GEORGE SALMON

[For this notice the Council is indebted to Sir Robert Ball.]

THE late Provost of Trinity College, Dublin, was the third and the greatest of three consecutive Provosts, each being highly distinguished in science. Humphry Lloyd, the brilliant experimentalist, and Jellett, well known for his optical and mathematical investigations, were the immediate predecessors of him of whom we now write, and who was admitted by universal consent to be one of the most famous mathematicians of the last century.

Had Salmon never done anything else than write his *Treatise on Conic Sections*, he would have richly earned the gratitude of all mathematicians. For more than half a century the book has been the standard authority on the subject. At the time it was written it was immeasurably superior to all other treatises on curves of the second order. Descartes had invented the method of coordinates. Poncelet, in his projective geometry, had developed the most beautiful and instructive branch of the subject, and it will be remembered that Salmon handsomely acknowledged the inspiration that Poncelet had been to him. Chasles had given to us those exquisite geometrical theories by which methods of wonderful power for geometrical investigation were provided. Graves, the late Bishop of Limerick, had contributed his theorem about the arcs of confocal conics, and it may well be doubted whether within the whole range of geometrical beauty there is anything more fascinating. Boole had just commenced three epoch-making investigations concerning invariants, which have originated a vast department of the modern mathematics. Many other illustrious names could, of course, be added, from the times of the ancient Greeks down to the middle of the last century, of the discoverers of the fundamental principles of the conic sections.

It was, however, reserved for the master mind of Salmon to create a great and comprehensive treatise which should be worthy of the subject. To the materials already provided by the labours of others Salmon added largely. He made the Cartesian coordinates flexible as they had never been before. He enormously increased their power as instruments of geometrical research, by the methods of abridged notation. He dovetailed the analytical methods with the methods of pure geometry. He gave to the projective geometry the position which its importance demanded. He showed how the method of reciprocal polars and the methods of projections established recondite theorems as the immediate consequences of elementary principles. In the later editions he gave the applications of invariants and covariants, and thus laid the foundation of the applications of these beautiful conceptions to geometrical investigation. By all this original work he provided the matrix in which the results of the previous investigators were incorporated and worked into a systematic whole, and his great *Treatise on Conic Sections* was the result. The style of this work is admitted to be the model of what a mathematical work should be. The excellence of its arrangement, the abounding wealth of illustrative examples, the philosophical manner in which the reader is led to a knowledge of the broad principles of geometry and algebra, the beauty and interest of the matters treated, the suppression of needless detail, the clear distinction between what is of real importance and what is merely casual—in all these points, it is not too much to say that Salmon's *Conics* is a book of unapproached merit. It superseded every other book of the kind when it appeared, and now nearly sixty years later its supremacy still remains unchallenged. I believe it may be added that Continental mathematicians, without a dissentient voice, coincide in this estimate of the value of this work. It is not, indeed, that other books on conic sections have not appeared. They have appeared, and I suppose may be reckoned by the score. No doubt they have their uses ; but I believe every one of the authors of these books has been glad to acknowledge his indebtedness to Salmon. Indeed, this great work has been, and still is, the quarry to which for many a long day every one has resorted who wanted to write on conics. There are huge reserves still in that quarry, and at least one author has confessed to me how saddening it has been to him time after time, whenever he thought he had discovered something good in the conics, to have found out that, if it was not wrong, it was already in Salmon.

Salmon used to lecture on mathematics in Trinity College, and the writer of this note enjoyed the inestimable privilege of attending these lectures in 1858–9. He was an admirable teacher. I particularly remember the course on conics. It was presumed that we had read or were reading his book, so the lectures often took the line of showing us the improvements or extensions which he was preparing for future editions. He would kindly encourage his pupils to make their comments, and even honour them by asking for their efforts to aid him in questions which were occupying him at the moment. He used to warn us that the good things in conics were mostly "threshed out," but still there were some leavings. It was, I think, in this way that Mr. Burnside, now Prof. W.

Snow Burnside, of Trinity College, who was also one of Dr. Salmon's pupils, originated some theorems and methods which duly appeared in later editions of the *Conics*. I also attended Salmon's lectures on "Elementary Theoretical Dynamics," and never shall I forget the delight with which we received his illustrations of the use of differential equations in dynamics.

The great success of the *Conics* naturally suggested that the same process which had led to such triumphs in curves of the second degree should be applied to curves of higher degrees. Hence arose Salmon's second book, *The Higher Plane Curves*. It will, I suppose, be admitted that this book, like the *Conics*, has no rival in the same subject. That there are geometrical beauties in the subject of higher plane curves will not be denied. For example, no one can deny beauty to the theorem stating that the four tangents drawn to a cubic from a variable point on the curve have a constant anharmonic ratio. Of course there are innumerable occasions, in practical as well as theoretical mechanics, when the theory of plane curves other than conics becomes of importance. I must, however, add, though I am perhaps expressing only my own opinion, when I say that the *Higher Plane Curves* is the least interesting of Salmon's four great works. It lacks the magic of the *Conics*. It has not the importance in nature possessed by the *Quadrics*, which form so large part of the *Geometry of Three Dimensions*. Nor does it lead to such splendid theories and conceptions as do the lessons in *Modern Higher Algebra*.

