OBITUARY

JOHN GREENLEES SEMPLE

John Greenlees Semple (known to all his close friends and colleagues as Jack) was born in Belfast on 10 June 1904, and died on 23 October 1985. He was third in a talented family of five (one girl and four boys), three of whom were to achieve professorial positions in British universities. He was educated at the Royal Belfast Academical Institution and at the Queen's University of Belfast, graduating with first class honours in mathematics in 1925. He then went up to St John's College, Cambridge, where he gained a distinction in the Mathematical Tripos of 1927 and proceeded to study for his doctorate under H. F. Baker. In this, he had the good fortune to enjoy the company and inspiration of the remarkable school of geometers that Baker had built up, whose members were to lead and influence the subject in this country for the next half-century. He won the Rayleigh Prize in 1929 and, in the same year, took up a lecturing position at the University of Edinburgh. In 1930 he obtained his Ph.D., was elected a Fellow of St John's College, and was appointed to the Chair of Pure Mathematics at the Queen's University of Belfast at the remarkably early age of twenty-six.

He stayed at Belfast for the next six years, contributing greatly to the development of the Mathematics Department there, and enhancing its reputation by his contributions to geometrical research. He established contacts with the mathematical community outside his own Department by initiating post-graduate lecture courses for secondary school teachers—a popular innovation at that time—and by developing close links between the activities of his own students and those of the corresponding groups at Trinity College, Dublin. He also took on more than his fair share of administrative duties, acting as Dean of the Faculty of Arts for three years and serving on the Senate of the University. He was elected a Member of the Royal Irish Academy in 1932. In 1936 he met and married Daphne Hummel, the daughter of Professor F. H. Hummel, an older colleague at Queen's, and together they moved to London, where Jack had been offered the Chair of Pure Mathematics at King's College. He occupied this position for the rest of his academic career.

At King's, he soon established a firm friendship with the then Head of Mathematics, George Temple, and together they ran a department that came to be known throughout the College—and the University—not only for its academic excellence but also for its informal and friendly atmosphere. This happy state of affairs was rudely interrupted by the outbreak of war in 1939 when, from fear of aerial bombardment, many London Colleges were evacuated to the provinces. The mathematicians from King's went to Bristol University and, on Temple's secondment to other duties, it fell to Semple to organise and oversee this exile. In an obituary notice which appeared in *The Times* newspaper of 5 November 1985, Temple paid generous tribute to the inspiring leadership with which Jack carried the Department through these war years.

King's College reopened in London in 1943 and some semblance of normality was resumed. Time and energy could again be found for research and related activities

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that had been somewhat curtailed during the early war years. One of Semple's main preoccupations at this time was his work as Secretary of the London Mathematical Society and as Editor of its *Journal*, positions which he held from 1944 to 1947. At the same time he was at work, with Leonard Roth, preparing the first and most highly regarded of the three authoritative text-books that bear his name **[17]**. Also in association with Roth, and with other London geometers such as T. L. Wren, J. W. Archbold and D. B. Scott, he founded the London Geometry Seminar, prototype of many special-subject intercollegiate seminars that were subsequently to grow up in London. These geometers, together with their research students, and later joined by P. Du Val, constituted one of the strongest schools of geometry in the world at that time. The seminar met regularly for some forty years and was visited by almost every geometer of any note. The meetings were well-known for the cut and thrust of their debate and for the general barracking that went on, and many a prospective speaker approached his impending ordeal with some trepidation; but it was all done with hugely good humour, and the charm and goodwill with which Semple presided over the proceedings (and the parties that invariably followed) ensured that visitors went away determined to come again and repeat the experience as soon as possible.

The 1950s were probably the happiest and most challenging of Semple's academic career. He had long been active in general College affairs and these activities were recognised, in 1953, by his election to the Fellowship of King's College, the highest honour that the College can bestow. In the same year, when Temple resigned to take up a Chair at Oxford, he became Head of the Mathematics Department and, with the able support of Temple's successor, Hermann Bondi, welded the Department into a very cohesive and forward-looking group. While Bondi was busy building up a strong research school in relativity and cosmology, Semple continued to develop and consolidate his own group in algebraic geometry. Because of its position in central London, and the consequent impossibility of extending its premises to any significant degree, King's College was not able to participate very much in the great expansion of universities that was taking place at this time. Despite this, Semple's Department continued to grow in reputation and, round about 1960, it was one of the most sought-after in the country, both by undergraduate and postgraduate students.

