PETER STEFAN

RONALD BROWN AND JAMES EELLS

Dr. Peter Stefan was killed in an accident while climbing by himself on Tryfan, in Snowdonia, on 18 June, 1978, at the age of 37.

Peter's schooling was in Bratislava; he took his first degree in 1965, and then lectured for three years at the Technical University of Prague. He was invited to a Conference on Dynamical Systems at Warwick in 1968. Having been active in the political upheavals in his country at that time, he decided after the Russian invasion to stay in England and work for a Ph.D. at the University of Warwick. He was appointed a temporary Assistant Lecturer at Manchester in 1969, and ICI Fellow at Warwick in 1970. In October, 1971, he was appointed to the post of Lecturer in Pure Mathematics at the University College of North Wales, Bangor. In 1976/7 he was given leave of absence to visit the Institut des Hautes Études Scientifiques.

All who met him were surely impressed by his lively intelligence and general interest. His opinions were his own, definite and showing culture and thought. Originally a Slovak, and maintaining a deep interest in the cultural and political life of Eastern Europe, he became a naturalised Briton in 1975, and said that he was looking forward to having a real vote. The air of freedom here was important to him, and was reflected in the passion for climbing which developed in his last two years.

He leaves a wife and a son by a former marriage.

Peter Stefan's main mathematical contributions are to singular foliations and their relationship to accessibility questions. We now describe these results.

An *arrow* on a smooth manifold M is a 1-parameter family (a^t) of diffeomorphisms between domains of M, such that if $a^t(x)$ is defined then so is every $a^s(x)$ for $0 \leqslant s \leqslant t$, and $a^0(x) = x$. If A is a set of arrows on M (symmetric in t), then we partition M as follows: Two points of M are said to be equivalent if we can reach one from the other by a composition $a_1^{t_1} \circ a_2^{t_2} \circ \ldots \circ a_p^{t_p}$ for $a_i \in A$ and $t_i \in \mathbb{R}$. These equivalence classes are called the *accessible sets of A*.

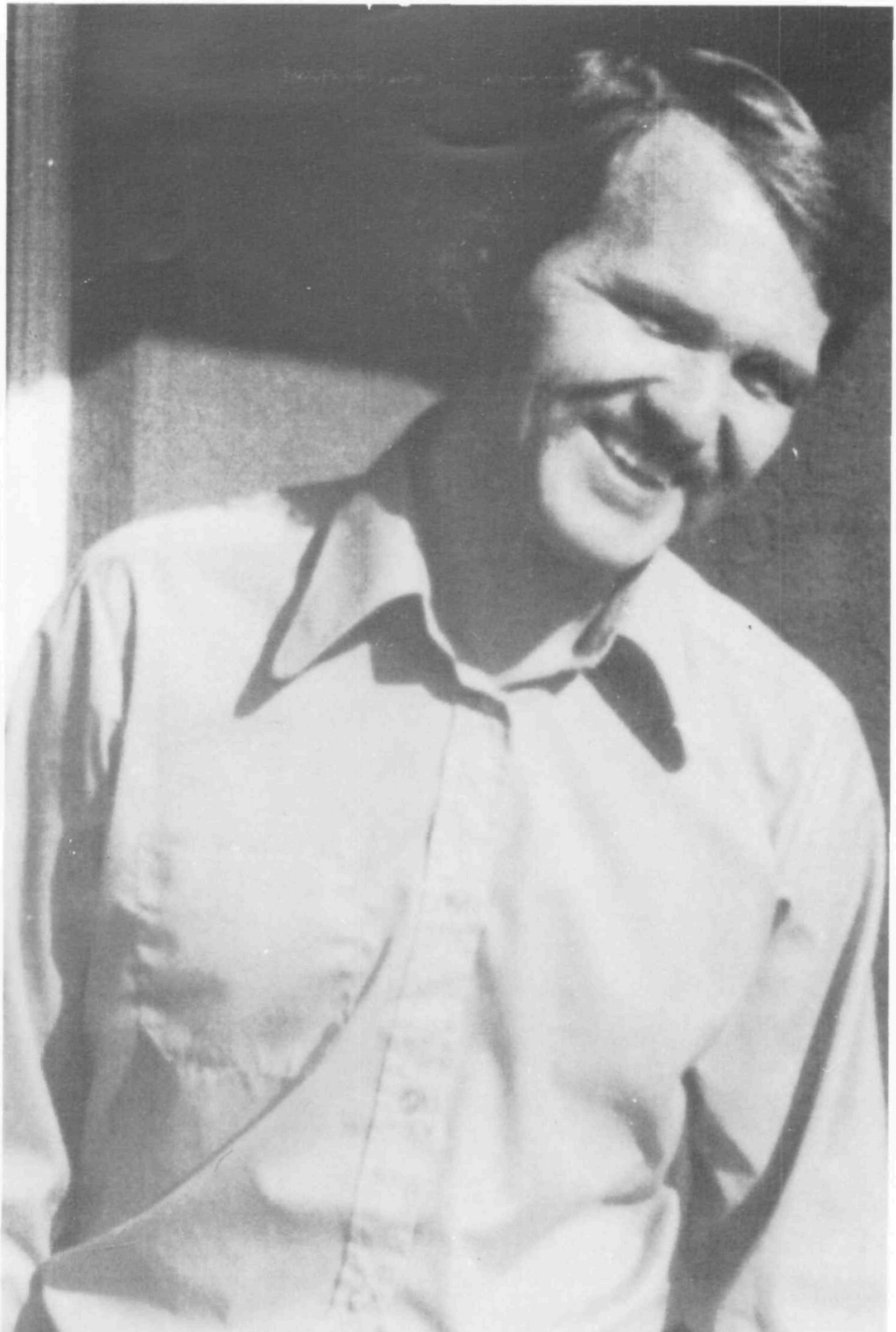
THEOREM. *The accessible sets of A are immersed submanifolds of M. They form a foliation with singularities (the dimensions of the leaves can vary). The tangent space to the leaf through $x \in M$ is spanned by*

