HERBERT WILLIAM RICHMOND

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Dr Herbert William Richmond, the eminent geometer, died in the Evelyn Nursing Home, Cambridge, on 22 April, 1948, at the age of eighty-four. At the time of his death he was Senior Fellow of King's College, Cambridge, in which college he had resided almost continuously for sixty-five years. Though crippled with rheumatism, he had been able, till within a few days of his death, to move about his rooms in a wheeled chair, to descend the staircase reasonably nimbly with the aid of crutches and to propel himself in his bath chair as far as the King's Fellows' Garden. Except for a somewhat marked deafness caused by his attendance at gun-trials at Eastney, Portsmouth, in 1916-1919, he was in full possession of all his faculties to the end, and to the end he maintained his interest in research and in his fellow-mathematicians and other friends. He died of heart failure following an attack of pneumonia.

Richmond was born on 17 July, 1863 at Tottenham, Middlesex. at Drapers' College (a school established by the Drapers' Company) of which his father, the Rev William Hall Richmond was headmaster. His mother was Charlotte Mary, née Ward, the daughter of Dr. Joseph Ward of Epsom. Herbert William was the eldest of the family having two younger brothers, George Ward and Alfred Mewburn, and two younger sisters, Margaret Evelyn and Ethel Mary. For five or six generations his forebears who bore his name Richmond had lived near Hexham on the Tyne, at Humshaugh or Haydon Bridge: small squires or parsons (or both). One of his other great great-grandfathers, William Hall, was a Fellow of St John's College, Cambridge; William Hall's brother was Provost of Trinity College, Dublin. On his mother's side his great-uncle Nathaniel Bagshawe Ward, F.R.S. (1791-1868), elder brother of the above-mentioned Dr. Joseph Ward, had been a doctor with a large practice in East London and an ardent nature-lover; he earned the gratitude of botanists by his discovery of the principle of the *Wardian Case*, invaluable for the introduction of species of plants to distant countries—tea, bananas, cinchona (quinine), and more recently, rubber.

When Richmond's father reached the proper age he entered, as was natural, the then recently founded University of Durham. He and his

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elder brother were among the earliest elected Fellows of the first English provincial university.

In September 1875 Richmond entered Merchant Taylors' School, which had been moved a few months earlier to Charterhouse Square, E.C., to a site left vacant by the removal of Charterhouse School to Godalming; he went up from Tottenham each day. The headmaster was the Rev. Dr. Baker. Richmond felt a deep gratitude to the chief mathematical master at Merchant Taylors', the Rev. J. A. L. Airey, who had graduated from Pembroke College, Cambridge, as Second Wrangler in 1846, and who by a curious chance had also taught Richmond's father in *his* school-days at Durham. Richmond remained at Merchant Taylors' until 1882, when he went up to King's College, Cambridge, having won an Eton scholarship thrown open on this occasion. He also won the Parkin Exhibition of his school. At King's he read mathematics,* his coach being Dr. Routh.

He was Barnes Scholar of the University of Cambridge in 1883. In 1885 he was placed Third Wrangler in Parts I and II of the Mathematical Tripos, taken (as they then were) together. Arthur Berry, also of King's and later to be his colleague for many years, was Senior Wrangler in that year; A. E. H. Love was Second Wrangler. Richmond was placed in Division I of Part III of the same Tripos six months later, in 1886. He took his B.A. degree in 1885, his M.A. in 1889. According to his own confession he was for a time sated with Tripos work, and for a while he dropped mathematics entirely, studying music instead. His enthusiasm for geometry, especially algebraic geometry, soon however revived, and a dissertation on that subject gained for him a Fellowship at King's in 1888 Nathaniel Wedd, a friend of his, was made a Fellow of King's the same year. In 1891 Richmond was made a college lecturer in mathematics; he retained this post until 1927. He became a university lecturer in 1901 and remained thus until 1919; he was also university lecturer under the new statutes from 1926 to 1928, when he retired. He was elected F.R.S. in 1911. He was President of the London Mathematical Society for 1920-1922. The honorary degree of LL.D. was bestowed on him by the University of St. Andrews in 1923. It will be seen that he was Fellow of King's for sixty years.