By contrast with the previous decade, the 1960s were academically less satisfactory for Semple. His teacher, H. F. Baker, had once written (in the preface to ⟨**1**⟩:

> 'The study of the fundamental notions of geometry is not itself geometry; this is more an Art than a Science, and requires the constant play of an agile imagination, and a delight in exploring the relations of geometrical figures; only so do the exact ideas find their value.'

This sums up an attitude towards geometry to which Semple had always been faithful and it is not surprising that he regarded with some disfavour the developments in algebraic geometry that were taking place at this time. The subject, for a while, seemed to have been completely overshadowed by the study of its own fundamentals. This led to a partial withdrawal of Semple's enthusiasm for research. His department continued to thrive, the Geometry Seminars went on, and he continued to work on problems for his own satisfaction; but, as his list of publications shows, he wrote nothing of any consequence for about ten years. The major part of his energy was directed instead, during this decade, towards administrative matters. He had always been deeply involved in the affairs of King's College (as distinct from purely departmental matters) but he now became actively involved also in the administrative work of the parent University of London. He served on the Academic Council, on the Court and on the Senate. He was much concerned, particularly, in advising on the development of those overseas colleges which were, at that time, still under the aegis of London University. To this end, he made visits to Rhodesia, the West Indies and Hong Kong, where he proved himself an able and sympathetic ambassador.

Semple retired from his Chair at King's College in 1969 and the occasion was marked by a symposium held in his honour which was addressed by a number of distinguished geometers and attended by many of his friends, colleagues and former pupils. His interest in geometrical research had by now begun to revive and he continued this interest for long after his retirement; he also continued to support, and occasionally give lectures to, the London Geometry Seminar. He was particularly pleased, in 1976, when the King's College London Association (the organisation of old students of the College) elected him as its President. This demonstrated the affection and esteem in which he was held, not only by the mathematical alumni, but also by generations of students and staff from a multitude of other interests and disciplines. Sadly, he was not able to enjoy this honour to the full. The onset of his illness considerably curtailed his activities. He was extremely irritated at this and fought bravely for nine years to overcome his increasing disabilities; but his condition worsened inexorably if gradually and, when the end came, it was little surprise to his family and friends.

Many colleagues have paid their tributes to the memory of Jack Semple. Foremost among the qualities for which he is remembered were his uprightness and determination, and the strong leadership that he was able to bring to projects of widely different sorts. In his own subject, he had in a high degree the gift of organising research, both for groups and for individuals; as a lecturer, he was much in demand and gave inspiration to many hundreds of students and other listeners; and, in his writing, he was able to pass on his love of geometry with an infectious enthusiasm that few other authors have managed to achieve. But, above all, he was a valued friend to so many of his colleagues and pupils and will be remembered by them for his good humour, his wisdom and the innumerable acts of kindness that he performed.

He enjoyed a happy home life, where he would indulge his main hobbies of reading and gardening, interrupted only by the occasional foray onto the golf course. He and Daphne (who survives him) had two children and took pride in their success, John in the medical profession and Jessie in the world of art.

Mathematical work

Semple's first geometrical work was concerned with Cremona transformations of n-dimensional space S_n ($n \geqslant 3$). These transformations give an incentive to the study of intersections, partial or complete, of hypersurfaces in S_n; and at the same time provide a tool by means of which this study may be further advanced. In the early decades of this century, it had become apparent that Cremona transformations of S_3 could be manufactured in ever increasing numbers and Semple's first paper [**1**] was an attempt to bring some kind of order into the profusion of examples that had already been discovered. He pointed out that, if a threefold locus is representable birationally on a space S_3 by means of a linear system of surfaces, and can also be projected birationally onto a second space S_3', then there is an induced Cremona transformation between S_3 and S_3'. By way of example, he considered the case where the linear system consists of cubic surfaces and showed how most of the important cubic transformations of space can be simply obtained by this method. In a second paper [2] he turned his attention to transformations of S_4 by means of quadrics and succeeded in obtaining a complete classification of them. His method here was to note that the surface F that would transform to a general plane of the image-space has necessarily to be a partial or complete intersection of two quadric hypersurfaces of S_4 and so belongs to one of a limited number of well-known types. He then developed a technique for finding, for each possible F, all the homaloidal systems of quadrics in which the variable intersection of two members is of the same type as F; and, using this, he was able to obtain his classification. By the same method he also obtained, in [4], a general classification of cubic transformations of S_4.