$$A(x) = \left\{ \frac{da^t(y)}{dt} : (a^t) \in A \text{ and } a^t(y) = x \right\}$$

if and only if A is homogeneous; that is, such that $(a^t)^*(x)A(x) \subset A(y)$ whenever $a^t(x) = y$.

This theorem is one of the main results in Stefan's thesis [2]. An announcement was given in [5]. Full details and applications appeared in [6]. Extension to the case where M is a Banach manifold, with a careful study of the relationship between homogeneity of A and integrability, was given in [9].

[BULL. LONDON MATH. SOC., 13 (1981), 170–172]



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facing p. 170

Singular foliations as in the Theorem can be generated by subgroups of the diffeomorphism group of M; this is applied in [7] to obtain a characterisation of k-determinancy of germs of smooth functions at the origin in $\mathbb{R}^n$. They also arise from arbitrary systems S of vector fields on M (an example developed independently by H. J. Sussmann [**15**, **16**]); intimately related is a theorem of W. L. Chow [**14**]:

If $[S]$ *is the smallest set of vector fields containing* S *and closed under the Lie bracket and if* $\dim [S]x = \dim M$, *then the accessible set of* S *containing* x *is open in* M.

Stefan gives two proofs in [**3**] and in [**5**]. In [**12**] (which was a preprint in 1974, and has been prepared for publication by D. R. Chillingworth) he gives more applications of his Theorem to generalisations of the classical Frobenius theorem, and also gives counter examples to claims in several papers that the condition on S that it be "locally of finite type" is sufficient for integrability.

In addition to its manifest geometric appeal, Stefan's Theorem has interpretations in control theory (as well as in Carathéodory's mathematical theory of entropy)—an area which interested Peter from the beginning of his studies in Prague, under the guidance of Professor J. Kurzweil. In [**1**], Stefan simplifies and completes the proof of Boltianskij's sufficiency conditions for the optimality of a time-optional control problem. In [**4**], Stefan gives another very short proof of Elliott's Lie-algebraic criterion for controllability on manifolds.

During his stay in France in 1976/7, Peter prepared two articles:

(1) An unpublished manuscript [**11**] on the Poincaré–Birkhoff fixed point theorem, about which he wrote "...what follows, apart from a few trivial observations, is simply an abbreviated (and occasionally more explicit) version of the proof given by Birkhoff...";

(2) [**10**], the main result of which we wish to state. Define an order $|\!-$ on the positive integers $\mathbb{N}$ as follows: Let $\mathbb{N} = A \cup B$, where $A = \{2^n l : n \geqslant 0, l \geqslant 3, l \text{ odd}\}$ and $B = \{2^m : m \geqslant 0\}$. Order A lexicographically with increasing n and l; order B with decreasing m, and let A precede B.

