# FREDERIC RIESZ

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In quick succession death has taken, in recent years, a heavy toll of the great and internationally recognized masters who, like bright stars, illuminated the mathematical sky of the first half of this century. Among them, Frederic Riesz, the great Hungarian mathematician, who died, 76 years old, on 28th February in Budapest, is perhaps the one whose name says most to the younger generation of analysts since he was foremost amongst the founders of Functional Analysis.

His life was long and full of success. Frigyes (Frederic) Riesz was born on 22nd January, 1880, at Györ (Hungary). His father Ignácz was a physician, and his mother was Szidónia *née* Nagel. His brother, Marcell Riesz, stands in his own right as a great mathematician. F. Riesz studied at Budapest, Göttingen, and Zürich. In Budapest, in 1902, he obtained the degree of Dr. phil. with a geometrical thesis [1]*. After two years of teaching at grammar schools he was appointed, in 1911, as Professor at the University of Kolozsvár which, in 1920, was moved to Szeged. Here—with A. Haar—he founded, in 1922, the János Bolyai Mathematical Institute and its famous journal, the *Acta Scientiarum Mathematicarum*. In 1946 he was called to the University of Budapest.

Recognition of his outstanding contributions to Mathematics did not fail. He was a member of the Hungarian Academy of Sciences and was awarded, in 1949, its Kossuth Prize. He was also a member of the Royal Physiographic Society in Lund (Sweden) and a corresponding member of the Paris Academy of Sciences; and he was an Honorary Doctor of the Universities Szeged, Budapest, and Paris. The London Mathematical Society is proud to have had him as a member since 1922 and to have published several of his papers†.

The mathematical productivity of F. Riesz was fairly large and always distinguished by its importance. We must not go here into details of his work and should rather point out its main trends and outstanding achievements. It concerns, almost always, real variable theory, usually in connection with Lebesgue measure and integral. His first famous result, in 1907, was the proof of what is now known as the Riesz-Fischer theorem [15] which is fundamental in the Fourier Analysis of Hilbert space. Then follow, from 1910 on, his pioneering researches in Functional

* The numbers refer to the (probably) complete list of publications at the end of this obituary. It is reprinted from *Mathematikai Lapok V* (Budapest, 1954), 260–266.

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Analysis which, I think, must be considered as his most outstanding achievement. He studied, in particular, the spaces of continuous functions [20] and the Lebesgue spaces $L^p$ [23], determined the form of the continuous linear functionals on them, established relationships between strong and weak convergence, and applied his results in a new fashion to moment problems and to linear equations in an infinity of unknowns. Concerning the latter he wrote a well-known account in the Collection Borel [27]. It follows the study of completely continuous operators [35] with applications to linear functional equations, in particular to integral equations, and their eigenvalues. Here he opens up, and leads deeply into, problems of much current interest. In this connection his several important contributions to the theory of Hilbert space ([65], [73], [77], [88]) must be mentioned.

The second great achievement of F. Riesz is the theory of subharmonic functions ([49], [54], [66]) which is his own creation and to which he contributed many of the first important results and applications. Connected with the name of F. Riesz is also the " constructive " approach to the Lebesgue integral ([40], [91]) which requires only the notion of measure zero and leads, starting from " simple functions ", by a completion process rapidly to the integral and its fundamental features. This approach is much preferred by modern writers to the slower measure-theoretic approach of Lebesgue himself and the geometrical one of Carathéodory. Here we should mention also his famous elegant proof, in 1931, of Lebesgue's theorem on the derivation of monotone functions ([69], [70]), his equally simple proof of Egoroff's theorem [58], and his elementary proof of the mean ergodic theorem ([80], [85]). He also contributed much to the theory of Fourier series, and there is his famous joint paper, with his brother Marcell, on the boundary values of analytical functions [37], together with several other important contributions to complex variable theory.

