acta Mathematica*, t. 7, 1885. In this memoir he introduced the important conceptions of "bifurcation" and "exchange of stabilities," explained their bearing upon the distinction previously emphasized by Thomson and Tait between ordinary and secular stability, and applied them to the problem of determining the conditions under which spheroidal and ellipsoidal figures of equilibrium of rotating masses of fluid become unstable. He further demonstrated the existence of a previously unknown pear-shaped figure, and suggested the possibility that the progressive deformation of this figure with increasing angular velocity might result in the breaking up of the rotating body into two detached masses. The bearing of these discoveries upon cosmic speculation as to the history of the Earth-Moon system and the origin of double stars is obvious. The fundamental conception involved in this theory is that of the dependence of the equilibrium configuration upon a variable parameter; in the special case the parameter is the angular velocity of the rotating body. As the parameter varies the quantities which determine the configuration, for example, the ellipticity of a rotating spheroid, also vary; and the totality of the configurations that can be specified by the same quantities and the parameter form what Poincaré calls a "linear series" of configurations. There may be several linear series, and when the parameter takes one or other of certain special values the same configuration may belong to two series. This possibility is expressed in the notion of bifiurcation, and Poincaré proved that if, as the parameter increases, a configuration is reached at which there is bifurcation, and if for values of the parameter smaller than the critical value the configurations of one series are stable, then for greater values they become unstable, and the property of stability passes to the configurations of the other series. Isolated problems which can now be recognized as exemplifying this theory had been solved before, particularly in the theory of elasticity, but the conceptions were for the first time formulated effectively in the memoir cited above, and they have since proved fruitful in many departments of mathematical physics.

Poincaré himself proceeded to develop the application of these conceptions to celestial mechanics. We have seen that he had begun to write on the problem of three bodies before the publication of the memoir on figures of equilibrium of rotating fluid. His further work on the subject issued in the publication in 1890 of a memoir "Sur le problème des trois corps et les équations de la dynamique" (*Acta Mathematica*, t. 13), for which he was awarded a prize founded by King Oskar II of Sweden and Norway. The methods and results of this memoir were afterwards amplified and presented as part of a systematic treatise in the work entitled

Les Méthodes nouvelles de la Mécanique céleste (t. I, 1892; t. II, 1894; t. III, 1899). G. W. Hill, in 1877, had proved the existence of a particular periodic solution of what is known as the restricted problem of three bodies, taking account of that inequality which in the lunar theory is called the "variation." Poincaré showed that Hill's solution is only one of a quadruple infinity of periodic solutions of the general problem, and that such solutions fall into families in respect of which the notions of bifurcation and exchange of stabilities find an application. He proved, further, that any motion of the bodies, if it is not exactly periodic, must differ from some one or other of those which can be expressed by periodic solutions by a very small deviation, so that a knowledge of the approximate periodic solution would suffice for the calculation of the actual motion during a very long period of time. It may happen that one of the bodies approaches closer and closer to a periodic orbit, traces a path which lies very near to this orbit during a great number of revolutions, but ultimately moves far away from it. Such solutions are called "asymptotic." The memoir of 1890 contains also the first introduction of the analytical conception of an integral invariant, an important but depressing theorem as to the non-existence of any uniform integrals other than those of energy, momentum, and moment of momentum, and a disconcerting but well supported statement to the effect that the series traditionally used by astronomers in the lunar and planetary theories are divergent. Of this memoir and treatise Sir George Darwin remarked, "It is probable that for half a century to come it will be the mine from which humbler investigators will excavate their materials."

