HAROLD STANLEY RUSE

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Harold Stanley Ruse, Emeritus Professor of the University of Leeds, died in Leeds General Infirmary on 20 October, 1974, three days after suffering a stroke, from which he did not regain consciousness. He had been a member of the Society since 1929, serving on the Council from 1938 to 1945 and being Vice-President in session 1942–43.

He was the third son of Frederick and Lydia Ruse of Hastings and was born on 12 February, 1905. He frequently returned to Hastings during university vacations to visit members of his family; his sister still lives there.

Educated at Hastings Grammar School and Jesus College, Oxford, he went in 1927 to the University of Edinburgh as Bruce of Grangehill research scholar. From 1928 to 1937 he was a lecturer in mathematics at the same university, spending session 1933–34 as a Rockefeller Research Fellow at Princeton University; he visited Princeton again in 1952–53.

In 1937 he became Professor of Mathematics in University College, Southampton and left there for Leeds in 1946, when he was appointed to the post of Professor of Pure Mathematics. He was Head of the Department of Mathematics in the University of Leeds from 1948 to 1968. On the creation of the School of Mathematics in 1968 he became its first Chairman, remaining in this position and continuing as Head of the Department of Pure Mathematics until his retirement in 1970.

He was elected to a Fellowship of the Royal Society of Edinburgh in 1931 and won the Keith Prize of this Society for the period 1935 to 1937. During session 1935–36 he was President of the Edinburgh Mathematical Society.

Ruse's most productive period as a research mathematician was during the time he spent in Edinburgh. Here he was a member of E. T. Whittaker's lively team of first rate mathematicians. With Whittaker's encouragement, he developed a keen interest in relativity, tensor calculus and the generalisation of problems of mathematical physics to non-Euclidean spaces. Thus he studied partial differential equations in relation to Riemannian spaces and this led to an interest in Riemannian geometry for its own sake: in other words, he became a differential geometer. He developed an elegant method of describing certain aspects of Riemannian spaces in terms of projective geometry and this proved fruitful in illuminating some technically complicated ideas.

However, his researches did not terminate when he left Edinburgh. Despite the responsibilities of his post in Southampton and despite the difficulties of maintaining a department during war-time restrictions and dangers (it must not be forgotten that for several years Southampton was very much a front-line town) he lost neither his creative powers nor his enthusiasm for mathematical research. It was of course impossible to carry on in the style of the Edinburgh days and certainly it was not in the national interest to do this, but when the war ended the opportunities for research returned and the unsolved problems were attacked with renewed vigour.

So it was at the height of his powers that Ruse went north to Leeds to his second appointment as a professor of mathematics. Here he found a small department,

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housed in No. 1 Beechgrove Terrace, a Victorian dwelling-house that may once have been desirable accommodation but now looked less than attractive with its black deposit from years of atmospheric pollution. The department, however, had great potential. Built up over the years by W. P. Milne and S. Brodetsky, it was now poised for a new era, in which one of the main themes was to be expansion. Research was to be given a boost by the return of members of staff from war service and some very good new appointments. C. W. Gilham, who had given long and devoted service to the department, continued as Senior Lecturer; he and Ruse soon established a close working relationship which was to be of great value to mathematics at Leeds. In 1948, Brodetsky, who had been Head of Department, retired and Ruse succeeded him in this position. The new Professor of Applied Mathematics was T. G. Cowling, who occupied this post with distinction until he too retired in 1970.

No department, however distinguished its members, can be successful unless its head is reliable, efficient and understanding. Harold Ruse had all these qualities and more. He was ever thoughtful for others, going out of his way to help students, graduates or members of staff as much as he possibly could. He always showed particular consideration for anyone in trouble, whatever it was and whoever was to blame; there were no exceptions. Mathematics students at the University of Leeds came to like and respect him. Above all he was to them a personal friend, a far cry from the remote, inaccessible and superior being that they might have expected him to be.