THEOREM (ŠARKOVSKII). *Let* $T : \mathbb{R} \to \mathbb{R}$ *be a continuous mapping which has a periodic orbit of period* n. *Then* T *has a periodic orbit of period* m *for every* $m \in \mathbb{N}$ *such that* $n \,|\!-\, m$.

Stefan gives a careful proof of that theorem. Then he obtains properties of minimal orbits of T, which apply to produce a substantial improvement of an estimate of Bowen–Franks.

Following his announcement of Šarkovskii's theorem, Peter wrote: "The main aim of these notes is to make the contents of [**1**] available to those who do not read Russian. The reader should be warned that this is not a translation: some new results, closely related to Šarkovskii's work, are presented in Section E and H, and the material of [**1**] has been rearranged and modified to suit my taste and to avoid one or two mistakes which have crept into Šarkovskii's argument".

The paper [**13**] has been prepared for publication by R. Brown. It is related to a question raised by J. H. C. Whitehead: Is every subcomplex of a 2-dimensional aspherical complex aspherical? In May, 1978, Stefan found a counter-example to an approach to this problem suggested by J. Huebschmann. This example is given in

[13], as well as most of a letter Peter wrote to Roger Fenn. Peter develops a method of Whitehead for describing the problem in terms of linkages, and uses group theoretic arguments to show that certain linkages do not arise in an aspherical situation.

Peter had a strong sense of what was important in mathematics, and was sometimes self-deprecating about his own work. Thus readers should not take literally either of the above quotations. However, they do illustrate Peter's approach to mathematics—his own and others'. Given a first-rate mathematical idea, he made it part of himself. That often required an exhaustive search for the right perspective in mathematical development, in exposition, technical accuracy, and historical viewpoint. His fine taste and judgement shine throughout his work.

Mathematical papers(†)

1. "On the proof of a theorem of V. G. Boltjanskii", *Cas. Pest. Mat.*, 93 (1968), 341–345.
2. "Accessibility and singular foliations", Ph.D. thesis, Warwick, 1973.
3. "Two proofs of Chow's theorem", in *Geometric methods in system theory*, (M. D. Q. Mayne and R. W. Brocket (*Editors*), D. Reidel, 1973).
4. (with J. B. Taylor) "A remark on a paper of D. L. Elliott", *J. Diff. Equations*, 15 (1974), 210–211.
5. "Accessibility and foliations with singularities", *Bull. Amer. Math. Soc.*, 80 (1974), 1142–1147.
6. "Accessible sets, orbits and foliations with singularities", *Proc. London Math. Soc.*, 29 (1974), 699–713.
7. "A remark on right *k*-determinancy", pre-print, Bangor, 1974.
8. "Accessibility and foliations", *Dynamical Systems—Warwick*, 1974, Lecture Notes in Mathematics, 458 (Springer, Berlin, 1975)
9. (with D. R. J. Chillingworth) "Integrability of singular distributions on Banach manifolds", *Math. Proc. Camb. Phil. Soc.*, 79 (1976), 117–128.
10. "A theorem of Šarkovskii on the existence of periodic orbits of continuous endomorphisms of the real line", *Commun. Math. Phys.*, 54 (1977), 237–248.
11. "The Poincaré–Birkhoff fixed point theorem", pre-print, IHES, 1977.
12. "Integrability of systems of vector fields", *J. London Math. Soc.*, (2), 21 (1980), 544–556.
13. "On Peiffer transformations, link diagrams and a question of J. H. C. Whitehead", in *Low-dimensional topology* (R. Brown and T. L. Thickstun (*Editors*), London Math. Soc. Lecture Notes) (to appear).

References

14. W. L. Chow, "Über Systeme von linearen partiellen Differentialgleichungen erster Ordnung", *Math. Ann.*, 117 (1939), 98–105.
15. H. J. Sussmann, "Orbits of families of vector fields and integrability of systems with singularities", *Bull. Amer. Math. Soc.*, 79 (1973), 197–199.
16. H. J. Sussmann, "Orbits of families of vector fields and integrability of distributions", *Trans. Amer. Math. Soc.*, 180 (1973), 171–188.

(†) Offprints of many of the papers listed are available from: The Secretary, School of Mathematics and Computer Science, University College of North Wales, Bangor, Gwynedd, LL57 2UW.