Richmond was a man of many and wide interests. It has already been mentioned that soon after taking his B.A. degree he devoted himself to a study of music (thereby sacrificing, as he has himself stated, his prospects

* These details of Richmond's provenance and early life are taken from some autobiographical notes he left behind with his papers. So far as possible his own words have been used.

of a Smith's prize). On the occasion of his eightieth birthday his colleagues of King's recognized his love of music by presenting him with a dictionary of music.

He was a keen and well-informed naturalist. Wild flowers and birds both delighted him. In his earlier years he regularly made the long journeys to the Orkneys and Shetlands for the sake of photographing sea-birds and their nests; his photography was of professional standard, and some of his pictures have been used as illustrations in standard works. He was also interested in the Fenland, both in its flora and fauna, as well as in its architecture. He was for some time on the Cambridge Committee of Management of Wicken Fen. As regards his architectural interests, the writers of this notice recall with pleasure architectural holidays they enjoyed with Richmond: one in the Hexham district, the country of his forebears, when (to recall an incident) they unexpectedly came upon the grave of the mathematician Weddle (known for "Weddle's rule" for integration and also for Weddle's quartic surface) in Corbridge Churchyard, and despatched a joint postcard to H. F. Baker to mark the occasion; another, viewing the castles and churches of the Cinque Ports and parts of Sussex; and another, including Croyland, King's Lynn, Castle Rising and Castle Acre, and the Perpendicular churches of Suffolk and Essex; besides many a shorter trip in the neighbourhood of Cambridge.

But shining all through his special interests was his genius for friendships with men of all academic generations. He endeared himself to all of them by his utter sincerity and unpretentiousness, his extreme modesty, his engaging humour (did he not once make fun of his own interests by inventing a notable wild flower, the "purple pigbane"?), his faithfulness, his power of identifying his own interests with theirs. He was a lovely person to know. His friend and former pupil, William Morton Page, C.B.E. (Fellow of King's, 1908), has written: "In spite of his shyness he had a remarkable gift for seeking out and befriending men who came up to King's from the lesser known schools, without friends or influence, and for helping them to form or develop their interests, particularly in music and in country pursuits. Many men will feel they have lost more than they can easily say." Amongst his friends who pre-deceased him there should not be forgotten H. Lob, a King's man who became a mathematical master at the Manchester Grammar School, who published a well-known paper in the *Mathematical Gazette* on inequalities deduced from the consideration that the centre of mass of a number of particles on a concave arc lies on the concave side of that arc, and who was killed in the air-raid on Manchester in January, 1941; and, in connection with his botanical interests, Sir Arthur W. Hill, F.R.S., Fellow of King's, 1901, Director of Kew Gardens, who also died in 1941.

Richmond never married. He was devoted to his sister, Mrs. Ethel Mary Norman, the wife of the Rev. W. A. Norman, and to their four daughters, his nieces. He showed also a continuing interest in the careers of his two nieces and nephew in Australia, the children of his brothers.

The writers of this notice are both a full generation junior to Richmond, and so cannot speak at first hand of his earlier mathematical activities at Cambridge*. But they have the advantage of access to certain notes left by Richmond about the directions of his earliest researches, and the reasons that attracted him to them.

According to these notes, *Algebraic geometry* was, and remained till his life's end, the mathematical subject which of all others appealed to him. It fascinated him in his undergraduate days as well as later. He felt that towards the end of his life it had been to some extent neglected. He therefore left on record the following brief sketch of its history during the last sixty years.

"In 1885 Cayley and Salmon had carried forward the investigations of the earlier German geometers, Hesse, Steiner, Plücker and others; and Salmon had expounded the subject in treatises which for clarity of style are still unrivalled. Further, the Italians, Corrado Segre and Castelnuovo were opening the way into a vast unexplored field, geometry of more than three dimensions. Rarely has a branch of science offered more inviting prospects to a novice hoping to undertake research.

"Yet I must now admit that my devotion to this one branch of mathematics has been in some degree unfortunate; for time has shown (what might perhaps have been foreseen) that the elementary algebraic methods, so effective and so admirable when applied to the simpler curves and surfaces, fail to produce results when applied to loci of higher order. Often, no doubt, no relations of an elementary nature exist, and the search for them is bound to end in disappointment. Nevertheless methods of elementary algebra may still be employed with success both in geometry proper and in applications such as arithmetical properties of rational functions. It is true that the scope of these methods is restricted, but there is compensation in the fact that when geometry is successful in solving a problem the solution is almost invariably both simple and beautiful.