Divergent series had no terrors for Poincaré, nor did he share Abel's opinion that such series are an invention of the devil. He knew that in many instances they had been employed with success for purposes of approximate numerical computation, especially in connection with irregular integrals of linear differential equations, and one of his memoirs (*Acta Mathematica*, t. 8, 1886) is devoted to the construction of a general theory of what are now called "asymptotic expansions" and their applications to the expression of such integrals. In another paper dating from the same year (*Bulletin de la Soc. Math. de France*, t. 14) he formulated a theory of the convergence of an infinite determinant, a novel type of infinite expression which had been introduced very shortly before by G. W. Hill in his researches on the lunar theory. In fact, Poincaré never gave up working at analysis, making important contributions among other things to the theory of functions of several variables, and the discussion of related questions concerning *analysis situs* or the connectivity of algebraic loci in space of any number of dimensions. One of his memoirs on this subject was communicated by him to this Society, at the meeting held in June, 1900, when he had occasion to be in London to take part in the conferences which led to the establishment of the International Catalogue of Scientific Literature. In another, perhaps the last that he wrote, he utilised the notions of *analysis situs* to extend his previous work on periodic solutions of the problem of three bodies (*Rendiconti d. Circ. mat. d. Palermo*, t. 33, fasc. 3, 1912). Although this memoir is unfinished, it affords a very remarkable example of Poincaré's mode of working, and the unexpected use that he could make of geometrical imagery.

Poincaré's activity in almost all departments of mathematical physics is well shown by the number of treatises which he published in the form of lectures delivered at the Sorbonne. The latest of these, the *Leçons de Mécanique céleste*, t. 3, *Théorie des Marées*, published in 1910, and the *Leçons sur les Hypothèses cosmogoniques*, published in 1911, deal with astronomical subjects, the latter containing a very interesting discussion of Laplace's nebular hypothesis; but the earlier volumes are occupied with Optics, Electricity, Capillarity, Vortex Motion, Elasticity, Conduction of Heat, Thermodynamics, Theory of the Potential, and Figures of Equilibrium of rotating fluid. Of these the *Électricité et Optique* in its two editions attracted the most attention, and had much influence in spreading Maxwell's ideas abroad in Europe. Besides all this work of expounding physical theories, always with a touch, or more than a touch, of novelty, Poincaré made important original contributions to theoretical Optics in two memoirs on the diffraction of light at a straight edge (*Acta Mathematica*, t. 16, 1892, and t. 20, 1897), but his chief work in physical subjects may be held to lie in the discussion of theorems concerning the existence of the functions which satisfy the partial differential equations of mathematical physics, and also satisfy prescribed conditions at assigned boundaries. The most famous of these is the one known under the name of Dirichlet's principle, to the effect that there exists a function which, with its differential coefficients of the first order, is finite and continuous at all points of the region of space bounded by a given surface, which satisfies Laplace's equation at all points within the region, and which takes at every point on the surface a prescribed value. One method of proof is to construct the function as the limit of a series of functions which all satisfy the differential equation, but do not satisfy the boundary condition exactly, only with continually improving approximation. This was the method developed by C. Neumann, and it seemed to be impossible to prove that the approximation really improves without limit unless the surface is everywhere convex. Poincaré overcame the difficulty by constructing a series of functions which do satisfy the boundary condition, but do not satisfy the equation. Such functions are the potentials of distributions of matter through the region, and the successive functions are invented to have the property that the density of the matter diminishes without limit. This is the method of sweeping (*balayage*), the matter being always, as it were, swept out of a portion of the region and distributed over the boundary of that portion. This method was propounded in a memoir published in the *American Journal of Mathematics*, Vol. 12, 1890, and afterwards expounded in the course of lectures entitled "Théorie du Potentiel Newtonien," 1899. Other famous memoirs by Poincaré on the partial differential equations of mathematical physics appeared in the *Rendiconti d. Circ. mat. d. Palermo*, t. 8, 1894, and in the *Acta Mathematica*, t. 20, 1897. The first of these contains a method for arriving at a series of quantities analogous to the frequencies of a vibrating system, and forms a step towards the theory of integral equations afterwards developed by Fredholm and others. At the end of the second are given suggestions, put forward as probable but not as proved, for the construction of functions, called by the author "fundamental," belonging to any given surface, and designed to play the same part in solutions of problems of potential theory related to the surface as spherical harmonics play in the solution of similar problems related to a sphere. Stekloff and others have developed these suggestions with some modifications so as to obtain a more complete theory. Poincaré seems to have been much interested in the theory of integral equations, and among his writings on this subject may be mentioned the first three of the lectures which he gave at Göttingen, in 1909, at the invitation of the University ("Sechs Vorträge über ausgewählte Gegenstände aus der reinen Mathematik und mathematischen Physik," Leipzig, 1910).