The work of F. Riesz is not only distinguished by the genuine importance of his results, but also by his aesthetic discernment in mathematical taste and diction. It is nowadays, as in other forms of art—and Pure Mathematics is an art rather than a Science—fashionable to be severely abstract to the extent of killing all artistic pleasure. The more leisurely mastership of F. Riesz's style, whether he writes in his native Hungarian, or in French or German, conveys such pleasure and is to the older mathematician a nostalgic reminder of what we are in danger to lose. For him there was no mere abstraction for the sake of a structure theory, and he was always turning back to the applications in some more concrete and substantial situation. We are lucky, indeed, that he gave us, in 1952, his masterly book *Leçons d'Analyse Fonctionelle* [96] which he wrote jointly with his outstanding pupil B. Sz.-Nagy. Here, in the first half written by himself, we find the old master picturing to us Real Analysis

as he saw it, lovingly, leisurely, and with the discerning eye of an artist. This book, I have no doubt, will remain a classic in the treasure house of mathematical literature. With it, and with all his other work, will live the memory of Frederic Riesz as a great and fertile mathematician for long in the history of our art.

## *List of Publications.*


1. " A negyedrendű elsőfajú térgörbén lévő pontkonfigurációk helyzetgeometriai tárgyalása ", Doktori értekezés, *Math. és Phys. Lapok*, 11 (1902), 293–309; 13 (1904), 191–204.
2. " Über einen Satz der Analysis Situs ", *Math. Annalen*, 59 (1904), 409–415.
3. " Sur la résolution approchée de certaines congruences ", *Comptes Rendus* (*Paris*), 139 (1904), 459–462.
4. Über mehrfache Ordnungstypen I ", *Math. Annalen*, 61 (1905), 406–421.
5. " Az analysis situsnak egy tételéről ", *Math. és Phys. Lapok*, 14 (1905), 13–24.
6. " Sur un théorème de M. Borel ", *Comptes Rendus* (*Paris*), 140 (1905), 224–226.
7. " Sur les ensembles discontinus ", *Comptes Rendus* (*Paris*), 141 (1905), 650–653.
8. " A térfogalom genesise ", *Math. és Phys. Lapok*, 15 (1906), 97–122; 16 (1907) 223–235.
9. " Sur les ensembles de fonctions ", *Comptes Rendus* (*Paris*), 143 (1906), 738–741.
10. " Új módszer a térbeli alakzatok ábrázolására ", *Math. és Phys. Lapok*, 15 (1906), 280–291; 16 (1907), 223–235.
11. " Die Genesis des Raumbegriffes ", *Math. u. Naturwiss. Berichte aus Ungarn*, 24 (1907), 309–353.
12. " Sur les systèmes orthogonaux de fonctions ", *Comptes Rendus* (*Paris*), 144 (1907), 615–619.
13. " Sur les systèmes orthogonaux de fonctions et l'équation de Fredholm ", *Comptes Rendus* (*Paris*), 144 (1907), 734–736.
14. " Sur une espèce de géométrie analytique des systèmes de fonctions sommables ", *Comptes Rendus* (*Paris*), 144 (1907), 1409–1411.
15. " Über orthogonale Funktionensysteme ", *Nachrichten Göttingen*, 1907, 116–122.
16. " Über die Approximation einer Funktion durch Polynome ", *Jahresbericht der deutschen Math.-Vereinigung*, 17 (1908), 196–211.
17. " Stetigkeitsbegriff und abstrakte Mengenlehre ", *Atti del VI congresso internazionale dei mat. Roma*, 2 (1908), 18–24.
18. " A lineár homogén integrálegyenletről ", *Math. és Term.-tud. Ért.*, 27 (1909), 220–240.
19. " Sur les suites de fonctions mesurables ", *Comptes Rendus* (*Paris*), 148 (1909), 1303–1305.
20. " Sur les opérations fonctionnelles linéaires ", *Comptes Rendus* (*Paris*), 149 (1909), 974–977.
21. " Sur certains systèmes d'équations fonctionnelles et l'approximation des fonctions continues ", *Comptes Rendus* (*Paris*), 150 (1910), 674–677.
22. " Integrálható függvények sorozatai ", *Math. és Phys. Lapok*, 19 (1910), 165–182 és 228–243.
23. " Untersuchungen über Systeme integrierbarer Funktionen ", *Math. Annalen*, 69 (1910), 449–497.
24. " Über quadratische Formen von unendlich vielen Veränderlichen ", *Nachrichten Göttingen*, 1910, 190–195.
25. " Sur certains systèmes singuliers d'équations intégrales ", *Annales de l'École Normale Supérieure*, 28 (1911), 53–62.
26. " Sur quelques points de la théorie des fonctions sommables ", *Comptes Rendus* (*Paris*), 154 (1912), 641–643.
27. *Les systèmes d'équations linéaires à une infinité d'inconnues* ", VI+182, Paris, 1913.
28. " Démonstration nouvelle d'une théorème concernant les opérations fonctionnelles linéaires ", *Annales de l'École Normale Supérieure*, 31 (1914), 9–14.