* One of us (E. A. M.) made his first acquaintance with Richmond (Lent Term, 1915) through his lectures on differential geometry for undergraduates; and very useful lectures they were. He feels to this day indebted to Richmond for these lectures, as he considers he owes to them his success in solving a very respectable problem in differential geometry set in the Trinity Senior Scholarships Examination in 1915. The other (F. P. W.) attended as an undergraduate Richmond's lectures on algebraic geometry for Schedule B of the Mathematical Tripos (Lent Term, 1914).

"The last sentence explains why so many of my published papers are very short. A result already known is obtained in a simple manner.

"Admittedly I have spent much time advocating old-fashioned methods which have fallen into undeserved neglect (in my opinion). This does not imply lack of appreciation of modern methods. But from this and other reasons I feel that my attitude has always been that of an amateur, and I have never been able to regard myself and my original work with the seriousness of my younger friends. I will not say that I have regarded geometry and mathematics as a hobby; but I have derived the pleasure that comes from a hobby rather than from a business." (February, 1946.)

With this extremely revealing and indeed moving preface, in Richmond's own words, we pass to an account of his actual papers.

Richmond's first paper (1) begins a closely connected series which exhibits clearly his development as a geometer; these papers form an important part of his output, and are therefore worth considering in some detail. They arise from the well-known theorem of Pascal (1640) on the collinearity of the points of intersection of pairs of opposite sides of a hexagon inscribed in a conic. Beginning with Steiner in 1828, a whole series of authors had dealt, piecemeal, with the intersection properties of the sixty Pascal lines arising from the sixty hexagons formed from the same six points on a conic; in 1877 Veronese summed up and extended all these results in a masterly memoir in the *Lincei Atti*. The same volume, however, contained a memoir by Cremona, which threw a new light on the subject. He remarked that a cubic surface with a node has on it six lines through the node, lying on a quadric cone. The plane of any two meets the cubic surface again in a third line; there are thus fifteen further lines on the cubic surface, and, if they are projected from the node upon a plane, it is clear that they become the fifteen joins of six points on a conic. The intersection properties of the lines on a cubic surface which had been studied by Salmon, Schläfli and others, lead directly to properties of the fifteen lines; and it appears that it is only when two lines in the three-dimensional figure intersect that the intersection of their projection has any significance in the Pascal figure. Thus the three-dimensional figure contains all the essential properties, and is far simpler than the original figure, in which any two lines meet, though their intersection may be irrelevant. Cremona's methods were purely "synthetic"; Richmond applied himself to their algebraic expression. His first paper (1) gave equations for the lines on a nodal cubic surface, but in (2) he had greatly improved his technique. His final result was as follows: The equation of a nodal cubic surface can be expressed in the form $\Sigma x_i^3 = 0$, $i = 1, \ldots, 6$, where the x_i are linear forms in the coordinates, connected by two linear relations $\Sigma x_i = 0$, $\Sigma k_i^2 x_i = 0$, the k_i being such that $\Sigma k_i = 0$, $\Sigma k_i^3 = 0$. The fifteen lines of the surface, not through the node, have equations such as

$$x_1+x_2 = x_3+x_4 = x_5+x_6 = 0;$$

the symmetry of the set is thus clearly exhibited, and the intersection properties can be found at once.

Richmond's next step (**8**, **10**, **20**) was to notice that no use had been made of the relations $\Sigma k_i^2 x_i = 0$, $\Sigma k_i^3 = 0$; all that is necessary for the Pascal configuration is that the fifteen lines should be the projection of fifteen lines in three dimensions whose equations are of the type

$$x_1+x_2 = x_3+x_4 = x_5+x_6 = 0,$$

where $$x_1+x_2+x_3+x_4+x_5+x_6 = 0.$$

Now these lines clearly lie on the cubic surface $\Sigma x_i^3 = 0$, to which form, as had been pointed out by Cremona, the equation of a *general* cubic surface can be reduced; and they are the lines obtained by omitting a double-six from the twenty-seven lines of the surface. On projection from a general point we no longer get the joins of six points of a conic, but if the vertex of projection is taken on the cubic surface, then, as had been remarked by Geiser, we get a certain set of fifteen bitangents of a plane quartic curve. In this case, a sixteenth bitangent arises from the tangent plane at the vertex of projection, and we have a symmetrical set of sixteen bitangents of a quartic, any fifteen of which exhibit all the properties of the Pascal hexagram.

