BENIAMINO SEGRE 1903–1977

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PATRICK DU VAL

Beniamino Segre was born in Turin on the 16 February, 1903, and received all his formal education there, obtaining a scholarship to the university when he was only sixteen. His teachers there included Peano, Fano, Fubini, Somigliana, and his relative (though not a very close one) Corrado Segre. He graduated in 1923, with a thesis [**1**] (the references are to the list of publications below) on the double curves of symmetroids in S_4; and the same year demonstrated the breadth of his interests by publishing a substantial paper [**2**] on hydrodynamics, investigating the origin of anti-cyclones. After a period as Assistant in Turin, in 1926 he obtained a Rockefeller scholarship in Paris, where he spent a year studying under Élie Cartan. On his return he was chosen as assistant to Severi in Rome, where he remained until 1931, when he was appointed Professor in Bologna. This was a period of great activity, and when he went to Bologna he was already known as the author of some 40 papers, many of considerable depth, covering many aspects not only of algebraic, but also of differential geometry, as well as topology and differential equations.

In 1932 he married Fernanda Coen, a native of Como; to the end of their lives the marriage was one of the deepest affection and understanding, and their friends are hardly able to think of either of them without the other.

In Bologna he continued to publish profusely, mainly (but by no means solely) on the classical problems of algebraic geometry, especially the theory of surfaces. But in 1938 the antisemitic policy of the Italian government deprived him of his position, and brought him and his wife, with three young children, to England as refugees. Here they lived for a time in London and Cambridge; but in 1940 he was interned as an enemy alien in the Isle of Man. This was probably the unhappiest period in the family's life, and later they were rarely willing to speak of it at all; during it, while Mrs. Segre was living in London with Leonard Roth and his Italian wife, who were old friends, their youngest child died; and when the *Andorra Star*, carrying prisoners of war and internees to Canada, was sunk in the Atlantic, she had no idea for some time whether her husband was on it or not. However, that phase of hysteria in our country passed off relatively soon, and most of the internees were released. Segre rejoined his family in London and later they returned to Cambridge, but it is perhaps hardly surprising that his list of publications shows a marked gap during the early war years. This period however was partly occupied in writing his monograph [**103**] on the non-singular cubic surface, describing in great detail, both projectively and topologically, the five real types, and introducing a new notation for the 27 lines on the surface.

In 1942 he was appointed to a teaching post (though hardly one commensurate with his abilities or his reputation) in Manchester, and active work began again. Several substantial contributions to algebraic geometry mark this phase, such as a detailed examination [**101**] of the behaviour of the intersections and tangential forms of variable algebraic varieties on passing to a limit which is singular or reducible, and a study [**102**] of the postulation of a curve assigned to have given multiplicity for

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algebraic primals of sufficiently high order. But it was probably his contact with Mordell and Mahler in Manchester that stimulated an interest in the arithmetic of algebraic varieties, and the parametric solution of diophantine equations. A number of papers [especially **104, 105, 106, 108, 113**] during this period were devoted to the determination of the rational points on a rational cubic surface (i.e. one defined by an equation with rational coefficients); in several very ingenious stages he established that a non-singular rational cubic surface with any rational points at all has an infinity of them, and gave parameteric equations for a number of typical cases, such that any rational values of the parameters give a rational point of the surface; and he also proved that if a rational cubic surface has one, two, or four double points it has an infinity of rational points, but that it can have three conjugate double points and no rational points at all. Classical geometry however continued to occupy him in this period, as is instanced by a paper [**107**] establishing upper bounds for the number of lines on a non-singular surface of order $n \geqslant 3$ (64, actually attained, for $n = 4$); and one, [**109**], on the quartic surface $x_1{}^4+x_2{}^4+x_3{}^4+x_4{}^4 = 0$ over the complex field, with a study of its real cases; which he followed up with one [**119**] on the rational points of any rational surface transformable into this over the complex field. The Manchester period also saw work on diophantine approximation [**116**], on close packing of circles [**110**], on the Tschirnhausen transformation of algebraic equations [**117**], treated, rather unexpectedly but very characteristically, by means of a study of the linear spaces lying on a quadric in a suitable space; on the conditions for the linear equivalence of two binary forms f, f', obtained from the geometry of the biaxial surface $f(x, y) = f'(x', y')$ [**118**]; and some projective properties of double-fours of lines in ordinary space.

In 1946 the family returned to Bologna, not I think without some heart searchings, at least on Mrs. Segre's part, since in spite of some early unhappiness they had come to love England, a love which remained (especially with her) to the end of their lives. In 1950 Segre was called to Rome to succeed Severi in the chair of Geometry, in which his predecessors also included Cremona, Castelnuovo, and Enriques; and he also succeeded Severi in the chair of Geometry in the Istituto Nazionale di Alta Matematica, which he retained till 1963. This was the beginning of the richest productive period of his genius, marked by many outstanding works, of which only the barest outline can be sketched here.

Probably his most important contribution to classical algebraic geometry is his extended study [**170**] in 1952 of the analysis of the singularities of an algebraic variety by means of dilatation, leading up to what is doubtless the definitive proof that every algebraic surface can be transformed by a finite sequence of dilatations into one without multiple points. Many attempts had been made in the past to prove that every algebraic surface can be freed from singularities by a birational transformation; but none of these had been found convincing except those of Walker and Zariski, both of whom however used methods much more brutal than dilatation, giving next to no information about the structure of the singularities while they were being removed.

Let V_v be an algebraic variety on which is a subvariety T_t, the suffixes indicating the dimensions. Dilatation is applicable when T_t is itself non-singular, and every point of it has the same multiplicity m for V_v; and consists in the transformation of V_v into the projective model V_v' of the complete system traced by all primals of any chosen order greater than that of T_t, passing through T_t. This transformation is regular at all points of V_v not on T_t, and in particular the singularities of V_v other than T_t are reproduced unchanged on V_v'; the image of T_t on the other hand is a primal T'_{v-1}

of V_v', generated by ∞^t algebraic varieties A_{v-t-1}, no two of which intersect, of order m, the images of the individual points of T_t; and no point of T'_{v-1} can have multiplicity $> m$ for V_v'. If $t = v-1$, every point of T'_{v-1} may be m-ple for V_v', and T'_{v-1} is then a non-singular variety, m-ple on V_v', which can be dilated in its turn; and one of the first results proved is that this process can be repeated only a finite number of times before we arrive at a transform of V_v on which the image of T_t has multiplicity $< m$, with at most a subvariety of dimension $< v-1$ of multiplicity m.