His next papers were concerned with representations of Grassmann manifolds on linear spaces. The Grassmann manifold $G(k,n)$ is the variety whose points represent, without exception, the linear spaces of dimension k that lie in a space of dimension n; it is of dimension $K=(n-k)(k+1)$ and belongs to a space of dimension

$$N=\binom{n+1}{k+1}-1.$$

Except in a few low-dimensional cases, N is large compared with K, and this makes for some difficulty in identifying and studying the subvarieties of $G(k,n)$ which correspond to interesting families of linear spaces; so one is led to seek alternative representations, preferably by points of K-dimensional *linear* spaces. Such representations are necessarily flawed, in that they possess exceptional elements, but—as Semple showed—this disadvantage has its compensations in the ease with which they enable one to handle quite complicated complexes and congruences. In his first paper on the subject [3], he shows how to represent the lines of four-dimensional space S_4 by means of the points of a six-dimensional space. He embeds two copies of S_4 as skew subspaces Σ, Σ' of a nine-dimensional space S_9. Each line l of S_4 thereby corresponds to a line λ in Σ and a line λ' in Σ'; and λ, λ' determine a unique three-dimensional join $[\lambda,\lambda']$. Then, selecting a fixed six-dimensional subspace S_6 of S_9, he takes L to be the point in which it is met by $[\lambda,\lambda']$. The resulting representation of the lines l of S_4 by the points L of S_6 is shown to be birational and induces a birational correspondence between $G(1,4)$ and S_6. The principal feature of the correspondence is that linear complexes, mapped by hyperplane sections of $G(1,4)$, correspond to quadric hypersurfaces of S_6 which (when S_6 is chosen in general position in S_9) pass through a rational quintic scroll. By choosing S_6 in special position (relative to Σ,Σ' and the natural collineation induced between them) other more special versions of the representation are obtainable, and Semple describes all of these. In a second paper [5] he extends these ideas to the general case; $G(k,n)$ is shown to be representable on a space of dimension K in such a way that hyperplane sections of $G(k,n)$ correspond to hypersurfaces of order $k+1$ belonging to a certain identified system. In a further paper [7] he demonstrates the usefulness of his representation for the case of the lines of ordinary space S_3, showing how the main types of congruences of the second and third orders are represented by simple well-known surfaces and putting clearly in evidence the presence of the various singular points and multiple rays that such congruences may possess.

Up to this point, Semple's published work had consisted mainly of results concerning particular varieties and configurations. In his next papers, however, he branched out into investigations of a more general kind. One of these [8] was concerned with the invariants of composite surfaces in higher space. Many years earlier, Severi ⟨6⟩ had obtained a complete set of formulae connecting the projective characters of two general irreducible surfaces ψ, ψ' which together make up a complete intersection of $r-2$ hypersurfaces of specified orders in r-dimensional space S_r ($r \geqslant 4$). Semple extended Severi's results to the case when any number of surfaces $\psi_1, \ldots, \psi_k$ are given (intersecting in specified ways) and found formulae for the projective characters of the residual surface ψ' for hypersurfaces of given orders through the ψ_i. He was thus enabled to assign *virtual characters* to a given composite surface $\psi_1 + \ldots + \psi_k$ in such a way that, when the latter is treated in Severi's equations as a single surface ψ having the characters in question, the residual surface ψ' has always the proper characters. The paper includes some notes on the case in which the composite surface is a limiting form of some irreducible surface, particularly with regard to the problem of determining the limiting envelope of such a surface. A similar problem is discussed in **[10]** where, after some analogous work on curves, the degeneration of a surface (on a given threefold) into a single multiple surface is considered, and the corresponding limiting envelope is identified.

Other papers from this period include a delightful contribution **[6]** to the subject of associated rational normal quartic curves in S_4 (see Room ⟨3⟩ for a full account); and a study **[11]** of the binodal singularities that are forced upon a surface when it is required to have contact of a given order with a given surface along a given curve. There was also an investigative essay **[12]** on the relation between the succession of multiple points on a space curve branch and that on its general plane projection; and another **[13]** on the behaviour of canonical, Jacobian and adjoint systems on a threefold when the latter is transformed by dilatation of a point or a curve. Mention should also be made of the elegant paper **[15]** in which it is shown how to derive the properties of Veneroni's axial cubic primal of S_5 by regarding it as a suitably chosen section of a determinantal cubic primal of S_8.