But, again, no use has been made of the second linear relation which must connect the x_i; giving this up, we see that the simplest possible interpretation of the algebra is to consider the cubic primal $\Sigma x_i^3 = 0$, $\Sigma x_i = 0$, in space of four dimensions (now known as Segre's cubic primal), containing a symmetrical set of fifteen planes. These planes can be obtained simply from six quite general solids $x_i = 0$ in the four dimensions, and the Pascal figure arises therefrom by the successive processes of section and projection. If the projection is done first, from a point on the cubic primal, we get a configuration of sixteen planes, which turn out to be the sixteen singular planes of a Kummer quartic surface.

This connecting up of different geometrical configurations, the clue to which lies in a common algebraic basis, combined with a readiness to work in as many dimensions as are necessary, is nowadays, of course, a standard method of investigation (see, for example, H. F. Baker, *Principles of Geometry*, vol. 4, *passim*); but Richmond was a pioneer in the field, and it is largely owing to him and his pupils that so great an advance has been made.

Many years later Richmond returned to Pascal's theorem (**56**, **57**), this time to investigate analogous properties of sets of $2n+2$ points on a rational normal curve of order n in n dimensions, making use of a generalization of a theorem given by Paul Serret in 1869 on the algebraic conditions for a set of points to be associated, in the sense that every curve of order s passing through all but one must pass through the remaining point.

Another matter which engaged Richmond's attention was the problem of "canonical forms" for the equation of a locus. He published only one paper (**19**) on the subject, but he dealt with it at some length in his lectures, and it was through his inspiration that Wakeford (*Proc. London Math. Soc.* (2), 18 (1920), 403) wrote one of his most brilliant papers.

Other investigations in algebraic geometry are listed in the Bibliography; mention can only be made here of a systematic account (**14**), with an interesting historical introduction, of the rational quartic curve in three dimensions, of which, unfortunately, only a first instalment appeared, and of his one page note (**30**) connected with the problem of the rationality or otherwise of the general cubic primal in four dimensions, a problem which does not appear to have been completely cleared up, even now.

In the more general birational theory of curves, developed so much by the Italian geometers, Richmond was less at home. He knew, of course, the literature of the function-theoretic treatment of the subject, and the geometrical interpretations which Noether had given by the introduction of his canonical curves, but his only papers on the matter (**23**, **27**) deal with Humbert's application of Fuchsian functions to the theory of algebraic curves; they are interesting, but they do not seem to lead anywhere in particular. Towards the end of his life, Richmond followed with pleasure the work of W. P. Milne and others on particular cases of contact primes of canonical curves, and published interesting notes (**51**, **55**) on the curves of genera 4 and 5; he also, in collaboration with his pupil Dr. F. Bath (**52**, **53**), investigated a new notation for the contact primes in the general case, in connection with the characteristics of multiple theta functions.

Early in his career, Richmond had paid some attention to differential geometry, in particular to minimal surfaces. He had noticed that, if the plane $lx+my+nz=p$ touches such a surface, then p, as a function of the independent variables l, m, n, is a solution, of the first degree in the variables, of Laplace's equation. This equation, being linear, is much simpler to deal with than the ordinary Lagrange equation for minimal surfaces, in terms of their point equation $z=f(x, y)$; and methods of solution have, of course, been much studied in various other connections. Richmond was thus able (**12**, **13**) to obtain many of the results given in Darboux's treatise, in a more systematic and a simpler way; in particular, his familiarity with the technique of algebraic coordinate geometry led him to take up the study of algebraic minimal surfaces, initiated by Sophus Lie, and incidentally to show that one of the surfaces discovered by Lie is non-existent. But Richmond appears to have abandoned this line of research; only quite late in his life did he return to the subject (73) in a suggestive note which points out the need for a synthesis of the rather scrappy and scattered literature.

A whole group of papers (**54, 60, 61, 63, 65, 66**) are concerned with chains of theorems, of which the best known, due to Clifford, may be stated as follows: Any three lines in a plane determine a circle (the circumcircle of the triangle formed by them); the four circles so determined from the threes of four lines have a point in common; the five points so arising from the fours of five lines lie on a circle; and so on indefinitely, circles and points arising alternately. Another such chain was enunciated (incompletely) by Frank Morley; and it was perhaps the study of Morley's paper, in connection with the obituary which Richmond wrote (**62**), that aroused, or at any rate revived, his interest in the matter. It is characteristic of Richmond that he was not content merely to refer the reader to the bibliographical note in Coolidge's *Treatise on the circle and the sphere*; he looked up the references, which needed correction, and added a valuable historical paragraph on the share of de Longchamps and Pesci in the discovery.