Under Ruse's energetic leadership mathematics at Leeds expanded, sometimes at a breath-taking rate. The small department, with an honours school of under 50 students and with research students something of a rarity, increased until it was too big to be a department and had to become a school; there are now nine professors, 160 honours students and 46 postgraduate students. No. 1 Beechgrove Terrace gave way to part of the second floor of the Parkinson building. This in its turn proved to be far from adequate and from 1967 the department, and then the school, occupied an extensive modern building. Particularly rapid developments took place in 1963, when new chairs in both Pure and Applied Mathematics were created, largely at Ruse's instigation. The reputation of Leeds as a centre of excellence for mathematics was considerably enhanced as a result.

An expansion of this magnitude and with such rapidity is not possible without sacrifice on the part of those who are involved. Characteristically Ruse did not spare himself. Because he was always so willing to undertake work of any kind to help others, he found that his services were much in demand, both inside and outside the department. The time available for mathematical research diminished and he had fewer and fewer opportunities to devote his undivided attention to it and to follow all the latest developments. It was his generosity and unselfish nature that brought this about, but, for one whose main work in life was mathematics, the price that had to be paid was a dear one. His multifarious duties and commitments, together with the natural process of growing old, diminished his flow of mathematical research papers, which brought him at times to despair. He became worried when he encountered words in lectures on differential geometry which were unfamiliar to him, but relieved when he found that he was not the only person in this position, especially if it transpired that the mysterious terminology was merely a fancy way of describing some simple and familiar idea. But he was never able to recover the fire of his Edinburgh days. Like Hardy, he seems to have found the process of growing old and losing his creative powers a bitter experience.

Soon after Ruse went to Leeds, his friend and fellow differential goemeter A. G. Walker (who had been a research student in Edinburgh) was appointed to the Chair of Mathematics in the University of Sheffield. Although their only joint publication is the book on Harmonic Spaces (**38**), which they wrote in conjunction with T. J. Willmore, there was much collaboration and exchange of ideas between Ruse and Walker. Together they instituted the Leeds–Sheffield Colloquium, which has evolved through various stages over the years as the number of universities in Yorkshire has grown and has now become the Yorkshire Pure Mathematics Colloquium and the Yorkshire Applied Mathematics Colloquium, taking in the Universities of Bradford, Hull, Leeds, Sheffield and York. Ruse and Walker were amongst those responsible for the inauguration of the British Mathematical Colloquium, which first met in Manchester in September 1949. Both played prominent parts in the organisation of this and were speakers at the meeting. Ruse went to as many meetings of the B.M.C. as he could, he was a regular attender at the International Congress of Mathematicians and he retained his interest in the Edinburgh Mathematical Society through visits to the St. Andrews Colloquium. As late as the summer of 1974 he attended the first Durham symposium, on differential geometry, preceding this by going to a short meeting at Liverpool held to mark the occasion of Walker's retirement. He always believed firmly in the value of meetings of mathematicians for the exchange of results and ideas.

Ruse was an excellent lecturer, who made a genuine effort to convey his subject to his audience. He had a feeling for mathematical elegance which showed up in his research papers as well as his lecture courses. This was particularly in evidence in those papers devoted to the geometrical interpretation of tensors and related objects. In (**12**), the first paper of this type, he applied the methods and notations of spinor algebra to obtain a large number of theorems on conics in a three-dimensional projective space. He followed this with papers (**14, 15**) in which the approach was from a different point of view; he applied the concepts and methods of three-dimensional projective geometry to four-dimensional Riemannian spaces, with particular reference to relativity, where the interpretation of four-dimensional space-time in terms of three-dimensional projective geometry seemed particularly appropriate.

