WINIFRED L. C. SARGENT

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Winifred L. C. Sargent, who died in October, 1979, was born on 8th May, 1905, at Ambergate in Derbyshire, the only child of her father's second marriage. Brought up in the Quaker Community at Fritchley close by, her early education was provided by her father and by a small private school for Friends' children. She entered the Friends boarding school at Ackworth (near Pontefract) in 1916 and three years later went on to the Mount School at York with a Joseph Rowntree entrance scholarship.

Since the Mount School was not strong on the mathematical side, she left and worked for her Higher School Certificate as a day girl at Herbert Strutts' School, Belper. The school, although only founded in 1909, had already achieved a high academic reputation, and it was from here that she gained a Derby County Scholarship in 1923 and a State Scholarship and Mary Ewart Scholarship to Newnham College, Cambridge, in 1924.

At Cambridge the dominant mathematical interest was mathematical analysis in its various forms. It was a type of mathematics that appealed to Miss Sargent and for which she had a natural aptitude. An Arthur Hugh Clough Scholarship in 1927 and a 1st Class B* degree were followed by a Mary Ewart travelling scholarship and a Goldsmiths Company Senior Studentship in 1928. Immediately after her first degree Miss Sargent embarked upon research work, but she did not consider her results were adequate and left the University to teach at Bolton High School.

In 1931, Miss Sargent was persuaded to become an Assistant Lecturer at Westfield College, moving to a similar position at Royal Holloway College five years later. In 1939, she became a research student of Professor Bosanquet and, although there was less than one year's detailed supervision (before the war intervened), from that time onwards Professor Bosanquet exercised a decisive influence on her work.

In 1941 she became a lecturer at Royal Holloway, and moved from there to Bedford College in 1948. Six years later she was awarded the degree of Sc.D. by Cambridge University and had the title of Reader conferred on her in respect of the position she had at Bedford College. She retired and cut all her ties with university life in 1967.

By the 1930's the theory and the applications of Lebesgue integration were well established. However, there were many other integrals and derivatives that had been defined for various reasons. The elucidation of the exact relations and properties of these new integrals and derivatives offered a wide and challenging field for research. It was in this field that Dr. Sargent did almost all her work.

[3] is a typical paper. In it the properties of Denjoy–Perron integrable functions are considered. For such a function the Cesàro-derivate $\mathrm{CD}f(x)$ is defined by

$$\mathrm{CD}f(x) = \lim_{h \to 0} \frac{2}{h^2} \int_x^{x+h} (f(t) - f(x))\,dt\,.$$

One result is that if $f(x)$ is C-continuous in $a \leqslant x \leqslant b$ and $\mathrm{CD}f(x)$ exists in

[Bull. London Math. Soc., 13 (1981), 173–176]

$a < x < b$ then for some ξ in $a < \xi < b$ we have

$$\frac{f(b)-f(a)}{b-a} = \mathrm{CD}f(\xi).$$

This, of course, is a direct analogue of Rolle's Theorem, but the techniques used in its proof are entirely different, requiring new and sophisticated arguments. In this paper as in much of Dr. Sargent's work, the arguments are pushed as far as they will go and counter examples given to show that the results are the best possible.

In [4] she developed properties of Burkill's Cesàro–Perron integrals that are analogous to properties of the Perron (or special Denjoy) integral. This arose out of work of Grimshaw and Bosanquet, and led her to the definition of a Cesàro–Denjoy integral equivalent to the Cesàro–Perron. [5] includes a direct and elegant proof of monotonocity of a function under certain conditions that is fundamental to the development of the Cesàro–Perron theory of integration.

[4] contains a mean value theorem which was later extended in [12] to the general case λ for C_λ functions. This last paper also contains a proof that C_λ-continuous functions are Darboux continuous. A second generalisation of [4] appears in [15] which includes a new integral $V_n D$ equivalent to $C_n D$ but based on the generalised derivative of de La Vallée Poussin.