The same careful verifying of references and presentation of the historical development of a subject is a valuable feature of Richmond's contributions to number theory. His first paper on the subject (**32**) deals with solutions of the equation $t^3+x^3+y^3+z^3=0$ in integers, positive or negative. As one would expect, he gives the business a geometrical twist; among the straight lines on the cubic surface represented by the above equation in homogeneous coordinates is a pair whose equations are $t+\omega x=0$, $z+\omega y=0$; $t+\omega^2 x=0$, $z+\omega^2 y=0$, where ω is one of the complex cube roots of unity; a transversal of these lines meets the surface again in a point whose coordinates can be expressed rationally in terms of two independent parameters. Thus rational, and therefore integral, solutions of the Diophantine equation can be obtained; in this way the well-known Euler formulae, as modified by Binet, can be found. They involve quartic polynomials in the parameters; it was now an obvious step, though the details require considerable skill, to adapt Clebsch's different one-one representation of a cubic surface on a plane to give another solution of the equation, involving only cubic polynomials. Also the join of two "rational" points of the surface meets it again in a third "rational" point, and thus, since any permutation of a solution gives a solution, we have a method of deriving new solutions from a known one. Combining these various remarks, Richmond was able to simplify considerably the labour of finding solutions, avoiding, as far as possible, the duplication of solutions from different values of the parameters; he gave a table of solutions, in which the integers are all less than 100.

Other papers (**35**, **37**, **48**, **67**) deal with Ryley's formulae for expressing any rational number as the sum of cubes of three rational numbers; here again Richmond was able to give a geometrical explanation of the origin of the formulae and to obtain a two-parameter solution. Of recent years the subject of Diophantine equations of this type has been much developed by L. J. Mordell, B. Segre and R. F. Whitehead; and, in particular, B. Segre has carried out in great detail a deep study of the rational points on algebraic loci. Richmond followed this work with great interest, and his short notes (**70**, **72**), published within a few years of his death, show that his mind was still active.

This account of Richmond's published papers scarcely gives the full flavour of the man, or of the inspiring nature of his lectures and conversation. Personal contact with him was needed to savour the true depth of his abilities. Perhaps none of Richmond's published work goes very deep. As he wrote in his short autobiographical note, quoted above, he was content for the most part to apply standard methods. His most pioneering contribution was the systematic use of n-dimensional projective geometry before it became fashionable in this country (though of course Cayley led the way). Richmond was without that urge for abstractness and the widest possible generality which is characteristic of much modern work, and which make some of that work almost unreadable. For that is the claim we make on behalf of Richmond: he can be read with pleasure, as well as with profit. And part of that arises from his interest in, and careful attention to, the historical origin of a theorem, its physical setting, as it were.

In the war of 1914-1918, Richmond (in 1916) joined the Anti-Aircraft Experimental Section of the Munitions Inventions Department, Ministry of Munitions, under A. V. Hill as Director and R. H. Fowler as Assistant Director. His joining this party he attributed to Sir Joseph Barcroft's mentioning him to A. V. Hill as a possible member—Barcroft and Hill both being Fellows of King's. Richmond first joined the party at the N.P.L., at Teddington, and later moved with it to the R.N. Gunnery School at H.M.S. *Excellent*, Whale Island, Portsmouth. Here Richmond worked until 1919, taking his share of much routine computing as well as in actual mathematical research in ballistics. In addition to this indoor work he shared in the observing of high-angle trials of anti-aircraft shells and fuzes, cycling out to Eastney or Hayling Island the long journey from Whale Island. Probably the younger members of the Section failed to recognize the proportionally greater effort required for this from a man well over fifty, but Richmond never sought the easier tasks on the score of his age. As he has left on record, it was co-operation rather than individual effort that counted in that work, but Richmond was apt to underestimate the influence which his example, of faithful sharing in many hum-drum tasks, had on many of the younger less patient members. They had before them the spectacle of a highly distinguished mathematician, an F.R.S., willing to take a hand in any job that came along, often enough thoughtlessly labelled "urgent" and requiring long spells of effort.

