integrity of his conduct throughout his long life as to his achievements as a mathematician.

In 1896 Castelnuovo married Elbina Enriques, the sister of his colleague and life-long friend. He leaves two sons and three daughters.

ARNOLD JOHANNES WILHELM SOMMERFELD

E. T. WHITTAKER.

Arnold Johannes Wilhelm Sommerfeld, Honorary Member of the Society, was born at Königsberg on 5 December, 1868, the son of a medical man, and was educated at the Gymnasium (where among his schoolfellows were Hermann Minkowski and Willy Wien) and University of that city. The University had long been famous for its Institute of Theoretical Physics, which had been founded by Franz Neumann (the veteran who, after having been left for dead on the battlefield of Ligny in 1815, died in his bed 80 years afterwards), and among Sommerfeld's teachers were Lindemann (who first proved the transcendence of π), Hurwitz, and Hilbert. In 1891 he obtained his doctor's degree: and in 1893 moved from Königsberg to Göttingen, where he came under the spell of Felix Klein.

Klein, who in the last years of the nineteenth century was the second greatest mathematician in the world (the first was Poincaré), was at this time much occupied in showing by example the value to dynamics, astronomy and physics of the new pure mathematics, especially the theory of functions of a complex variable. Sommerfeld, who became his assistant in 1894, was taken into partnership in a splendid piece of work of this character, namely the book *Ueber die Theorie des Kreisels,* of which the first two volumes were published in 1897-98*. But even before this, in 1896, Sommerfeld under Klein's influence had put forth some remarkable papers† on the mathematical theory of diffraction. He considered the two-dimensional problem (the light being emitted by a line parallel to the z-axis, and the screen being bounded by two parallels to the z-axis) and showed that it depended on integrating the equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+k^2 u=0$$

on a two-sheeted Riemann surface. A rigorous solution was obtained in the form of a complex integral. The theory of the many-valued solutions of the partial differential equations of mathematical physics, which was created in these papers, was a very great advance.

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* Leipzig, Teubner.

† *Math. Ann.*, 47, 317; *Proc. London Math. Soc.* (1), 28, 395.

After this he turned to problems of electrodynamics, particularly the propagation of electromagnetic waves along wires* and the fields due to moving electrons. The culmination of his work in the latter subject was reached in 1911†, when he obtained the following invariant expression for the field due to an electron moving in any manner: let

$$\mathfrak{R} = (x-x_0,\ y-y_0,\ z-z_0,\ l-l_0),$$

where (x, y, z, l) are the coordinates in space-time of the electron, and (x_0, y_0, z_0, l_0) those of the observer, and $l = ict$: let the element of proper-time of the electron be

$$d\tau = \frac{1}{ic}\{(dx)^2+(dy)^2+(dz)^2+(dl)^2\}^{\frac{1}{2}}:$$

let $\dot{\mathfrak{R}}$ and $\ddot{\mathfrak{R}}$ be the first and second derivatives of $\mathfrak{R}$ with respect to τ: let $\mathfrak{N} = (\mathfrak{R}[\dot{\mathfrak{R}}\ddot{\mathfrak{R}}])$: and let f be the six-vector whose spatial components are the three components of the magnetic intensity, while the other components are $-i \times$ the three components of the electron intensity. Then

$$4\pi f = \frac{e}{(\mathfrak{R}\,\dot{\mathfrak{R}})^3}\,[\mathfrak{R},\ e^2\,\dot{\mathfrak{R}}-\mathfrak{N}].$$

This expression is independent of the choice of coordinates.

In 1897 Sommerfeld became professor of mathematics in the mining academy at Clausthal, in the Harz mountains, and in the same year he married Johanna, daughter of Dr. Ernst Höpfner of Göttingen. In 1900 he exchanged his chair for that of technical mechanics in the Technische Hochschule at Aachen: and in 1906 he was appointed to succeed Boltzmann as professor of theoretical physics at Munich, where he remained for the rest of his life.