If Poincaré's original contributions to theoretical physics are concerned rather with the discussion of the equations than with the construction of comprehensive theories of phenomena, as a critic and expositor of such theories he was unrivalled. Particular reference may be made to the brilliant tract of 80 pages, entitled *La théorie de Maxwell et les oscillations Hertziennes*, which he contributed to the *Scientia* series in 1899. This tract was one of the earliest of his popular writings, but the most celebrated of them is the volume entitled *La Science et l'Hypothèse*, which has been translated into six languages, and has had a circulation of 20,000 copies. His philosophical writings, of which this book is the best known example, seem to have begun in 1887 with a discussion of the foundations of Geometry, but rapidly extended to a review of the foundations of physical science. The particular notion that is associated with his name is that of the possibility of alternative laws of nature enunciated in the Introduction to the *Electricité et Optique*, in the words—" Si donc un phénomène comporte une explication mécanique complète, il en comportera une infinité d'autres qui rendront également bien compte de toutes les particularités révélées par l'expérience." The idea that the laws of nature which we arrive at gradually, building on the foundations laid by our predecessors, may constitute merely one of an infinite number of conceivable rational schemes for coordinating and interpreting experience, may be regarded as a development of the views of Kirchhoff and Mach, but the special form which it received at the hands of Poincaré is all his own, and the wonderfully simple and vivid style of his writings secured for his ideas a wider circle of readers than falls to the lot of most mathematicians and philosophers.

Poincaré was honoured during his lifetime. In his own country he was elected a member of the Académie des Sciences in 1887, subsequently receiving every honour that can be conferred upon a French man of science by his countrymen. The Royal Society of Sciences of Göttingen was quick to realise that a new star had risen in the mathematical firmament and elected him a foreign member in 1883. Afterwards very many foreign academies and scientific societies did the same, and he was the recipient of many medals and other distinctions. In England the Cambridge Philosophical Society elected him an Honorary Member in 1890, and our own Society and the Manchester Literary and Philosophical Society followed in 1892. In 1894 he became a Foreign Member of the Royal Society and an Associate of the Royal Astronomical Society. He received the gold medal of the Royal Astronomical Society in 1900, and in 1901 was the first recipient of the recently founded Sylvester medal of the Royal Society. Both Cambridge and Oxford conferred Honorary Degrees upon him. A few months before his death he gave, by invitation of the University of London, a series of lectures which were attended by many English mathematicians. Several accounts of his life and writings have been published, the most complete being that by E. Lebon in the series entitled *Savants du Jour* (2nd ed., 1912). There can be no doubt that France was proud of him, and moreover curious about him. This curiosity led to his giving a most interesting account of the way in which he made discoveries in mathematics. He held that mathematical reasoning does not consist in piling syllogism on syllogism, but essentially in the order in which the syllogisms are placed; and he described how, in circumstances in which he might be occupied with anything rather than mathematics, the combination of ideas which would prove to be effective in the development of some theory would suddenly flash into his mind with a conviction of absolute certainty. He had a very retentive memory, especially for anything he heard; and it is said of him that he associated symbols with sounds, his mind acting rather by audition than by visualising. Calculators have often been visualisers, but Poincaré distrusted his powers of calculation and preferred to work by a combination of analytical and geometrical conceptions, often expressed in extremely simple terms, towards necessary general conclusions. The record of his life shows that he was not one of those who sit by the roadside waiting for inspiration. He was always at work, ever acquiring fresh knowledge by assimilating the work of others, and constantly giving verbal expression to the form in which the acquired knowledge presented itself to his mind and the relation in which it stood to the things that he had known before. It is given to few men to dream even one such dream as the invention of automorphic functions, the formulation of the geometrical criterion of stability, or the discovery of families of periodic orbits. There was needed for these a mind stored with knowledge of groups and functions and series and differential equations, of general theorems concerning the equations of dynamics, of harmonic analysis, of non-Euclidean geometry and geometry of many dimensions, of *analysis situs*. All this knowledge and much besides he could bring to bear upon any matter to which it could be applied. But there was another requisite—the indefinable thing that we call genius. His right is recognized now, and it is not likely that future generations will revise the judgment, to rank among the greatest mathematicians of all time.

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