In addition to the routines of computation in ballistics, Richmond with one of the writers of this notice (E. A. M.) wrote a memoir entitled "The equivalent constant wind" which showed in pioneer fashion how to calculate the effects on high-angle trajectories of winds whose speed and direction varied with height. In particular they evolved the formula (a definite integral involving the horizontal component of the shell's velocity) for the first-order effects of a *cross-wind* variable with height, a formula which had not been previously isolated.

Again, Richmond took part, with R. H. Fowler, C. N. H. Lock and others, in some notable experiments on spinning shells which were carried out in 1919 after the war proper had ended. The results of these experiments, together with the analysis of aero-dynamic forces they led to, were afterwards published in the *Phil. Trans.* in a memoir (**33**) communicated to the Society by Richmond, and this block of work became classical. The methods then pioneered have now been worked into routine gunfire trials, and had a considerable influence on the course of ballistic research even in the war of 1939-1945. Richmond attended all the firings, and incurred as a consequence a certain measure of deafness Both his ears suffered concussion, one badly, and the increasing deafness to which this led became somewhat of a burden to him in later years. He found he could not follow proceedings on committees, and in consequence, in 1923 or so, he resigned from the Council of the London Mathematical Society and other Committees.

Richmond was accustomed to look back with pleasure and satisfaction on his association with A. V. Hill, R. H. Fowler and others in this period of 1916-1919. It was then that one of us (E. A. M.) first got to know Richmond well (sharing lodgings with him in Portsmouth for a time), and to appreciate his sterling qualities. Amongst the members of the anti-aircraft section who should have been mentioned in E. A. M.'s obituary notice* of R. H. Fowler was one with whom Richmond always maintained a firm friendship

* See *Obituary Notices of the Royal Society*, 1945, 5, 61.

—George Green, actuary to the Commercial Insurance Company, who in 1917-1919 was invalided out of the army in consequence of wounds incurred in the Battle of the Somme and who later joined A. V. Hill. It was Richmond's wish that Green's name should not be omitted in any notice of Richmond's war-work.

After the dispersal of the Anti-Aircraft Experimental Section in 1919, the work of editing the collected researches of the Section in the form of the confidential *Text-Book of A.A. Gunnery* (Vol. I, 1925; vol. II, 1924) was entrusted to Richmond, and an office under his charge was set up in Cambridge for the purpose. Richmond had the arduous task of harmonizing a great many contributions by different writers, but he carried it through to what has proved to be a very successful conclusion; the *Text-Book* was much used in the war of 1939-1945. Only those who knew at first hand the details of the work could appreciate the self-sacrificing labours which Richmond devoted to this job, at a time when he might reasonably have been enjoying well-earned leisure for his own geometrical researches. He delivered his Presidential address to the London Mathematical Society on November 9, 1922, under the title "The mathematical problems of shell-flight".

Bibliography

(1) 1889. "A symmetrical system of equations of the lines on a cubic surface which has a conical point", *Quart. J. of Math.* 23, 170–179.

(2) 1891. "On Pascal's hexagram", *Trans. Cambridge Phil. Soc.* 15 (1894), 267–302.

(3) 1892. "Note on the sum of functions of quantities which are in arithmetical progression", *Messenger of Math.* 21, 29–34.

(4) 1892. "The sum of the cubes of the coefficients in $(1-x)^{2n}$", *Messenger of Math.* 21, 77–78.

(5) 1892. "Cuspidal quartics", *Quart. J. of Math.* 26, 5–26.

(6) 1893. "A construction for a regular polygon of seventeen sides", *Quart. J of Math.* 26, 206–207.

(7) 1899. "On the condition that five straight lines situated in a space of four dimensions should lie on a quadric", *Proc. Cambridge Phil. Soc.* 10 (1900), 210–212.

(8) 1899. "The figure formed from six points in space of four dimensions", *Math. Annalen* 53, 161–176.

(9) 1899. "Note on the invariants of a binary sextic", *Quart. J. of Math.* 31, 57–59.

(10) 1899. "On the figure of six points in space of four dimensions", *Quart. J. of Math.* 31, 125–160.

(11) 1899. "On the expansions in powers of arc of the coordinates of points on a curve in Euclidean space of many dimensions", *Quart. J. of Math.* 31, 315–320.

(12) 1900. "On minimal surfaces", *Trans. Cambridge Phil. Soc.* 18 (1900), 324–332.