Applying this to surfaces, let m be the highest multiplicity of any point for V_2; by fairly trivial preliminary dilatations of points we can ensure that there is no isolated m-ple point, and that the constituents of the m-ple curve are non-singular and non-intersecting. Then by a finite sequence of dilatations we can remove these, leaving at most isolated points of multiplicity as high as m, which we proceed to dilate in turn. The difficulty here of course is that this may produce further m-ple curves, as for instance the dilatation of a tacnode familiarly produces a double line on the transformed surface, and the whole process has to begin again. The proof that it can begin again only a finite number of times before we arrive at a surface with no point of multiplicity as high as m is too complicated to be summarised here, but it is carried out with complete rigour. It makes substantial use of the concept, where V_v is a primal on a non-singular variety U_{v+1} (not an essentially restrictive hypothesis, as we can begin with the projection of V_v into S_{v+1}), of primals W_v of U_{v+1} with behaviour "associated" to V_v at a point P of the latter; which means that if, in terms of a local co-ordinate system $(x_0, \ldots, x_v)$ at P on U_{v+1}, V_v and W_v have polynomial equations $f = 0$, $g = 0$, then for some polynomial b, not vanishing at P, bg is in the polynomial ideal with basis

$$\left(\frac{\partial f}{\partial x_0}, \ldots, \frac{\partial f}{\partial x_v}\right).$$

If P is an m-ple point of V_v it is at least, and in general precisely, an $(m-1)$-ple point of W_v; and if it is precisely $(m-1)$-ple, when P is dilated, the image of W_v has behaviour associated to that of V_v at all points of the image of P; and by comparing the effects of a sequence of dilatations on V and W it is possible to prove that if certain results hold for $(m-1)$-ple points, they hold also for m-ple points, so that by induction they only need to be proved for $m = 2$. Finally, having freed the surface of m-ple points, we remove those of next highest multiplicity in the same way, and so on.

Another major contribution to algebraic geometry was Segre's systematic development of a formal calculus of systems of equivalence on an algebraic variety V of v dimensions. These had been introduced by Severi as long ago as 1909 as the natural generalisation to all dimensions from $v-1$ to 0 of the classical idea of linear systems of primals on V. Severi himself pointed out the existence on a surface of an invariant series of equivalence, of sets of points, of order $I+4$ (I being the Zeuthen–Segre invariant); and Segre as early as 1934 **[66]** found on any threefold a similar invariant series of sets of points, and an invariant system of equivalence of curves. Todd in 1937 defined in terms of Jacobians a canonical system of equivalence for all dimensions from $v-1$ to 0; and these included those found by Severi for $v = 2$ and by Segre for $v = 3$. Segre in 1953–54 returned to the subject in two very substantial papers **[178, 186]**. Systems of equivalence, including virtual differences in the usual way, form an additive group, which becomes a commutative ring on defining the product of two elements as their virtual intersection, of codimension equal to the sum of the codimensions of the factors (codimension i on V meaning dimension $v-i$), and the product being zero if its virtual dimension is negative. Systems of mixed dimension are written as sequences $\{M\} = M_0, M_1, \ldots, M_v$ (the suffixes indicating codimension on V), where of course any of the terms may be zero; and these multiply like formal power series modulo the $(v+1)$th power of the maximal ideal. M_0 can only be zero or a multiple of V itself, which is the unity of the ring, since for any subvariety M of V, the intersection $(M.V) = M$. If $M_0 = V$, $\{M\}$ is a unit of the ring, and has a well defined inverse denoted by $\{\tilde{M}\}$, obtainable as a binomial series; for instance if $\{M\} = V, -M_1, 0, \ldots, 0$, $\{\tilde{M}\} = V, M_1, M_1{}^2, \ldots, M_1{}^v$, the sequence of characteristic systems of M_1 ($M_1{}^i$ denoting the virtual intersection of i members of the system M_1). On every irreducible subvariety P of V there is of course a similar ring of equivalence; the additive group of this has a natural homomorphism into that on V, since if two subvarieties of P are equivalent on P they are so on V, but not conversely; the multiplicative groups of the two rings are on the other hand completely different, since if M, N are subvarieties of P of dimension m, n, their virtual intersections on V and on P have dimension $m+n-v$, $m+n-p$ respectively; products in the two rings are accordingly denoted by $(M.N)_V$, $(M.N)_P$.

After all this formal calculus (considerably developed) the major step forward is the definition, for every irreducible p-dimensional subvariety P, of a sequence $\{P_V\} = P, P_{V_1}, \ldots, P_{V_p}$ (the numerical suffixes denoting now codimension on P) of systems of equivalence on P, called the sequence of immersion of P in V. If P is a primal of V (i.e. $p = v-1$), writing $\{P\} = V, P, 0, \ldots, 0$, $\{P_V\} = (P.\{\tilde{P}\})_V$, the sequence of characteristic systems of P with alternate positive and negative signs; and if $P \subset M \subset V$ (i.e. P is a subvariety of M, itself a subvariety of V),

$$\{P_V\} = (\{P_M\}.\{M_V\})_M,$$

which defines the sequences of immersion for $p = v-2, v-3, \ldots$ in turn; and this definition of $\{P_V\}$ turns out to be independent of the choice of the intermediate subvariety M. Now let $A^1, \ldots, A^s$ $(v-p \leqslant s \leqslant p)$ be s primals of V, which all contain P, and intersect residually in a pure variety Q of the proper dimension $v-s$; then $(A^1 \ldots A^s)-Q$ (called the functional equivalence system of P for these primals) is equal to the term of dimension $v-s$ in the product $(\{P_V\}.\{A^1\} \ldots \{A^s\})$, where $\{A^i\} = V, A^i, 0, \ldots, 0$. An elaborate calculus of these sequences of immersion is built up, and extended to cover varieties with singular loci; in particular, an expression is obtained for the virtual locus of improper double points on a subvariety of dimension $\geqslant \frac{1}{2}v$.

But the climax is the unexpectedly simple definition of the canonical sequence of systems of equivalence on V: identifying V with the diagonal locus on its direct product $V \times V$ with itself, the sequence of immersion $\{V_{V\times V}\}$ of the diagonal locus in the direct product is an invariant of V in the sense of depending only on the internal structure of V and not on its relation to any other variety; then defining $\{V^*\}$ to be the sequence derived from the inverse $\{\tilde{V}_{V\times V}\}$ of the invariant sequence just defined by changing the signs of all the terms of odd codimension, Segre proved that the terms $V_1{}^*, \ldots, V_v{}^*$ of this sequence are the same as the canonical systems of equivalence defined quite differently by Todd. In the second paper referred to he examined the changes in these canonical systems and sequences of immersion when any subvariety of V is dilated, generalising widely the familiar result that when a point of a surface is dilated, the resulting exceptional curve is added to the canonical system.