Semple's next publication was the text-book **[17]** that he wrote in collaboration with Roth. This has come to be regarded as one of the most useful source-books, in English, for the techniques and results of classical algebraic geometry. The coverage is extremely wide. The early chapters constitute a highly condensed course on higher plane curves and rational correspondences, with a detailed presentation of the properties of linear systems and the general techniques of transformation and representation. These are followed by chapters on the projective characters of surfaces, on line geometry and on enumerative geometry, and the work concludes with accounts of the invariantive geometry of curves and surfaces. The book has stood up well to the passage of time (despite some shortcomings, of which the authors were only too well aware). It was out of print for a while, but popular demand led to its being reprinted in 1985; this new edition was sold out within months and yet another reprint was produced in 1986, demonstrating that the latest generation of geometers still appreciate the importance of this fine work. A particular feature of the book is its extended treatment of enumerative geometry. This reflected a life-long fascination that Semple had for the pioneering work of Schubert ⟨4⟩ on the enumerative calculus; and he made it the subject of many lectures and seminars that he was invited to give. The need to establish Schubert's results rigorously was pointed out by Hilbert (as the fifteenth of his famous problems) and, by 1940, the theoretical foundations of the methods had been secured. There still remained, however, the practical job of determining the validity (or otherwise) of a host of Schubert's formulae and numerical results. Apart from linear spaces (representable on Grassmannians) the only geometric variable that had been studied in any detail was the complete conic. Semple set about extending this repertoire, adding his own contribution [**18**] to the existing work on conics, and investigating the enumerative geometry of complete quadrics [**16**, **21**], complete collineations [**20**] and triangles [**23**].

Another geometric variable to attract his attention was the *curve-element*. Such an element E_k, of order k, consists of a sequence $O, O_1, \ldots, O_k$ of $k+1$ consecutive points in some projective space S_r. The *origin* O of the E_k is an actual point of S_r while, for $i \geqslant 1$, each O_i is proximate to (in the first neighbourhood of) its immediate predecessor O_{i-1}. The fundamental question that has to be addressed, from the point of view of enumerative geometry, is whether the curve-element is a bona fide geometric variable, that is, whether the family of all the E_k in S_r can be represented on the points of some non-singular algebraic variety. This is in fact the case, but the question is not quite as straightforward as one might at first imagine. In the case of E_1, a point O_1 in the first neighbourhood of O is (by definition) a point of the variety obtained by blowing up S_r at O; whereas a point O'_1 in the neighbourhood of some other point O' has to be located on a different variety, namely that obtained by blowing up S_r at O'. The problem then is to glue together all the varieties obtained from S_r, by blowing it up at different points, to form a model of all the E_1 of S_r. This, in fact, is not particularly difficult to do; but the case of E_2 (and higher order elements) raises questions of a much more intricate nature and was eventually worked out, in full generality, by Du Val ⟨**2**⟩. Semple's contribution, in [**24**], was to obtain the details of the representation of the E_2 in S_r, with full information about the models (and bases on them) for the cases $r = 2$ and $r = 3$. The same paper contains some tentative suggestions towards an analogous theory for surface-elements.

Semple produced two more text-books [**22**, **27**] in the 1950s, both in collaboration with Kneebone. The first of these was intended primarily for undergraduates reading for a first degree in mathematics and will be remembered by generations of London students, in particular, as the text that best fitted in with their geometry syllabus. Unlike many other texts, it has no hide-bound adherence to either analytic or synthetic methods, but proceeds by a happy combination of the two; and therein, surely, lies its charm. The other book [**27**] is an introductory account of the theory of algebraic curves. It was in competition with a number of other books on the same subject which appeared at about the same time and, probably for this reason, it did not enjoy the same popularity as Semple's two earlier text-books. It cannot be denied, however, that it was a substantial and scholarly contribution to the subject, its particular strength lying in the wealth of information to be found in the examples appended to each chapter. Semple's other published work during this period included a short account [**19**] of the inflexional properties of the general projection, on S_7, of the Segre variety V_4^6 of S_8 (product of two planes); and two joint papers, one with Kirby [**25**] on the local form of the equations of a dilatation, and the other with Gibson [**26**] on the Cayley model representation of certain systems of conics and twisted cubics of ordinary space.