(13) 1900. "On the simplest algebraic minimal curves, and the derived real minimal surfaces", *Trans. Cambridge Phil. Soc.* 19 (1904), 69–82.

(14) 1900. "Rational space-curves of the fourth order", *Trans. Cambridge Phil. Soc.* 19 (1904), 132–150.

(15) 1900. "Curves without double points", *Nature,* 63, 58.

(16 1900. "On the inflexions of a binodal quartic curve", *Quart. J. of Math.* 32, 63–65.

(17) 1900. "On extensions of the property of the orthocentre", *Quart. J. of Math.* 32, 251–256.
(18) 1901. "Note on the inflexions of curves with double points", *Proc. London Math. Soc.* (1), 33, 218–226.
(19) 1902. "On canonical forms", *Quart. J. of Math.* 33, 331–340.
(20) 1902. "Concerning the locus $\Sigma x_r^3 = 0$; $\Sigma x_r = 0$; $(r = 1, 2, 3, 4, 5, 6)$", *Quart. J. of Math.* 34, 117–154.
(21) 1902. "The Hessian in covariant geometry", *Quart. J. of Math.* 34, 154.
(22) 1902. "The volume of a tetrahedron in elliptic space", *Quart. J. of Math.* 34, 175–177.
(23) 1903. "On automorphic functions and the general theory of algebraic curves", *Proc. Cambridge Phil. Soc.* 12, 78–81.
(24) 1903. (With T. STUART.) "The inflexion conic of a trinodal quartic curve", *Proc. London Math. Soc.* (2), 1, 129–131.
(25) 1906. "On the reduction of the general ternary quintic to Hilbert's canonical form", *Proc. Cambridge Phil. Soc.* 13, 296–297.
(26) 1908. "On the property of a double-six of lines, and its meaning in hypergeometry", *Proc. Cambridge Phil. Soc.* 14, 475–477.
(27) 1909. "On automorphic functions in relation to the general theory of algebraic curves", *Quart. J. of Math.* 40, 258–274.
(28) 1909. "To construct a regular polygon of 17 sides", *Math. Annalen* 67, 458–461.
(30) 1909. "On the parametric representation of the coordinates of points on a cubic surface in space of four dimensions", *Proc. Cambridge Phil. Soc.* 15, 116.
(31) 1919. "A geometrical proof of Professor Morley's extension of Feuerbach's theorem", *Proc. Edinburgh Math. Soc.* 38 (1920), 2–5.
(32) 1920. "On integers which satisfy the equation $t^3 \pm x^3 \pm y^3 \pm z^3 = 0$", *Trans. Cambridge Phil. Soc.* 22 (1923), 389–403.
(33) 1921. (With R. H. FOWLER, E. G. GALLOP and C. N. H. LOCK.) "The aerodynamics of a spinning shell", *Phil. Trans. A*, 221, 295–387. [Abstract: *Proc. Royal Soc. A*, 98, 199.]
(34) 1922. "A note on apolarity", *Proc. London Math. Soc.* (2), 20, 105–106.
(35) 1922. "Every positive rational number is a sum of cubes of three such numbers", *Messenger of Math.* 51, 171–175.
(36) 1922. "An elementary note upon Waring's problem for cubes, positive and negative", *Messenger of Math.* 51, 177–186.
(37) 1922. "On analogues of Waring's problem for rational numbers", *Proc. London Math. Soc.* (2), 21, 401–409.
(38) 1923. "On an extension of Wallace's pedal property of the circumcircle", *Proc. Cambridge Phil. Soc.* 22 (1925), 34–41.
(39) 1923. "On the real twisted cubic curves which are geodesic upon a quadric surface", *Proc. Cambridge Phil. Soc.* 22 (1925), 42–48.
(40) 1923. "On the electrostatic field of a plane or circular grating formed of thick rounded bars", *Proc. London Math. Soc.* (2), 22, 389–403.
(41) 1923. "Note on the use of the Schwarz-Christoffel transformation in electrostatics (and hydrodynamics)", *Proc. London Math. Soc.* (2), 22, 483–494.
(42) 1924. (With F. BATH.) "On loci which have two systems of generating spaces", *Proc. Cambridge Phil. Soc.* 22 (1925), 319–324.
(43) 1926. "Note upon some properties of the curve of striction", *Proc. Edinburgh Math. Soc.* 44, 90–97.
(44) 1927. "On the meaning of an equation in dual coordinates", *Proc. Edinburgh Math. Soc.* (2), 1, 39–40.
(45) 1927. "On the uninodal quartic curve", *Proc. Edinburgh Math. Soc.* (2), 1, 31–38.
(46) 1929. (With H. LOB.) "On Malfatti's problem", *Proc. London Math. Soc.* (2), 30, 287–304.