There is little doubt however that what Segre will chiefly be remembered for is his study, which increasingly occupied the latter half of his life, of geometries in which the ground field is not that of complex numbers, and especially of those in which it is a Galois field. His work on the arithmetic of algebraic varieties had accustomed him to comparing geometries over different fields, in this case those of rational, real, and complex numbers. In 1950 he gave a series of lectures in London, republished in book form **[166]**, which together with many of his arithmetical results on cubic surfaces and so forth, contained explicit statements of very general questions (with answers to some of them) about the extension or change of geometrical results with extension of the ground field, and also a general study of quadrics over an arbitrary field, with particular emphasis on fields with characteristic $p \neq 0$, especially Galois fields, to which we shall return shortly. Already in 1948 he had published his "Lezioni di Geometria Moderna" **[142]**, of which in 1960 he brought out an English translation "Lectures on Modern Geometry" **[233]**, enlarged to nearly twice the length (and including an admirable appendix by Lombardo–Radice on finite non-Desargesian planes). At first sight the original form of the book seemed to follow the traditional pattern, with an algebraic introduction on groups, rings, fields, and ideals; the construction of projective geometry out of linear algebra; and the complementary construction of what he calls "Graphic space" from the axioms of incidence, Fano's axiom (that there are at least three points on any line), and Desargue's theorem, with or without Pappus's (or Pascal's) theorem, leading up in the usual way to the construction of a co-ordinate system belonging to a well defined field or corpus, the latter term being used when the commutative law of multiplication is not included in the definition. But in fact in the algebraic introduction all kinds of fields and corpora are considered; in the linear algebra, emphasis is placed on left or right geometries over a non-commutative corpus; and in the synthetic development, the equivalence of Pappus's (or Pascal's) theorem to the commutative law is brought into the foreground; and there is a short chapter (much expanded in the English version) on geometry over a finite field.

In 1958 he published a paper **[216]** which is a detailed investigation of quadric reguli, conics (defined as plane sections of reguli), and the residual section of a regulus by a plane through a generator (which is not generally a line) in the geometry over a non-commutative corpus. Just as a non-identical projectivity on a line can have an infinity of fixed points, so two distinct reguli can have an infinity of common generators, and two distinct conics an infinity of common points. Curiously, however, he ostentatiously makes no use of the Maclagan–Wedderburn theorem that every finite corpus is commutative, hoping to be able to prove this from purely geometrical considerations; though in this he was only partially successful, and I cannot discover that he ever returned to this problem later. (This paper reappears almost unchanged as the final chapter of **[233]**.)

But from the middle 1950's Segre's attention was increasingly concentrated on geometry over a Galois field, of order $q = p^h$ (p prime) and the developing discipline now known as combinatorics, which blends with Galois geometry, as a means of handling finite subsets of the finite set of points in space. His work here has proved the inspiration of a rapidly developing school, which is expanding the topic in all directions.

In 1959 a substantial paper **[221]** brought together the results of a number of publications during the previous five years, which were improved on in **[222]**, **[225]**. Non-singular quadrics, classified in $S_{r,q}$ (r dimensional space over the Galois field of order q) according to the maximum dimension $k-1$ of linear spaces lying on them, offer only one type for even r and two for odd r, in contrast to the $[\frac{1}{2}(r+3)]$ types in real S_r; we have in fact the three types: I. $r = 2k$; II. $r = 2k-1$; III. $r = 2k+1$. Thus in the plane every conic has $q+1$ points, and in S_3 we have quadrics with two families each of $q+1$ generating lines, and hence $(q+1)^2$ points, and quadrics with q^2+1 points and no lines, but neither conics nor quadric surfaces with no points in the ground field, which might have been expected by analogy with real geometry. As in ordinary geometry, if a quadric has any double points, these are all points of an S_{k-1} $(1 \leqslant k \leqslant r)$ and the quadric is a cone projecting from S_{k-1} a non-singular quadric in an S_{r-k} skew to S_{k-1}. The discovery of the different types is, like so much of the argument in this topic, largely a matter of counting; thus on the line there are altogether q^2+q+1 "quadrics" or point pairs, of which $q+1$ are the repeated points of the line, and $\frac{1}{2}q(q+1)$ pairs of distinct points; the remaining $\frac{1}{2}q(q-1)$ are thus conjugate pairs, reducible only in a quadratic extension of the ground field. Next, if a non-singular conic has any points on it it has just $q+1$; the number of such conics is found from the fact that a conic is uniquely determined by five points of which no three are collinear, so that we have only to divide the number of such pentads in the plane by the number $\binom{q+1}{5}$ of them on each conic; it then turns out that these, with the q^2+q+1 repeated lines, $\frac{1}{2}q(q+1)(q^2+q+1)$ pairs of distinct lines, and $\frac{1}{2}q(q-1)$ pairs of conjugate lines through each of the q^2+q+1 points, exhaust all the $(q^6-1)/(q-1)$ "quadrics" in the plane, so that there are no conics containing no points. This gives us the above three types for $r = 1, 2$; and the classification is extended to $r \geqslant 3$ by induction, since every quadric has some points on it (as its plane sections have) and the tangent prime at any point of a non-singular quadric cuts it in a cone projecting from this point a quadric of one of the three specified types, with $r-2$, $k-1$ in place of r, k.

These results, and the expressions found for the number of points and of linear spaces of all dimensions up to the maximum on quadrics of each type, are valid for all $q = p^h$. But there are a number of quite anomalous and unexpected results when q is even (i.e. $p = 2$). To begin with, in this case a projectivity on a line is involutory if and only if it is parabolic, and the three involutions which interchange four given distinct points of the line by pairs all have the same single united point. Further, as with characteristic 2 there is no difference between a skew symmetric matrix and a symmetric one whose diagonal elements are zero, the polarity with respect to a quadric is the same thing as a null polarity; and the associated linear complex of lines consists of the tangent lines of the quadric. Hence also, as a null polarity in even dimensions always has at least one singular point, every non-singular quadric in $S_{r,\,2^h}$ for even r determines a unique point, called its nucleus, every line through which is a tangent; in particular, the tangents to a conic are not, as with other ground fields, the figure dual to a conic, but are a pencil of lines. The polarity depends only on the product terms in the equation (which of course have to be written without the traditional coefficient 2) and not on the square terms; so that the family of tangents is common to q^{r+1} quadrics, which when r is odd include both types II, III. Most of these results hold also, mutatis mutandis, for any ground field of characteristic 2, not necessarily finite. In **[205]** Segre went on to examine what a number of other familiar figures look like with a ground field of characteristic 2, such as plane cubics (on which a quadratic involution has in general two united points, but in special cases only one, instead of the familiar four), twisted cubics, a type of plane quartic which has all the lines of a pencil as bitangents, and the curve of intersection of two quadrics, revealing many unexpected properties.