There now came the fallow period of ten years during which Semple published practically nothing. He was, in fact, on the point of retirement, and looking over some of his earliest work, when the urge came upon him to reconsider some of the problems raised there. The present writer had the pleasure of collaborating with him in this work. Our first paper [**29**] described a method of classifying Cremona transformations using what we called *sub-homaloidal* systems of hypersurfaces. These are systems of hypersurfaces (Φ) in space S_N, of freedom n ($\leqslant N$) with the property that n

independent generic members of (Φ) have a free intersection which is a linear space of dimension $N-n$. For $n = N$, this reduces to the familiar notion of a homaloidal system; while, for $n < N$, one can obtain a *variable* homaloidal system simply by taking sections of (Φ) by a variable n-dimensional space. This idea provides a natural setting, therefore, for describing the variation and specialisation of homaloidal systems (together with their associated Cremona transformations). The paper contains a number of examples of such sub-homaloidal systems and includes, in particular, a description of the specialisation of the reciprocal transformation of S_n (under which lines are transformed into rational normal curves of order n through $n+1$ points $P_0, \ldots, P_n$) into a specialised type for which the P_i are consecutive along a linear branch.

There followed two other papers [**31**, **33**] on particular Cremona transformations, both in six-dimensional space. The first was that generated by quadric hypersurfaces containing a normal elliptic scrollar surface of order seven, while the other was by means of quadrics through a rational surface of order eight, projective model of the system of plane quartic curves with eight base points. The former prompted a further paper [**32**], written jointly with Du Val, concerning the normal elliptic scrolls whose points can be represented by the *unordered* pairs of values of an elliptic parameter. Of special interest here is the introduction of a numerical character (called the *torque*) for such scrolls of even order; it is shown, for example, that the quartic scroll in ordinary space possesses elliptic quartic curves bisecant to its generators if and only if the torque has certain particular values. This work was extended, in [**35**], to k-dimensional varieties whose points correspond to unordered sets of k values of an elliptic parameter; the paper includes a classification of normal elliptic scrollar varieties of dimension k, defining and describing the k most general types of such varieties, and serving as a generalisation of the classical work of Segre ⟨**5**⟩ on elliptic ruled surfaces.

While these last few papers were in progress, Semple and I were also working on a book [**34**] about generalised Clifford parallelism. This was inspired by a memoir of Wong ⟨**7**⟩ in which he had extended Clifford's original concept of parallelism in three-dimensional elliptic space to elliptic space of any odd dimension. Semple was struck by the idea that, instead of developing the theory metrically as Wong had done, we could usefully generalise Clifford's own geometrical characterisation of the parallelism. We accordingly took, as our definition: *given an ambient* $(2n-1)$-*dimensional projective space* S_{2n-1} *and a non-singular* (*absolute*) *quadric therein, two* $(n-1)$-*dimensional spaces* Π_1, Π_2 *are Clifford parallel* (*relative to the said quadric*) *if and only if they and their polar spaces* Π'_1, Π'_2 *are four generators of a regulus*—a regulus being the Segre product variety of a line and an $(n-1)$-space. This turned out to be a most fruitful approach and, using it, we were able to repeat and extend Wong's results, including the fact that only spaces of dimensions three, seven and fifteen can be fibred by space-filling systems of Clifford parallels; and explicit geometrical constructions for these three systems were described. As the work developed, it became more and more apparent that the essential objects of study, so far as this parallelism is concerned, are not the individual spaces Π_i, but the *pairs* of polar spaces (Π_i, Π'_i), and this led us to look at various varieties whose points represent such polar pairs. This revealed a remarkably rich field of geometrical study and the second half of [**34**] was devoted to a description thereof.

After his retirement, Semple reverted to his earlier interest in enumerative problems and set himself the task of collecting together the existing work on the geometry of complete conics. This led to the production of a long essay in which he reviewed particularly the work of Severi and van der Waerden and incorporated a large number of examples that he had investigated over the years. The essay was the last thing that he wrote and was originally circulated privately. Soon after its production he was taken seriously ill; but he expressed a desire to see the work in print and his friends were happy to assist in seeing to its publication [36]. It was an irony of fate that, having espoused the cause of enumerative geometry for so much of his life, he was struck down at just that moment when other geometers had at last begun to revive interest in the subject.

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