Questions of summability factors leading on to properties of Kernels are given in [7], [13], [16], [17] and [18]. These papers contain numerous important results, but attention must be drawn to the idea of a normed linear space which is a "β-set in itself" a generalisation of being of "2nd Category in itself" which is used to establish a property of transforms of Denjoy integrable functions.

[18] is another paper with many important results, including the following concerning Cauchy–Lebesgue integrals.

A necessary and sufficient condition that $k_s(t)$ should be such that

$$\lim_{s\to\infty}\left(\int_0^\infty x(t)k_s(t)dt\right) = \int_0^\infty x(t)dt$$

whenever $x(t)$ is such that the right hand side exists and is finite, is that

(i) $\exists a(s) \geqslant 0$ for all $s \geqslant 0$ such that $k_s(t)$ is measurable and essentially bounded over $0 \leqslant t < a(s)$ and of bounded variation over $a(s) \leqslant t$,

(ii) $\exists a, s_0, M$ such that for $s > s_0$, $k_s(t)$ is essentially bounded by M over $0 \leqslant t < a$ and has total variation less than M over $a \leqslant t$,

(iii) for $\lambda > 0$, $\displaystyle\lim_{s\to\infty}\int_0^\lambda (1-k_s(t))dt = 0.$

Dr. Sargent also contributed to the theory of fractional integration and differentiation in [11], [14], [19].

The last three papers, [22], [23], [24], are concerned with the properties of BK-spaces—that is spaces of complex sequences in which the mapping from a sequence to one of its terms is continuous. Dr. Sargent established a number of interesting properties both in terms of mappings between BK-spaces and of individual BK-spaces.

For example, if $y \in Y$, a BK-space, and y_n is the sequence formed from the first n terms of y followed by zeros, then a bounded subset B is conditionally compact if and only if $\|y_n\| \to \|y\|$ uniformly on $B+(-B)$.

Dr. Sargent's work is marked by its exceptional lucidity, its exactness of expression and by the decisiveness of her results. She made important contributions to a field in which the complexity of the structure can only be revealed by subtle arguments. Her work was in many ways an expression of her character. Although she was self-effacing and totally lacking in any ambition for self-advancement, yet she was independent and would not tolerate anything which she thought to be second-best. She never attempted to publicise her work, and could only rarely be persuaded to address a seminar—but when she did the result was brilliant. It seems that she never attended a mathematical conference, but she did attend every one of the weekly seminars in Analysis held by Professor Bosanquet from their inception in 1947 until her retirement in 1967.

Dr. Sargent had few interests outside mathematics but she had a passion for walking and enjoyed tennis in her younger days. She will be remembered by her friends for her dedication to the search for mathematical truth, her absolute integrity and above all for the determination with which she imposed on herself and on those she taught the highest and most meticulous standards of accuracy and precision.

List of published works

1. "On Young's criteria for the convergence of Fourier Series and their conjugates", *Proc. Camb. Phil. Soc.*, 25 (1929), 26–30.
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6. "On the integrability of a product", *J. London Math. Soc.*, 23 (1948), 28–34.
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10. "On fractional integrals of a function integrable in the Cesàro–Perron sense", *Proc. London Math. Soc.* (2), 51 (1949), 46–80.
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18. "On the transform $y_x(s) = \int_0^\infty x(t)k_s(t)dt$", *J. London Math. Soc.*, 30 (1955), 401–416.
19. "Some summability factor theorems for infinite integrals", *J. London Math. Soc.*, 32 (1957), 387–396.

20. "On some cases of distinction between integrals and series", *Proc. London Math. Soc.* (3), 7 (1957), 249–264.
21. "Some sequence spaces related to the ℓ^p spaces", *J. London Math. Soc.*, 35 (1960), 161–171.
22. "Some analogues and extensions of Marinkiewicz's interpolation theorem", *Proc. London Math. Soc.* (3), 11 (1961), 457–468.
23. "On sectionally bounded BK-spaces", *Math. Z.*, 83 (1964), 57–66.
24. "On compact matrix transformations between sectionally bounded BK-spheres", *J. London Math. Soc.*, 41 (1966), 79–87.