Now for every q, the $q+1$ points of a conic are such that no three of them lie on a line; and for even q, the $q+2$ points obtained by adjoining the nucleus to the conic have the same property, since as every line through the nucleus is a tangent, it lies on no chord of the conic. This led to the definition of a k-arc as a set of k points in the plane, no three of which are collinear; the k-arc is complete if it is not contained in any $(k+1)$-arc, and if k has its maximum value, namely $q+1$ for odd q and $q+2$ for even q, the k-arc is called an oval.

Now let us go back to 1954. Throughout Segre's life he responded to mathematical challenges, the enormous geometrical knowledge he developed in his early years enabling him to do so. Two Finnish astronomers, Järnefelt and Kustaanheimo had written some papers on approximating Euclidean space by finite space, and in 1949 they conjectured that, for q an odd prime, a $(q+1)$-arc is a conic. This conjecture was thought implausible by the reviewer in *Mathematical Reviews*. It was Segre's proof [**191**] of this conjecture, for any odd q, that gave the impetus to all later research in this field. It was immediately recognised that the result was significant, not because of its depth (the proof is very short and completely elementary) but because it was the first characterisation of an algebraic variety in a finite space by a purely combinatorial property. It was also a perfect example of Segre's method. He picked the right property which characterises conics over the complex numbers, proved it true for $(q+1)$-arcs, and hence showed that this property characterises conics in the finite case. This result also showed that theorems in finite geometry do not necessarily consist merely of analogues to results in complex geometry.

For even q, the situation is more complicated. The only cases in which a $(q+2)$-arc necessarily consists of a conic with its nucleus are $q = 2, 4, 8$, and possibly $q = 64$ (this last case is still unresolved). A $(q+1)$-arc can always be completed to an oval or $(q+2)$-arc.

For any q (odd or even) a q-arc is always incomplete; and for all $q \geqslant 7$ there are complete k-arcs which are not ovals, and are contained in no conic; for instance, there are complete 6-arcs for $q = 7, 8, 9$, complete 7-arcs for $q = 9$, and complete 8-arcs for $q = 11$.

Similarly for $r \geqslant 3$ a K-arc in S_r is defined to be a set of K points of which no subset of $r+2$ lie in a prime. For even q, hardly anything is known about these; but for sufficiently large odd q, in relation to given dimension r and an arbitrary integer $c \geqslant 0$, every $(q+1-c)$-arc in $S_{r,q}$ is contained in a normal rational curve of order r, and is of course the whole of it if $c = 0$.

Another generalisation of the idea of k-arcs in the plane to S_r $(r \geqslant 3)$ is the K-cap (calotta), a set of K points of which no three are collinear. This compares naturally, not with any curve, but with a quadric; though except for the convex or elliptic quadric in S_3 (type II) the cap cannot consist of all points of the quadric, since in all other cases the latter has lines on it, only two points of any one of which can belong to the cap. For $r \geqslant 4$, the value of K for a K-cap on a non-singular quadric of each of the three types cannot exceed:

I. $2(q^k+1)(q^k-1)/(q^2-1)$;

II. $2(q^{k-1}+1)(q^k-1)/(q^2-1)$; and

III. $2(q^{k+1}+1)(q^k-1)/(q^2-1)$

$(k-1$ being still the maximum dimension of linear spaces lying on the quadric).

A K-cap for which K has its maximum value (for given r and q) is called an ovaloid. For $q = 2$, this maximum is q^r, every ovaloid consisting of all the points of $S_{r,2}$ except those of a particular prime. For $r = 3$ and odd q, the maximum for K is q^2+1, and every ovaloid is an elliptic quadric; for even $q > 2$ the maximum is likewise q^2+1, but it is only for $q = 4$ that every ovaloid is a quadric; and for all q there are complete caps with fewer points than the maximum, which accordingly are not ovaloids. Simple as these results look, the arguments by which they are obtained are often very ingenious, and the problems become rapidly more difficult with increase of r, even for $r = 4$. Not all these results on arcs and caps were obtained by Segre himself, but they are the work of a school entirely inspired by him, and of course include far more than we have attempted to summarise here.

These results and many others in Galois geometry, laboriously obtained over a number of years, are assembled in a systematic account of the whole subject as it stood in 1959 [**221**], which is indeed more like a monograph than a paper. Perhaps the most profound result in this is the connexion between plane k-arcs and algebraic curves. It is shown that the lines in $S_{2,q}$ unisecant to a k-arc belong to an algebraic envelope of class $t = q+2-k$ when q is even, and of class $2t$ when q is odd. There was an important sequel [**259**] in 1967. In 1948, Weil had completed the proof for an absolutely irreducible curve of arbitrary genus g, of what had already been shown by Hasse for an elliptic curve, that the number N of points on its non-singular model satisfies $|N-(q+1)| \leqslant 2g\sqrt{q}$. Segre used this result to find good lower bounds on k for a k-arc to be necessarily contained in an oval. Specifically, it was shown that if a k-arc K satisfies $k > q-\frac{1}{4}\sqrt{q}+7/4$ for odd q, or $k > q-\sqrt{q}+1$ for even q, then K is necessarily contained in an oval. An equivalent way of expressing this is that the number of points on a complete arc other than an oval is bounded above by these numbers. For $q = 8$, $[q-\sqrt{q}+1] = 6$, and this is indeed the maximum number of points on a complete k-arc in $S_{2,8}$ other than an oval. In turn these results led to greatly improved upper bounds for $m(r, q)$, the maximum number of points in $S_{r,q}$, no three of which are collinear. The solution of this problem, apart from its intrinsic interest, would have applications in Statistics and Coding theory. As observed above, $m(2,q) = q+(q, 2)$; $m(3,q) = q^2+1$ for $q > 2$; and $m(2,r) = 2^r$. Otherwise the only results known are that $m(4, 3) = 20$ and $m(5, 3) = 56$. It is surprising that such a simply posed and apparently elementary problem has been solved in so few cases.

The problem of the number N of points on an algebraic curve was also considered in [**246**]. Although the Hasse–Weil theorem gives very good bounds for N, it is still a problem on which much remains to be done. Segre gave an algorithm for determining N when the curve has equation $a_0 x_0{}^n+a_1 x_1{}^n+a_2 x_2{}^n = 0$, and in fact determined N for many fields when $n \leqslant 7$, thus verifying the Hasse–Weil theorem by elementary methods in these cases. It is noteworthy how often $\sqrt{q}$ appears in this and other problems as some sort of error bound.