The central idea here is that at any point P of a Riemannian 4-space the tangent space gives rise to a 3-dimensional projective space S_3, in which the homogeneous coordinates of a point correspond to the components of a contravariant vector at P. The tangent space is a centred affine space and S_3 is obtained as the hyperplane at infinity in this space. A change of coordinates in the Riemannian space induces in a natural way a linear transformation in the projective space. A covariant vector at P gives rise to a plane in S_3; if the covariant vector has components ω_i ($i = 1, 2, 3, 4$) then the plane has equation $\omega_i X^i = 0$ (where the summation convention is used). A symmetric second order covariant tensor at P gives rise to a quadric in S_3; if the components of the tensor are a_{ij}, the equation of the quadric is $a_{ij} X^i X^j = 0$. In particular, the fundamental tensor at P gives a non-degenerate quadric; for space-time this corresponds neatly to the null cone. A skew-symmetric second-order covariant tensor with components b_{ij} determines a linear complex in S_3, given by $b_{ij} p^{ij} = 0$, where $p^{ij} = X^i Y^j - X^j Y^i$ are the Plücker coordinates of the line joining the points X^i and Y^i. The curvature tensor, whose components R_{hijk} are skew-symmetric in the suffixes h, i and in the suffixes j, k determines a quadratic complex, given by $R_{hijk} p^{hi} p^{jk} = 0$.

These geometrical ideas seem to have had their origin in relativity (**14**), but perhaps more significant is the way in which they have come to be of importance in relativity itself. The work done by Ruse plays a vital part in the classification of certain types of space-time configurations (see A. Z. Petrov, *Einstein Spaces*, Pergamon Press, 1969). The idea of expressing the curvature tensor of a 4-dimensional space in terms of a 6×6 symmetric matrix, given explicitly by Ruse (see (**22**), p. 8), is important here. Further papers (**18, 19, 28**) also contain material of interest in this connection, even though they are not explicitly on relativity.

Ruse developed his methods in other directions. Whilst the restriction to four-dimensional Riemannian spaces is less appropriate for differential geometry than for relativity, he was nevertheless able to make useful contributions to the theory of harmonic Riemannian spaces and related topics (**18, 19, 20, 21, 22, 28, 30, 31**) where the elegance of his approach was often illuminating. Furthermore the ideas could be applied to spaces of dimension greater than 4. A good example of the insight sometimes achieved through Ruse's methods is to be found in his paper on multivectors and "catalytic" tensors (**24**). This contained a re-examination of E. T. Whittaker's work in spinor-calculus, in which it appeared that spinor-calculus could in certain circumstances be used to construct, by the operations of addition, multiplication and contraction, new tensors which in tensor-calculus could be constructed only by solving tensor equations (actually by a process akin to taking square roots); furthermore spinor-calculus could be used to obtain directly a tensor which could not be obtained by tensor analysis except through the use of an unrelated tensor, which played a "catalytic" rôle: that is, its significance in the problem resembled the part played by a catalyst in a chemical reaction, in that it was necessary to the process but disappeared in the final result. However, Ruse was able to show that spinor-calculus was not after all necessary for the constructions described by Whittaker and that Whittaker's catalyst could in fact be omitted from consideration if the problem were approached differently. It was the geometrical methods that brought this to light, and incidentally it showed that the methods could be used to discover tensor formulae involving covariant derivatives.

Ruse's earliest papers were essentially on the theme of extending parts of classical analysis to the realm of general relativity. Some of the papers (for example (**1, 3**)) are basically on pure mathematics, whilst others have a strong flavour of mathematical physics. However there was usually much more mathematics than physics. Thus the paper "Gauss' theorem in general space-time" (**13**) generalised the work of Whittaker, who had extended the classical theorem of Gauss to the case of a static gravitational field, but Ruse admitted that his generalisation may not be of physical significance.

Laplace's equation and other partial differential equations feature prominently in Ruse's work (**2, 4, 5, 11, 16, 17, 36**). Indeed his last paper but one was on Laplace's equation in a simply harmonic manifold and it was out of one of his earliest works (**4**) that the concept of harmonic spaces arose. The theory of these spaces is one of the most significant of British contributions to local differential geometry and Ruse played a major part in its development.