The genius of Salmon has, however, lightened up the subject of the higher plane curves as probably no other writer could have done. Here, again, the admirable simplicity of his methods minimizes the difficulties in a most difficult and intricate subject. The present generation are so accustomed to the artifices rendered familiar by Salmon that we are often hardly conscious of all we owe to him. Descartes, no doubt, taught us how to represent the coordinates of a point on a curve, with the help of the rectangular axes; but it was Salmon who saw how to avail himself of the privilege that those axes might be placed anywhere. He sometimes put the origin on the curve and made the tangent one of the axes, and at once a great simplification comes into the equation of the curve. It is Salmon also who has applied and developed his abridged notation and made symbols represent ideas rather than numerical magnitudes. To Salmon also we, in great measure, owe the first great application of the theory of invariants and covariants to the study of cubics and other curves. The subject is truly a terrific one.

Throughout his life Salmon was intimately associated by correspondence with the greatest of contemporary mathematicians. Among these, perhaps, the chief was Cayley, and an indication of this remarkable friendship is given in connexion with the book now under consideration. By the time a second edition of the *Higher Plane Curves* was called for, Salmon had become a Professor of Divinity, and had thus made engagements which, in his own words, "left me no leisure to make acquaintance with recent mathematical discoveries, or even to keep up any memory of what I had previously known." In this emergency he applied to Professor Cayley to obtain advice as to the choice of some younger mathematician, who would undertake the editing of the work. To his agreeable surprise, Professor Cayley offered to do this himself. This offer was gratefully accepted, and the second edition was the joint work of Salmon and Cayley. The opening chapter is by Cayley, and the opening words of that chapter may well be quoted as showing the deep impression which the principles of the projective geometry had by this time made on the two great mathematicians:

"We have in the plane a special line, the line infinity, and on this line two special (imaginary) points, the circular points at infinity. A geometrical theorem has either no relation to the special line and points, and it is then *descriptive;* or it has a relation to them, and it is then *metrical.*"

In the third edition, 1879, Dr. Salmon also availed himself of the aid of Mr. G. L. Cathcart, Fellow of Trinity College, Dublin. Mr. Cathcart has, indeed, for many years mainly undertaken the work of editing all successive issues of Dr. Salmon's mathematical works.

It was in 1862 that Salmon published the great work known as *A Treatise on the Analytic Geometry of Three Dimensions*. Those of us who had to make some acquaintance with ellipsoids and hyperboloids before 1862 will recall the impossibility of obtaining any book on solid geometry which we, who had been reared on Salmon's *Conics*, could regard as quite satisfactory. We had to struggle as well as we could with such books as there were. We had Frost, and we had some French book of which I have forgotten the name. But the appearance of the work which we have been accustomed familiarly to call the "Surfaces," at once gave the student what he so greatly wanted. Here we find the methods adopted with much success in the conics applied to the quadrics. In the chapters on the general theory of surfaces the methods characteristic of Salmon's line of thought are specially to be noted. In the higher parts of the subject we specially remember the striking discovery of the 27 right lines on the surface of the third degree. With reference to this great geometrical advance Salmon remarks in a foot-note: "Cayley first showed that a definite number of right lines must lie on the surface; the determination of that number as above, and the discussions in Art. 458, were supplied by me."

As in the case of the other books, the "Surfaces" has grown in each successive edition. The publication of this book, no doubt, stimulated the study of the subject, and the great developments of the theoretical parts of the subject became incorporated. In the advanced parts of the subject, efforts of the geometrical imagination are called for to which no adequate response can be made with present intelligence. We may be quite satisfied with the demonstration that on the reciprocal of the general surface of the n-th order there will be a nodal curve of the degree $\frac{1}{2}n(n-1)(n-2)(n^3-n^2+n-12)$, but no finite intelligence can give geometrical vividness to such a statement. A large part of the second half of this book, especially in the later editions, is devoted to most elaborate investigations of this kind, which, while they are marvellous expositions of mathematical ingenuity, transcend the powers of human conception almost as much as do the theorems of four dimensional space, and more than the most important investigations of non-Euclidean geometry. And here it may be noted that of four dimensional space or non-Euclidean geometry Salmon has never treated. It is, perhaps, too much to say that he regarded such speculations with scorn, but I have heard him say, jestingly, that he reserved such themes for the next world.