Another remarkable contribution in the mid-1960's to the relation between geometries over different fields was the paper [**243**] in which Segre studied the representation, in the geometry over a field γ, of that over another field δ, which is an algebraic extension of γ; a familiar case of this of course is the mapping of an r-dimensional complex space on a $2r$-dimensional variety in real geometry, of which the complex plane offers a sufficient illustration. Three models are considered: I, the ∞^4 of real lines in S_5 that meet two skew and conjugate imaginary planes, which constitutes a fibring of the real S_5; II, the Grassmannian of this line system in real S_5, which is the real sheet W_4 of the $V_4{}^6$ in S_8, direct product of two planes, in the case where each generating plane of either system is conjugate complex to a plane of the other system, their intersection being the unique real point of either plane; and III, the map of W_4 on real S_4 by projection from the ambient S_3 of the convex quadric surface which is the map on W_4 of the points of the line at infinity in the complex plane; this is the same as the map obtained by taking as affine co-ordinates in S_4 the real and imaginary parts of the affine co-ordinates in the complex plane; points not at infinity in the complex plane are mapped by points not at infinity in the real S_4, but the images of the points at infinity of the plane are the lines of a linear congruence with conjugate imaginary directrix lines in the prime at infinity in S_4.

Now let δ be any separable algebraic extension of order n of γ, say $\delta = \gamma(x)$, where x is a root of a polynomial equation $f(x) = 0$, irreducible over γ. Model I of S_{r-1} over δ is obtained by taking in the geometry over the splitting field δ^* of $f(x)$ (which in general of course is larger than δ, but coincides with δ if $n = 2$, or if γ, and hence also δ, are finite) n copies of S_{r-1}, which are conjugate over γ (i.e. permuted transitively by the automorphisms of δ^* over γ, which permute transitively the roots of $f(x) = 0$) and which are linearly independent, i.e. span S_{nr-1}; the S_{n-1}'s that join sets of conjugate points of these are in the geometry over γ, and in this constitute a fibring of S_{nr-1}, whose fibres are in one-one correspondence with the points of S_{r-1} over δ. Models II, III are obtained from this much as in the familiar example outlined above: in particular, in model III, the images of points not at infinity of S_{r-1} over δ are the points not at infinity of $S_{n(r-1)}$ over γ, but those of the points at infinity are the $(n-1)$-dimensional fibres of a fibring of the prime at infinity of $S_{n(r-1)}$, which is in fact model I for $r-1$ in place of r.

But now, conversely, if, in the geometry over γ, Φ is any fibring of $S_{n(r-1)-1}$ by S_{n-1}'s, with the property that it induces a similar fibring on the join of any two or more of its fibres, we can construct abstractly an r-dimensional graphic space, whose points are the points not at infinity of $S_{n(r-1)}$, and the fibres of Φ in the prime at infinity, and whose S_k's are the S_{nk}'s whose S_{nk-1}'s at infinity are fibred by Φ, together with the $S_{n(k+1)-1}$'s at infinity fibred by Φ; and this satisfies the axioms of incidence, but not (*a priori*) Pappus's or even Desargues's theorem. If Φ has n conjugate directrix spaces, we obtain in this way only what we already have, the geometry over a field extension of γ; but for $n = 2$, and for a variety of fields γ, including the real number field, and some with characteristic $p \neq 0$, Segre succeeds, by methods too complicated to be summarised here, in constructing fibrings Φ without directrix spaces, and models of non-desarguesian graphic planes.

Segre's last great contribution to Galois geometry was a 200-page paper **[251]** in 1965, devoted to the case in which $q = p^h$ is a perfect square, i.e. h is even, so that $x \to x^{\sqrt{q}}$ is an involutory automorphism of $GF(q)$; this can be treated to a great extent as the analogue of the interchange of conjugate complex elements in the complex number field; and $x^{\sqrt{q}}$ is accordingly denoted by $\bar{x}$. Apart from some initial generalities, the paper is concerned with the geometry of a Hermitian variety, i.e. the zeros of a Hermitian form $\bar{X}AX^*$, where $X = (x_0, \ldots, x_n)$, the star denotes matrix transposition, and $A = \bar{A}^*$. The Hermitian form can be reduced by a linear transformation to the canonical form $\bar{x}_0 x_0 + \ldots + \bar{x}_{n-t} x_{n-t}$, and the variety is correspondingly denoted by $U^t_{n,q}$. In contrast with what happens over the complex field, in the finite case Hermitian varieties are algebraic, and provide examples of varieties of arbitrarily high order. For $t = 0$, $U_{n,q} = U^0_{n,q}$ is the set of self conjugate points of a polarity, whence many properties can readily be obtained. In particular, the number of points and subspaces on $U_{n,q}$ can be calculated; indeed, as for quadrics, there are formulas for the number of sections of different type by any subspace. $U_{1,q}$ consists of $\surd q+1$ points, $U^1_{1,q}$ of a single point. $U_{2,q}$ consists of $q\surd q+1$ points, such that the tangent line at each point meets $U_{2,q}$ there only (in a $U^1_{1,q}$) and every other line meets $U_{2,q}$ in a $U_{1,q}$. For example, $U_{2,4}$ is a cubic curve, consisting exactly of nine inflexions, in the same configuration as the nine inflexions of a non-singular cubic curve over the complex numbers. $U_{3,q}$ consists of $(q+1)(q\surd q+1)$ points on $(\surd q+1)(q\surd q+1)$ lines, such that every tangent plane meets $U_{3,q}$ in $\surd q+1$ concurrent lines, and every other plane meets it in a $U_{2,q}$. So $U_{3,4}$ consists of 45 points on 27 lines; its collineation group has order 51,840, and is of course isomorphic with the permutation group of the 27 lines on a cubic surface in complex geometry. Over the complex numbers however the collineation group of a cubic surface has order at most 648; this discrepancy arises from the fact that every tangent plane meets $U_{3,4}$ in three concurrent lines, whereas over the complex numbers at most 18 of the 45 tritangent planes can intersect the cubic surface in three concurrent lines. The occurrence of the 27 lines on a cubic surface and on $U_{3,4}$ is a deceptive analogy; for a general quintic primal in complex S_4 contains 2875 lines, considerably less than the 66,625 lines lying on $U_{4,16}$. A significant property of the general $U_{n,q}$ is that it contains no more generators (subspaces of maximal dimension) when the ground field is extended.

The projective unitary group is described in considerable detail, for singular as well as non-singular varieties. Segre also works out all the polarities commuting with a Hermitian polarity.

A great deal of structure is obtained by defining a *distance* on $U_{n,q}$. Let P_1, P_2 be non-conjugate points of $U_{n,q}$, and let P_i' be the intersection of $P_1 P_2$ with the polar prime of P_i $(i = 1, 2)$. Then the distance $\{P_1 P_2\}$ is defined to be the cross-ratio $\{P_1, P_2'; P_2, P_1'\}$. The dual notion is that of the *angle* between two primes. This leads to a wealth of properties, which have not yet been applied.