The origin was Ruse's attempt, in 1930, to generalise the "elementary" solution of Laplace's equation

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}=0$$

to a general Riemannian space. For the classical form of the equation quoted above, the elementary solution is $V = 1/\sqrt{(2\Omega)}$, where

$$\Omega = \tfrac{1}{2}\{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2\}$$

and it is valid in any neighbourhood not containing the point (x_0, y_0, z_0). The general form for Laplace's equation in a Riemannian space, in which the fundamental tensor has components g_{ij} and the corresponding contravariant tensor has components g^{ij}, is

$$\Delta_2 V \equiv g^{ij}\left(\frac{\partial^2 V}{\partial x^i \partial x^j} - \Gamma^k{}_{ij}\frac{\partial V}{\partial x^k}\right) = 0,$$

where the summation convention applies and $\Gamma^k{}_{ij}$ are the Christoffel symbols formed from the g_{ij}. Ruse appeared to have determined the elementary solution in the form

$$V = A\int_a^s \frac{J}{\sqrt{g}\sqrt{g_0}}\frac{ds}{s^{n-1}},$$

where s is the geodesic distance from the point P_0 of coordinates $(x_0{}^i)$ to the point (x^i), g is the determinant of the components g_{ij}, g_0 is the value of g at P and J is the determinant of the $n \times n$ matrix whose $(i, j)^{th}$ element is

$$\frac{\partial^2(\frac{1}{2}s^2)}{\partial x^i \partial x_0{}^j}.$$

However, there was a flaw, namely the implicit assumption that the integrand is a function of s alone. This is so for a space of constant curvature, but is not necessarily true; Ruse's only joint paper (**16**), in which the co-author was his former Edinburgh colleague E. T. Copson, draws attention to the error and explains it in detail (a necessary process, for the error is not immediately obvious). Copson and Ruse described a space as "centrally harmonic" with respect to a point P_0 if Laplace's equation $\Delta_2 V = 0$ has a solution depending only upon the geodesic distance from P_0, and "completely harmonic" if the space is centrally harmonic with respect to each point. They proved some results on centrally harmonic spaces, showing that every Schur space is centrally harmonic with respect to the origin but not every centrally harmonic space is a Schur space. However, little seems to have been done on spaces which are just centrally harmonic; the main focus of attention has been on "completely" harmonic spaces. Copson and Ruse observed that a space of constant curvature is completely harmonic. They derived a sequence of conditions on the curvature tensor of a centrally harmonic space, the first of which shows that a completely harmonic space is an Einstein space; the sequence is not only infinite but becomes increasingly complicated, so it is hard to decide whether the conditions imply that the space is essentially of constant curvature. The matter was left by Copson and Ruse as a reasonable conjecture but an unanswered question.

The concept of a harmonic space and the conjecture attracted some attention; it was clear that the problem was a nontrivial one. Associated with it was the problem of simply harmonic spaces (see A. G. Walker, "Note on a distance invariant and the calculation of Ruse's invariant", *Proc. Edinburgh Math. Soc.* (2), 7 (1942), 16–26).

The most convenient way to define these is to use Ruse's invariant ρ, defined by

$$\rho = \frac{\sqrt{g}\sqrt{g_0}}{J}$$

and featuring in the paper (**4**) on the elementary solution of Laplace's equation. For a completely harmonic space, ρ is a function of s alone; a simply harmonic space is one for which $\rho = 1$ for every pair of points. At first it seemed reasonable to conjecture that every simply harmonic space is flat.

It proved to be the case that completely harmonic spaces are not necessarily of constant curvature and that simply harmonic spaces are not necessarily flat. Whilst Ruse himself did not produce the necessary counter-examples, his paper (**20**) suggested strongly that there were grounds for believing that the conjecture might prove to be false.