(47) 1929. (With H. LOB.) "On a neglected principle in elementary trigonometry", *Proc. London Math. Soc.* (2), 31, 355–369.

(48) 1930. "On rational solutions of $x^3+y^3+z^3 = R$", *Proc. Edinburgh Math. Soc.* (2), 2, 92–100.

(49) 1931. "The tritangent circles of a circular quartic curve", *Proc. Edinburgh Math. Soc.* (2), 3, 46–52.

(50) 1932. "On Cayley's problem of seven lines", *J. London Math. Soc.* 7, 113–117.

(51) 1932. "On the canonical curve of genus five", *Proc. Cambridge Phil. Soc.* 28, 174–189.

(52) 1932. (With F. BATH.) "A notation for the contact primes and contact quadrics of any canonical curve; and for the characteristics of multiple theta functions", *Journal London Math. Soc.* 7, 183–192.

(53) 1932. (With F. BATH.) "A notation for the contact primes and contact quadrics of any canonical curve; and for the characteristics of multiple theta functions", Second paper, *Proc. London Math. Soc.* (2), 38, 49–71.

(54) 1933. "An extension of Morley's chain of theorems on circles", *Proc. Cambridge Phil. Soc.* 29, 165–172.

(55) 1935. "On the tritangent planes of a quadri-cubic curve", *Proc. Edinburgh Math. Soc.* (2), 4, 159–169.

(56) 1936. "On extensions of Pascal's theorem", *Proc. Edinburgh Math. Soc.* (2), 5, 55–62.

(57) 1937. "On extensions of Pascal's theorem (second paper). Paul Serret's theorem", *Proc. Edinburgh Math. Soc.* (2), 5, 155–158.

(58) 1937. "On a certain (4, 2) congruence of lines", *Journal London Math. Soc.* 12, 105–112.

(59) 1937. "A note on the four integer cube theorem", *Journal London Math. Soc.* 12, 206.

(60) 1937. "On certain chains of geometrical theorems", *Quart. J. of Math.* 9 (1938), 34–40.

(61) 1939. "A note on the 'Morley-Pesci-De Longchamps' chain of theorems", *Journal London Math. Soc.* 14, 78–80.

(62) 1939. Frank Morley. (Obituary notice.) *Journal London Math. Soc.* 14, 73–78.

(63) 1939. "An extension of De Longchamps' chain of theorems", *Proc. Edinburgh Math. Soc.* (2), 6, 78–84.

(64) 1940. "A proof of Aronhold's theorem upon quartic curves", *Proc. Edinburgh Math. Soc.* (2), 6, 190–191.

(65) 1941. "On a chain of theorems due to Homersham Cox", *Journal London Math. Soc.* 16, 105–107. Addendum: *Journal London Math. Soc.* 16, 208.

(66) 1941. "A chain of theorems for lines in space", *Journal London Math. Soc.* 16, 108–112.

(67) 1942. "A note upon Professor Mordell's paper (On Ryley's solution of $x^3+y^3+z^3=n$)", *Journal London Math. Soc.* 17, 196–199.

(68) 1944. "Cube root", *Math. Gaz.* 28, 20–21.

(69) 1944. "On certain formulae for numerical approximation", *Journal London Math. Soc.* 19, 31–38.

(70) 1944. "Notes on a problem of the 'Waring' type", *Journal London Math. Soc.* 19, 38–41.

(71) 1944. "On the Newton-Raphson method of approximation", *Edinburgh Math. Notes,* 34, 5–8.

(72) 1944. "On the Diophantine equation $F \equiv ax^4+by^4+cz^4+dw^4=0$, the product $abcd$ being a square number", *Journal London Math. Soc.* 19, 193–194.

(73) 1944. "On minimal surfaces", *Journal London Math. Soc.* 19, 229–241.

(74) 1947. "On isoptic families of curves", *Edinburgh Math. Notes* 36, 18–21.

(75) 1948. "A note upon an arithmetical property of quartic surfaces", *Journal London Math. Soc.* 23, 6–8.