As further evidence that this paper has yet to be properly mined, one may cite the following: towards the end, Segre considers the existence on $U_{3,q}$ of a subset L of the lines on the variety, such that there are just m of them through each point of it. It is shown that for odd q, $m = q+1$ or $(\surd q+1)/2$, but the proof extends easily to even q (when of course the latter alternative is absent, as $\surd q+1$ is odd). When $m = q+1$, L consists of all the lines on $U_{3,q}$; when $m = (\surd q+1)/2$, L is called a hemisystem. As $m \neq 1$, the points of $U_{3,q}$ cannot be partitioned by a subset of lines. It is shown that for $q = 9$ hemisystems do exist. They consist of 56 lines, any skew pair among them having a pair of skew transversals in the set. This immediately implies the existence of a $2-(56, 11, 2)$ design, which was frequently rediscovered from 1970 on.

Throughout his life Segre published a large number of papers (most of them quite short) on the differential geometry of algebraic varieties and on such varieties as solutions of differential equations. A systematic survey of his results in this field is contained in his Ergebnisse volume **[208]**, published in 1957, with a second enlarged edition **[272]** in 1971. Unfortunately I am too little at home in this topic to be able to give any serious account of it. He also wrote a very substantial textbook in two volumes **[167, 206]** with a five-year interval between them, on differential forms and their integrals. This is probably the most complete and systematic account of this whole topic in existence, starting with very clear accounts of the Grassmann algebra, the differential and integral calculus involved, and the theory of functionals, through an extremely lucid account of homology and cohomology theory, to the work of de Rham and Hodge, and the most advanced applications of the theory. Unfortunately, this work was only published in a small lithographed edition, and hence is not very widely available.

In 1975 his attention was drawn to the four-colour problem by a paper by Gardner, quoting from McGregor a map in 110 regions, which it was claimed required at least five colours. He seems not to have been sure whether Gardner intended this seriously, but anyway he gave an explicit colouring of the map in question with four colours, and that was that [**283**]. He followed this up with a paper [**284**] in which he attacked the problem by relating the enumerative properties of the map to configurations in Galois space, without however quite attaining to the proof he had hoped for.

Segre retired from his University chair in 1973, but continued very active in the affairs of the Accademia dei Lincei, and other Italian and international scientific organisations. Neither he nor his wife had much liked living in Rome, preferring their native Piedmont and Lombardy; and some time in the later 1960's they abandoned their flat near the University and settled in Frascati with their two married children in adjacent houses, and lived happily surrounded by grandchildren.

In 1973 there was a conference on Combinatorial Theories in Rome, jointly organised by the Accademia dei Lincei and the American Mathematical Society; there were some 75 talks, and more than 150 mathematicians attended. Although not officially named as such, it was generally regarded as a tribute to Segre's 70th birthday; it was in some sense a climax to his mathematical career, and a recognition of his influence. The occasion was also marked by a collection of papers in his honour, published in the Annali di Matematica.

In the post-war period Segre made many journeys abroad to international gatherings of all kinds, including a number to this country; for instance, he represented the Lincei at the tercentenary celebrations of the Royal Society; and he was generally (in later years I think always) accompanied by his gracious and charming wife. But in the autumn of 1976 she died very suddenly, and though he continued to throw himself into academic business, he never really recovered from her loss. He made a last visit to England in the summer of 1977, his daughter taking her mother's place as his companion, when he took part in a symposium on Combinatorics in Royal Holloway College, and also received an honorary degree from the University of Sussex. He died on the 22 October, 1977.

Segre was vice-president of the Accademia dei Lincei 1965–1967 and 1973–1976, and was president 1967–1973, and again in the last year of his life. Under his direction the Academy greatly increased its strength and scope; he was the prime mover in the creation of the Centro Linceo Interdisciplinare; and he initiated the grandiose publication of the collected papers of Severi (which are even more numerous than his own); he also used his influence to great effect in enabling many foreign mathematicians to visit Italy, especially those from Eastern Europe. From 1974 till his death he was president of Accademia dei XL; he was a member of the Accademia Ponteficie delle Scienze, and in the last year of his life president of the Société Européenne de Culture. Besides Sussex, he was an honorary doctor of Bologna, and of Bratislava; and was an honorary member, foreign member, or corresponding member, of a great number of learned societies in a variety of countries. Besides the London Mathematical Society, whose honorary membership was conferred in 1963, those included the Scientific Academies of Bologna, Turin, Palermo and Modena, the Istituto Lombardo, the Accademia Petrarca and the Accademia Ligure in Italy; the Académie des Sciences de l'Institut de France, the Académie des Sciences of Toulouse, the Société Royale des Sciences of Liège, the Académie Royale de Bélgique, and the Academia Nacional of Buenos Aires. He was also a member of the Executive Committee of the International Mathematical Union, and of the Presidential Board of the Associazione Italia-U.R.S.S., and at various times president of the Società Italiana di Logica e Filosofia delle Scienze, and vice-president of the Groupement des Mathematiciens d'Expression. Among other honours he was awarded were the gold Medal of the Società Italiana dei XL, that of the Benmeriti della Scuola della Cultura e dell'Arte, the City of Bologna Prize, the Golden Pen of the President of the Council of Ministers, the order of Cavaliere Gran Croce OMRI, and that of Chevalier de la Légion d'Honneur.

Segre could sometimes seem, especially to anyone meeting him for the first time, conversationally somewhat forbidding, but in the company of friends he could relax very convivially. He was fond of children, and most children liked him at first sight. I know our own children loved him; and I vividly recall an occasion when he and his wife were staying with us, seeing him return from a walk with the children demonstrating a Russian dance all the way up our suburban street, to the astonishment of the neighbours. He was devoted to his own children and grandchildren, and always seemed at his happiest in their company.

Segre was an excellent listener to lectures, contributing vigorously to the discussion, and often improving more or less impromptu on the results put forward by the lecturer. For instance in 1966 he gave a series of lectures in Sussex, which resulted in **[259]**; and Steinhaus, who was also there, gave a lecture in which he conjectured that every closed curve in space had a pair of parallel tangents; the very next day Segre gave a lecture on his solution of this problem; his solution did in fact contain an error, which he rectified in the four papers **[262]**, **[264]**–**[266]** which arose out of this encounter. Again, in 1969 Mumford gave a series of lectures in Varenna on varieties defined as intersections of quadric primals; Segre felt that many of the results could be obtained without recourse to cohomological methods, and this led to **[269]**.