Another conjecture arose from the work of A. Lichnerowicz (" Sur les espaces riemanniens complètement harmoniques ", *Bull. Soc. Math. France*, 72 (1944), 146–168). This was that completely harmonic spaces are symmetric in Cartan's sense: that is, the covariant derivative of the curvature tensor is zero. Thinking about this, Ruse produced a brand new idea. He noticed that the condition for a space to be symmetric, namely that the components R_{hijk} of the curvature tensor satisfy

$$R_{hijk,p} = 0,$$

where the comma denotes covariant differentiation, could be replaced by

$$R_{hijk,p} = \kappa_p R_{hijk},$$

where κ_p are the components of some covariant vector field, without basically upsetting the algebraic situation: in the case of a metric satisfying either of these conditions, the requirement for the space to be completely harmonic reduces to a finite set of algebraic equations. This of course does not produce examples of completely harmonic spaces, but merely indicates that, from one point of view at least, they are algebraically possible. What Ruse went on to show was that not only is this true, but in fact spaces of the required kind do exist. Such spaces came to be called " spaces of recurrent curvature " and for some time a good deal of interest centred around the idea of recurrence of tensors, both in relation to harmonic spaces and to other considerations, notably the theory of parallel fields of planes (**27**, **29**).

Recurrent spaces are now established in relativity as well as geometry, where they have some significant applications. It is interesting to note how they arose in differential geometry from a problem that had its roots in relativity and have turned out to be of importance in relativity itself. In the same way, Ruse's geometrical ideas were born in relativity and, having evolved outside it, returned with a new significance. One would not, however, regard differential geometry as a catalyst for relativity!

Full details of the theory of harmonic spaces may be obtained from the book (**38**) which Ruse wrote jointly with A. G. Walker and T. J. Willmore; it is worth noting that there are other types of spaces which are harmonic. Ruse's work opened up fascinating and challenging new areas of differential geometry, which attracted the attention of many mathematicians, including A. Lichnerowicz, A. G. Walker, T. J. Willmore, A. J. Ledger and A. C. Allamigeon and other colleagues of Professor Lichnerowicz in Paris. Interest became especially keen once it had been established that harmonic spaces are non-trivial. By no means all the problems concerning such spaces have been solved, even in the positive definite case.

Ruse's many friends were greatly saddened by his sudden and unexpected death. At the meeting in Durham in July he had seemed well enough and in some ways was more relaxed than he had been at one time. He was glad to meet old friends, including several who had worked on harmonic spaces. He had had an operation for cataract some two or three years earlier, but he still kept up his swimming, which was an exercise he always enjoyed. He kept his day to day interest in mathematics going through his continuing contact with the School of Mathematics at Leeds. The day before he collapsed he attended an algebra seminar in the Department of Pure Mathematics.

He had good friends outside the mathematical world and was well-liked and respected by the Brethren of the Community of the Resurrection at Mirfield. His acquaintance with the Community dated from his early days in Leeds, when he stayed for a time as a guest in the Hostel of the Resurrection. Subsequently be became warden of one of the University's halls of residence, Woodsley Hall, where he showed unfailing tact and courtesy to all involved in the difficult task of running a student community in days of severe rationing. To the students themselves he was the ideal warden, presiding firmly yet with compassion and understanding, so that he obtained their full cooperation and made the Hall a happy one. He relinquished this duty when it became abundantly clear that nobody could be expected to cope with all the duties he had acquired.

His association with the Community of the Resurrection continued throughout and ultimately it came to play a big part in his life. He was a regular attender at the Anglican Chaplaincy, he was appointed to the Anglican Chaplaincy Committee of the University of Leeds, and was also a member of the Parochial Church Council and the Deanery Synod.

My thanks are due to Mrs. M. M. Turner, for many years a priceless asset to Harold Ruse as his efficient and understanding secretary, to Professor A. G. Walker and to Professor J. V. Armitage, for help in preparing this notice.

List of publications: *H. S. Ruse*

Papers

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2. " The potential of an electron in a space-time of constant curvature ", *Quart. J. Math. Oxford Ser.*, 1 (1930), 146–155.
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5. " Generalised solutions of Laplace's equation ", *Proc. Edinburgh Math. Soc.* (2), 2 (1930–31), 181–188.
6. " Note on refraction and reflection in general relativity ", *Atti pontif. Accad. Sc.*, 84 (1931), 662–672.
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Book

38. *Harmonic Spaces*, jointly with A. G. Walker and T. J. Willmore. (Edizioni Cremonese, Roma, 1962).