Of course, in the geometrical use of the imaginaries, Salmon greatly delighted. I remember well the zest with which he perplexed us at lectures by showing how a line could be perpendicular to itself, and how on the same line (of course it is $x+\sqrt{-1}\,y=0$) the distance between any two points was zero. But, nevertheless, so far as I remember, he has not in any of his books made use of the complex variable in the way in which it is now of such importance in modern mathematics. It should also be noted that Salmon's methods depended but little upon the differential or integral calculus. The subjects that engrossed his attention, and which he made of such absorbing interest, were essentially treated by geometrical and algebraical processes. When he feels constrained to render some account of a subject like Monge's differential equation relating to curvature, he introduces it with a sort of apology as being only a concession to some presumed desire on the part of the reader. As one of his old pupils, now a distinguished mathematician, and one of Salmon's warmest admirers, once said to me, "Salmon is not a calculus man."

Those who, like myself, are old enough to remember when Salmon's *Lessons introductory to Modern Higher Algebra* first appeared, will not need to be reminded of the impression made by the appearance of that book. It was the first announcement to many mathematicians that an algebra existed which was not the theory of equations. As to the origin of the subject we may quote Salmon's own words: "What I have called modern algebra may be said to have taken its origin from a paper in the *Cambridge Mathematical Journal* for November, 1841, when Dr. Boole established the principles just stated (*i.e.* that the discriminant of a binary quantic is an invariant) and made some important applications of them. Subsequently, Professor Cayley proposed to himself the problem to determine *a priori* what function of the coefficients of a given equation possesses this property of invariance: that, when the equation is linearly transformed, the same function of the new coefficients is equal to the given function multiplied by a quantity independent of the coefficients."

Salmon was primarily attracted to this magnificent theory by its remarkable applications to the theories of conics and other curves, as well as surfaces. He thought at first that he could include in appendices to his geometrical works all that was necessary for his object. But the the interest of the subject grew upon him, and as he made further acquaintance with Cayley's famous series of memoirs on quantics, Salmon decided to prepare his well-known work on *Modern Higher Algebra*, with the dedication: "To A. Cayley, Esq., and J. J. Sylvester, Esq.: I beg to inscribe this attempt to render some of their discoveries better known, in acknowlegdment of the obligations I am under not only to their published writings, but also to their instructive correspondence."

It is in this book that Salmon has undertaken some of the most formidable calculations that any pure mathematician has ever faced. The portentous invariant E alone requires several pages for its expression. He has told us that, when the work was done, he proceeded to check it by testing whether the sum of the coefficients was zero. He was dismayed by finding a balance of several thousands, but, divining a possible error, he halved this balance, and finding this was exactly one of the two coefficients, he immediately surmised a wrong sign, which proved to be the case, and the verification was complete. The tables of symmetric functions and of eliminants, which are given in the appendix, are perhaps more generally useful than the value of E, which has, indeed, not been reprinted in the later editions.

In the latter half of Salmon's career, he was no longer the ardent mathematician. During the first part of this later period he was the distinguished Regius Professor of Divinity, and during the later part he was the great Provost. Books not a few issued from him during the years when he ceased to cultivate mathematics, but they are not books with which the Mathematical Society has concern. To Salmon was accorded the distinction of winning fame as a theologian which was only second to his fame as a mathematician. He used often to say he had forgotten all his mathematics, but till quite recently, though not, I think, quite up to the end of his life, there was one branch of calculation which had a great fascination for him. It was the determination of the number of figures in the recurring periods in the reciprocals of prime numbers. For many years, when he attended a concert (and of music he was extremely fond), or when he was attending a meeting of the Church Synod, he used to spend every spare moment in scribbling figures. Often it would be thought that he could not divide his attention between his figures and the business in hand; but when it came to be his turn to speak (he spoke on such occasions extremely well and to the point) it would be found that he had missed nothing urged by previous speakers. He used often to say that his work at these figures he regarded as frivolous or useless. He even tried to break himself of what he deemed a pernicious habit. "When I get up to 20,000, I will stop," he used to say: and so he did—but then, on having to sit out some unusually dull speech, or on the occasion of a confinement to the home for some slight ailment, his resolution would break down, and he would begin again. I think the last time I heard about it he was approaching the primes near 50,000.

In his earlier years chess was Salmon's principal recreation, and he was a first-rate player. He was one of those chosen to play against Murphy, on the celebrated occasion when Murphy challenged twelve European players simultaneously, and I believe Salmon succeeded in obtaining a draw. His love of music we have already mentioned, and he was a great reader of current literature of all kinds, especially novels. His humour was delightful, and it seasoned his gravest works. He inspired the affection, as well as the respect, of all who had the good fortune of coming into contact with him. In the University of Dublin, Salmon is venerated as the most distinguished Provost who has presided over Trinity College in the three hundred years of its history.