Occasionally these encounters could arouse a combative streak in him. There is an anecdote in circulation of a lecture by Hodge in Oxford, which ended with Segre and another member of the audience occupying opposite ends of the blackboard, and holding forth quite independently. I myself well remember a lecture by Severi in Harvard, after the 1950 Congress, which was constantly interrupted by Lefschetz in strong disagreement; the situation developed with Segre at the blackboard, firmly explaining what he thought was the resolution of the difference, while Severi and Lefschetz continued to shout each other down in French.

I would like to express my thanks to Dr. James Hirschfeld for very substantial help in writing the above, mainly, though by no means exclusively, in the treatment of Galois geometries.

SEGRE'S MATHEMATICAL PUBLICATIONS

(This list does not include book reviews, obituaries, biographical essays, and other publications which are not part of the author's Scientific work. A more complete list, down to 1975, will be found in the Annuario, 1975, of the Accademia Nazionale dei XL.)

1923

1. "Genera della curva doppia per la varietà di S_4 che annulla un determinante simmetrico", *Atti Acc. Sc. Torino*, 58, 100–108.

2. "Sul moto sferico vorticoso di un fluide incompressibile", *Ann. di Mat.*, (4) 1, 31–55.

1924

3. "Dei sistemi lineari tangenti ad un qualunque sistema di forme", *Rend. Acc. Naz. Lincei*, (5) 33, 182–185.
4. "Sui complessi algebrici di rette di S_n", *Rend. Acc. Naz. Lincei*, (5) 33, 218–222.
5. "Una proprietà caratteristica di tre sistemi ∞^1 di superficie", *Ann. Acc. Sc. Torino*, 59, 666–671.

1925

6. "I complessi quadratici di rette di S_4", *Rend. Acc. Naz. Lincei*, (6) 2, 476–479.
7. "Intorno ad una proprietà dei determinanti simmetrici del 6° ordine", *Rend. Acc. Naz. Lincei*, (6) 2, 539–542.

1926

8. "I sistemi semplicement infiniti di superficie (in particolare piani o sfere) e le loro traiettorie ortogonali", *Atti Acc. Sc. Torino*, 61, 286–293.
9. "Una generalizzazione della trasformazione di Koenigs", *Rend. Acc. Naz. Lincei*, (6) 4, 438–442.
10. "Généralisation de la transformation de Laplace", *C. R. Acad. Sci. Paris*, 183, 1248–1250.

1927

11. "Le piramide iscritte e circoscritte alle quadriche di S_4 e una notevole configurazione di rette dello spazio ordinario", *Mem. Acc. Naz. Lincei*, (6) 2, 204–229.
12. "Sur l'intégration d'un certain système d'équations differentielles", *C. R. Acad. Sci. Paris*, 184, 268–270.
13. "Sur les diagrammes de probabilité", *C. R. Acad. Sci. Paris*, 184, 573–574.
14. "La cubique indicatrice de l'élément linéaire projectif d'une surface", *C. R. Acad. Sci. Paris*, 184, 729–731.
15. "Sur les transformations des réseaux R", *C. R. Acad. Sci. Paris*, 184, 1396–1398.
16. "Les systèmes conjugués et autoconjugués d'espèce ν et leur transformation de Laplace", *Ann. Éc. Norm. Sup.*, (3) 44, 153–212.

1928

17. "Les systèmes conjugues de 2ᵉ espèce en involution ou grilles", *Ann. Fac. Sc. Toulouse*, (3) 20, 1–46.
18. "Sulle curve algebriche le cui tangenti appartengono al massimo numero di complessi lineari independenti", *Mem. Acc. Naz. Lincei*, (6) 2, 578–592.
19. "Le congruenze K e la trasformazione F delle superficie dello spazio ordinario", *Rend. Circ. Mat. Palermo*, 52, 345–372.
20. "Sobre algunas representaciones reales del plano complejo", *Rev. Mat. Hispano-Americana*, 6, 137–146.
21. "Sui moduli delle curve poligonali e sopra un complemento al teorema di esistenza di Riemann", *Math. Ann.*, 100, 537–551.
22. "Sui moduli della curve algebriche", *Atti Congr. Intern. Math. Bologna*, vol. IV, 129–131.
23. (with F. Severi) "Un paradosso topologico". *Rend. Acc. Naz. Lincei*, (6) 9, 5–8.
24. "Esercizi e complementi di analisi algebrica." (Lithograph.)

1929

25. (with F. Severi) "Ancora sopra un paradosso topologico", *Rend. Acc. Naz. Lincei*, (6) 9, 117–122.
26. "Costruzione di una curva semplice sghemba di Jordan, incontrata da tutte le semirette uscenti da un punto esterno", *Rend. Acc. Naz. Lincei*, (6) 9, 136–142.
27. "Studio dei complessi quadratici di rette di S_4", *Atti Ist. Veneto*, 58, 595–649.
28. "Sui sistemi continui di curve piane con tacnodo", *Rend. Acc. Naz. Lincei*, (6) 9, 970–974.
29. "Sui moduli delle curve algebriche", *Ann. di Mat.*, (4) 7, 71–102.
30. "Esistenza e dimensione di sistemi continui di curve piane algebriche con dati caratteri", *Rend. Acc. Naz. Lincei*, (6) 10, 31–38.
31. "Quartiche piane e superficie cubiche", *Boll. U.M.I.*, (1) 8, 203–210.
32. "Intorno alle congruenze di rette che ammettono reti coniugati ad invarianti uguali", *Boll. U.M.I.*, (1) 8, 242–244.
33. "Sulle equazioni differenziali autoaggiunte del 4° e 5° ordine", *Boll. U.M.I.*, (1) 8, 244–245.
34. "Esistenza di sistemi continui distinti di curve piane algebriche con dati numeri plückeriani", *Rend. Acc. Naz. Lincei*, (6) 10, 557–560.

1930

35. "Sulle corrispondenze (2, 2) cicliche", *Boll. U.M.I.*, (1) 9, 1–3.
36. "Sulla costruzione delle bisestuple di rette", *Rend. Acc. Naz. Lincei*, (6) 11, 448–449.
37. "Sulla quartica osculatrice in un punto ad una curva sghemba", *Boll. U.M.I.*, (1) 9, 159–162.
38. (with F. Severi) "L'inviluppo di un sistema più volte infinito di curve piane", *Ann. di Mat.*, (4) 8, 173–199.
39. "Sulla caratterizzazione delle curve di diramazione dei piani multipli generali", *Mem. Acc. d'Italia* 1, fasc. 4, pp. 31.
40. "Sulle congruenze di rette che ammettono reti coniugati ad invarianti uguali", *Mem. Acc. d'Italia* 1, fasc. 5, pp. 39.
41. "Esercizi e complementi di analisi algebrica", 2nd ed. (Lithograph.)