29. " Sur les polynomes trigonométriques ", *Comptes Rendus (Paris)*, 158 (1914), 1657–1661.
30. " Über ein Problem des Herrn Carathéodory ", *Journal für Math.*, 146 (1916), 83–87.
31. " Über die Randwerte einer analytischen Funktion ", Riesz Marcellel együtt, *Comptes rendus du 4 congrès des math. scand., Stockholm* (1916), 27–44.
32. " Lineáris függvényegyenletekről ", *Math. és Term.-tud, Ért.*, 35 (1917), 544–578.
33. " Megadott tagokkal kezdődő hatványsorokról ", *Math. és Term.-tud, Ért.*, 35 (1917), 605–632.
34. " Végtelen sorozatok integrálásáról ", *Math. és Phys. Lapok*, 26 (1917), 67–73.
35. " Über lineare Funktionalgleichungen ", *Acta Math.*, 41 (1918), 71–98.
36. " Über Integration unendlicher Folgen ", *Jahresbericht der deutschen Math.-Vereinigung*, 26 (1918), 274–278.
37. " Über die Fourierkoeffizienten einer stetigen Funktion von beschränkter Schwankung ", *Math. Zeitschrift*, 2 (1918), 312–315.
38. " Folytonos és korlátos ingadozású függvény Fourier-féle együtthatóiról ", *Math. és Phys. Lapok*, 27 (1918), 67–71.
39. " Über Potenzreihen mit vorgeschriebenen Anfangsgliedern ", *Acta Math.*, 42 (1920), 145–171.
40. " Sur l'intégrale de Lebesgue ", *Acta Math.*, 42 (1920), 191–205.
41. " Folytonos függvényoperációkról ", *Math. és Term.-tud. Ért.*, 37 (1920), 29–36.
42. " Analytikus függvény kerületi értékeiről ", Szegő Gáborral együtt, *Math. és Term.-tud. Ért.*, 38 (1921), 113–127.
43. " Über einige funktionentheoretische Ungleichungen ", Fejér Lipóttal együtt, *Math. Zeitschrift*, 11 (1921), 305–314.
44. " Sur le théorème de M. Egoroff et sur les opérations fonctionnelles linéaires ", *Acta Sc. Math.*, 1 (1922–23), 18–26.
45. " Sur les valeurs moyennes du module des fonctions harmoniques et des fonctions analytiques ", *Acta Sc. Math.*, 1 (1922–23), 27–32.
46. " Sur les suites de fonctions analytiques ", *Acta Sc. Math.*, 1 (1922–23), 88–97.
47. " Über die Randwerte einer analytischen Funktion ", *Math. Zeitschrift*, 18 (1923), 87–95.
48. " Über eine Verallgemeinerung der Parsevalschen Formel ", *Math. Zeitschrift*, 18 (1923), 117–124.
49. " Über subharmonische Funktionen und ihre Rolle in der Funktionentheorie und in der Potentialtheorie ", *Acta Sc. Math.*, 2 (1924–26), 87–100.
50. " Jelentés az 1924, évi Kőnig Gyula jutalomról ", *Math. és Phys. Lapok*, 32 (1925), 1–6.
51. " Elemi módszerek a felsőbb matematikában ", *Math. és Phys. Lapok*, 32 (1925), 112–124.
52. " Über die erste Randwertaufgabe für $\Delta u = 0$ ", Radó Tiborral együtt, *Math. Zeitschrift*, 22 (1925), 41–44.
53. " Sur une inégalité de M. Littlewood dans la théorie des fonctions ", *Proc. London Math. Soc.* (2), 23 (1925), 36–39.
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56. " Sur la convergence en moyenne (I) ", *Acta Sc. Math.*, 4 (1928–29), 58–64.
57. " Su alcune disuguaglianze ", *Bolletino dell'Unione Mat. Ital.*, 7 (1928), 77–79.
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59. " Sur la convergence en moyenne (II) ", *Acta Sc. Math.*, 4 (1928–29), 182–185.
60. " A lineáris függvényoperációk szétbontásáról ", *Mat. és Fiz. Lapok*, 36 (1929), 1–9.
61. " Sur la décomposition des opérations fonctionnelles linéaires ", *Atti del cong. int. dei mat. Bologna*, 1928, 3 (1930), 143–148.
62. " Sur l'approximation des fonctions continues et des fonctions sommables ", *Bulletin Calcutta Math. Soc.*, 20 (1930), 55–58.