He was closely associated with two fundamental discoveries made in connection with X-rays. In the early years of the century, many attempts had been made to test the hypothesis that X-rays are waves, by trying to obtain diffraction-effects with them: but no certain conclusions were drawn. In 1912 Sommerfeld compared photometric measurements made by P. P. Koch with the predictions of theory, and found evidence of genuine diffraction corresponding to a wave-length of 0.3 Å. Thereupon P. P. Ewald, who had just taken his doctorate at Munich, suggested that some notion of the dimensions of the atom-lattices in crystals might be obtained from observations of the transmission of X-rays through the crystals: and Max von Laue, who was a junior lecturer at Munich, in contact with Sommerfeld and Ewald, saw that if the X-rays had a wave-length of the

* *Ann. de Phys.*, 67 (1899), 233.

† *München Sitz.* (1911), 1, at p. 56.

order found by Sommerfeld, then the crystal-lattice would act as a three-dimensional diffraction-grating, so to speak, for X-rays. Sommerfeld saw at once the importance of the idea and arranged for two trained workers in his department to carry out the experiments, which were completely successful and created a new science.

From 1911 onwards Sommerfeld was concerned chiefly with quantum theory. In his first paper* on this subject, he referred to the name *Quantum of Action* which had been given by Planck to the quantity h occurring in his law of radiation, and proposed the hypothesis that *in every purely molecular process, a certain definite amount of Action is absorbed or emitted, namely the amount* $h/2\pi$. An attempt which he, in partnership with his assistant Debye, made in 1913 to account for the photo-electric effect on this principle was not successful: but in 1915, simultaneously with William Wilson†, he published‡ an important extension of the quantum rule, which made it now possible to determine the steady states of systems which have more than one degree of freedom: and he successfully applied the new theory to explain the fine-structure of the atomic hydrogen spectrum, introducing a new quantum number, the *azimuthal* quantum number: he also took into account the relativist increase of the electron's mass with its velocity. He and Debye went on to a general study of the quantification of systems which admit of the separation of variables: and in 1916 they gave§ a quantum theory of the Zeeman effect, which introduced a third quantum number, the *magnetic* quantum number. It was found that so far as concerned the number, position, and state of polarisation of the Zeeman components, the quantum theory gave, for single lines which are not members of doublets or triplets, exactly the same results as Lorentz's original theory which was based on classical electrodynamics: this happened because Planck's constant h cancelled out in the part of the expression for the frequency which depended on the applied magnetic field. The general theory of *selection-principles*, *i.e.* statements that transitions of the active electron from certain orbits to certain other orbits never take place, was initiated in 1918 by A. Rubinowicz, a Pole working with Sommerfeld at Munich: and in 1920 Sommerfeld introduced|| a fourth quantum number, the *inner quantum number*, which has different values for the two terms of a doublet, or the three terms of a triplet. The necessity for explaining the existence of the inner quantum number led to the discovery of electron-spin: and it was found that the disposition of the electrons in any atom could be described, and the Newlands-Mendeléev periodic law could be explained, in terms of the four quantum numbers

* *Verh. d. Phys. Ges.*, 13 (1911), 1074.

† *Phil. Mag.*, 29 (1915), 795; 31 (1916), 156.

‡ *München Sitz.* (1915), 425, 459; *Ann. d. Phys.*, 51 (1916), 1.

§ *Phys. Zs.*, 17 (1916), 491, 507.

|| *Ann. d. Phys.*, 63 (1920), 221.

(namely the principal quantum number originally introduced by Bohr and the three others discovered by Sommerfeld). The progressive development of quantum spectroscopy was described in the six successive editions of Sommerfeld's famous book *Atombau und Spectrallinien*, which had an immense influence.

The greatest achievement of Sommerfeld's later life was his theory of electrons in metals. A fairly satisfactory explanation of the metallic conduction of heat and electricity, and of such phenomena as the Peltier effect, had been published in 1904-5 by Lorentz, making the assumptions of the classical theory. Sommerfeld now assumed that the conducting electrons in a metal may be regarded as an electron-gas which is degenerate in the sense of Fermi's statistics. This made it possible to account for several hitherto unexplained anomalies: *e.g.*, that the specific heat of a metal is usually what would be expected if it were due to the metallic atoms alone, the specific heat of the Fermi electron-gas being extremely small. The theory of the Peltier, Thomson and Volta effects, and of thermionic phenomena, could be treated on this assumption in a satisfactory way.

Sommerfeld's famous school at Munich came to an end in the Hitlerite Germany of 1940. His death, on 26 April, 1951, was the consequence of a street accident.