1931

42. "Intorno alla teoria delle superficie proiettivamente deformabili e alle equazioni differenziali ad esse collegate", *Mem Acc. d'Italia* 2., fasc. 3, 1–143.
43. "Questioni geometriche legate colla teoria delle funzioni di due variabili complesse", *Rend. Sem. Mat. Univ. Roma*, (2) 7, 59–107.
44. "Intorno al problema di Poincaré della rappresentazione pseudo-conforme", *Rend. Acc. Naz. Lincei*, (6) 13, 676–698.
45. "Sulla completezza della serie caratteristica di un sistema continuo di curve irriducibili tracciate su di una superficie algebrica", *Rend. Circ. Mat. Palermo*, 55, 443–449.
46. "Questioni geometriche legate colla teoria delle funzioni di due variabili complesse", *Boll. U.M.I.*, (1) 10, 269–274.
47. "Coordinate", *Enciclopedia Trecciani*, vol. XI, 294–303.

1932

48. "La geometria in Italia, dal Cremona ai giorni nostri", Inaugural lecture in Bologna, 13 Jan. 1932; *Ann. di Mat.*, (4) 11, 1–16.
49. "Sulla possibilità di generare una quadrica mediante due schiere proiettive di quadriche", *Boll. U.M.I.*, (1) 11, 3–8.
50. "Determinazione di certi gruppi covarianti di due o piu serie lineari", *Rend. Circ. Mat. Palermo*, 65, 214–222.
51. "Sulle condizioni per la regolarità di un sistema lineare di forme", *Rend. Acc. Naz. Lincei*, (6) 16, 114–120.
52. "Sulle superficie algebriche aventi il sistema canonico composto con una involvuzione", *Rend. Acc. Naz. Lincei*, (6) 16, 316–320.
53. "Alcuni resultati di geometria algebrica", *Boll. U.M.I.*, (1) 11, 265–267.
54. "Questioni geometriche legate alla teoria delle funzioni di due variabili complesse", *Atti Soc. Ital. Progr. Sc.*, 20, 27.

1933

55. "Proprietà in grande di curve algebriche, dedotte da proprietà in piccolo", *Mem. Acc. d'Italia*, 4, 9–16.
56. "Intorno alla geometria sopra una varietà algebrica", *Rend. Acc. Naz. Lincei*, (6) 17, 207–211.
57. "Intorno ad un teorema di Kakeya", *Boll. U.M.I.*, (1) 12, 123–130.
58. "Sulla serie caratteristica d'una superficie sopra una varietà algebrica a quattro dimensioni", *Rend. Acc. Naz. Lincei*, (6) 17, 917–918.
59. "Sulle curve algebriche che ammettono come trasformata razionale una curva piana dello stesso ordine, priva di punti multipli", *Math. Ann.*, 109, 1–3.
60. "Sui gruppi di S_k associati di un S_r", *Rend. Acc. Sc. Bologna*, (2) 38, 27–33.
61. "Determinazione geometrico-funzionale di gruppi di punti covarianti, relativi a dati sistemi lineari di curve sopra una superficie algebrica", *Rend. Acc. Naz. Lincei*, (6) 18, 297–302, 382–385, 445–451.

1934

62. "Gli scorrimenti nella geometria non euclidea degli iperspazi ed alcune notevoli corrispondenze proiettive", *Ann. di Mat.*, (4) 12, 327–347.
63. "Sulla teoria delle equazioni algebriche a coefficient reali", *Mem. Acc. d'Italia*, 5, 323–346.
64. "Intorno alla parità di alcuni caratteri di una varietà algebrica di dimensione dispari", *Boll. U.M.I.*, (1) 13, 93–95.
65. "Sugli integrali di differenziali binomii", *Rend. Acc. Naz. Lincei*, (6) 19, 279–283.
66. "Nuovi contributi alla geometria sulle varietà algebriche", *Mem. Acc. d'Italia*, 5, 479–576.

67. "Sui moduli delle superficie algebriche irregolari", *Rend. Acc. Naz. Lincei*, (6) 19, 488–494.
68. "Intorno alle ovali iscritte in un poligono regolare", *Boll. U.M.I.*, (1) 13, 275–279.
69. "Sui circoli geodetici di una superficie a curvatura totale costante, che contengono nell'interno una linea assegnata", *Boll. U.M.I.*, (1) 13, 279–283.
70. "Il teorema sul minimo numero di vertici di un ovale, ed alcune sue estensioni", *Atti Acc. Sc. Torino*, 70, 116–122.

1935

71. "Proprietà in grande delle linee piane convesse", *Rend. Acc. Naz. Lincei*, (6) 20, 407–410, 455–458; 21, 11–14.
72. "Alcune proprietà in grande delle linee convesse", *Mem. Acc. Sc. Bologna*, (9) 2.
73. "Il teorema di Meusnier nella geometria differenziale degli insiemi", *Mem. Acc. d'Italia*, 6, 1205–1220.
74. "I birapporti sulle superficie non sviluppabili dello spazio, e le condizioni geometriche per l'equivalenza proiettiva fra queste", *Rend. Acc. Naz. Lincei*, (6) 21, 656–660, 692–697.
75. "Risposta alla questione N. 2", *Boll. U.M.I.*, (1) 14, 266–268.
76. "Sugli elementi curvilinei che hanno communi le origini ed i relativi spazi osculatori", *Rend. Acc. Naz. Lincei*, (6) 22, 393–399.
77. "Le linee proiettive ed un invariante di una curva su di una superficie", *Rend. Acc. Naz. Lincei*, (6) 22, 400–405.

1936

78. "Intorno alle parti fisse del sistema canonico sopra una superficie algebrica", *Rend. Acc. Sc. Bologna*, pp. 4.
79. "Quelques résultats nouveaux dans la géométrie sur une V_3 algébrique", *Mem. couronné Acad. Royale de Belgique*, (2) 16, 3–99.
80. "Un problema di geometria numerativa", *Boll. U.M.I.*, (1) 15, 49–55.
81. "Sulle varietà di Veronese a due indici", *Rend. Acc. Naz. Lincei*, (6) 23, 303–309, 391–397.
82. "Intorno alle ovali sghembe, e su di un'estensione del teorema di Cavalieri–Lagrange alle funzioni di due variabili", *Rend. Acc. Naz. Lincei*, (6) 23, 654–656; *Mem. Acc. d'Italia*, 7, 365–397.
83. "Un teorema sopra le superficie algebriche con due fasci unisecantisi, ed una relazione fra gli angoli sotto cui si incontrano due curve algebriche tracciate su di una sfera", *Boll. U.M.I.*, (1) 15, 169–172.
84. "Invarianti topologici relativi ai punti uniti delle trasformazioni regolari fra varietà sovrapposte", *Rend. Acc. Naz. Lincei*, (6) 24, 195–200.
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