**63.** " Sur les valeurs moyennes des fonctions ", *Journal London Math. Soc.*, 5 (1930), 120–121.
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**65.** " Über die linearen Transformationen des komplexen Hilbertschen Raumes ", *Acta Sc. Math.*, 5 (1930–32), 23–34.
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**71.** " Über Sätze von Stone und Bochner ", *Acta Sc. Math.*, 6 (1932–34), 184–198.
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**73.** " Zur Theorie des Hilbertschen Raumes ", *Acta Sc. Math.*, 7 (1934–35), 34–38.
**74.** " A Lebesgue-féle integrálról, mint a differenciálás műveletének megfordításáról ", *Mat. és Fiz. Lapok*, 42 (1935), 1–24.
**75.** " Sur les fonctions des transformations hermitiennes dans l'espace de Hilbert ", *Acta Sc. Math.*, 7, 147–159.
**76.** " Sur l'intégrale de Lebesgue comme l'opération inverse de la dérivation ", *Annali di Pisa*, II, 5 (1936), 191–212.
**77.** " The integral representation of unbounded self-adjoint transformations in Hilbert space ", E. R. Lorch-csal, *Trans. American Math. Soc.*, 39 (1936), 331–340.
**78.** " Eine Ungleichung für harmonische Funktionen ", *Monatschefte für Math. u. Phys.*, 43 (1936), 401–406.
**79.** " A lineáris operációk általános elméletének néhány alapveto fogalomalkotásáról ", *Math. Term.-tud. Ért.*, 56 (1937), 1–46.
**80.** " Some mean ergodic theorems ", *Journal London Math. Soc.*, 13 (1938), 274–278.
**81.** " A Jordan-féle görbetételről ", *Math. Term.-tud. Ért.*, 57 (1938), 477–482.
**82.** " Sur le théorème de Jordan ", *Acta Sc. Math.*, 9 (1939), 154–162.
**83.** " Sur quelques notions fondamentales dans la théorie générale des opérations linéaires ", *Annals of Math.*, 41 (1940), 176–206.
**84.** " Sur la théorie ergodique des espaces abstraits ", *Acta Sc. Math.*, 10 (1941), 1–20.
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**88.** " Über Kontraktionen des Hilbertschen Raumes ", Sz.-Nagy Bélával együtt, *Acta Sc. Math.*, 10 (1942), 202–205.
**89.** " Sur la théorie ergodique ", *Commentarii Math. Helv.*, 17 (1944–45), 221–239.
**90.** " On a recent generalization of G. D. Birkhoff's ergodic theorem ", *Acta Sc. Math.*, 11 (1948, 193–200.
**91.** " L'évolution de la notion d'intégrale de Lebesgue ", *Ann. de l'Institut Fourier, Grenoble* (1949), 29–42.
**92.** " Lebesgue integrálfogalmának fejlődése ", *Mat. Lapok*, 1 (1950), 79–90.
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**94.** *Ua. franciául*, uo. 216–224.
**95.** " Sur la représentation des opérations fonctionnelles linéaires par des integrales de Stieltjes ", *Fysiogr. Sällsk. Lund. Förhdl.* (1952), 147–152.
**96.** *Leçons d'Analyse Fonctionnelle* (Szőkefalvi Nagy Bélával együtt)" (Budapest 1952), Akadémiai Kiadó, VIII+448 o., Átdolgozott 2, kiadás 1953, VIII+455 o.; 3, kiadás 1955, VIII+488 o.